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A Pure Theory of Aggregate Price Determination

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Abstract

This article considers aggregate price determination related to the neutrality of money. When the true cost of living can be defined as the function of prices in the overlapping generations (OLG) model, the marginal cost of a firm solely depends on the current and future prices. Further, the sequence of equilibrium price becomes independent of the quantity of money. Hence, money becomes non-neutral. However, when people hold the extraneous belief that prices proportionately increase with money, the belief also becomes self-fulfilling so far as the increment of money and the true cost of living are low enough to guarantee full employment.

Key Words: Marginal cost, True cost of living, Neutrality of money, Credibility of money, Rational extraneous belief

JEL Classification: E24, E31, E40

1 Introduction

As Keynes [5] points out, there appears to be a serious crack between macroeconomics and microeconomics on the mechanism of price determination.[1] In microeconomics, prices are governed by marginal costs. However, macroeconomics emphasizes the role of money in the process of aggregate price determination. How are these theses related to each other? The present paper explores this problem.

Such a crack is deeply connected to the neutrality of money. Otaki [9], [10] has already shown that the equilibrium sequence of the aggregate price can be independent of the quantity of money, using the standard deterministic two-period overlapping generations (OLG) model of production economy. On the other hand, Lucas [7] proves that the quantity theory of money strictly holds in certainty.[2]

These seemingly contradictory results suggest the following theoretical hypothesis: The assumption that prices are determined by marginal costs means that the equilibrium production and employment level is an interior solution. That is, the economy is located at imperfect employment.

The interior solution emerges from to the lack of purchasing power of money. By some plausible assumption, the true cost of living becomes the

[1] In Keynes' words, "For it is far from being consistent with the general tenor of the classical theory, which has taught us to believe that prices are governed by marginal prime cost in terms of money and that money-wages largely govern marginal prime cost. Thus if money-wages change, one would have expected the classical school to argue that prices would change in almost the same proportion, leaving the real wage and the level of unemployment practically the same as before, any small gain or loss to labour being at the expense of profit of other elements of marginal cost which have been left unaltered. They seem, however, to have been diverted from this line of thought, partly by the settled conviction that labour is in a position to determine its own real wage and partly, perhaps, by preoccupation with the idea that the prices depend on the quantity of money."

[2] Although Lucas [7] uses a dynamic stochastic general equilibrium model, uncertainty is not essential for proving the neutrality of money [Theorem 2]. In addition, whether firms are price makers or takers is also irrelevant to the theory extended in the article. As Otaki [9] and [10] show, the concept of monopolistic competition plays a crucial role in proving the welfare-improving effect of the expansionary monetary policy and the existence of involuntary unemployment in a frictionless economy.

function of the current and future prices independent of quantities. Hence, the nominal reservation wage also depends on the same factors. Thus, when the equilibrium current price is equal to the marginal cost, the equilibrium price sequence becomes independent of the quantity of money. If the quantity of money is sufficiently relatively small to the equilibrium price level determined beforehand, some individuals are unemployed, and the interior equilibrium emerges without any price stickiness. In other words, money is non-neutral unless some additional conditions are satisfied.[3]

The boundary solution, namely, the quantity theory of money, can be attained by the following two conditions. First, people hold the extraneous belief that the price level proportionately varies with the quantity of money.[4] Second, the increased rate of money, that is, the inflation rate, is sufficiently modest to reduce the true cost of living, and every individual wishes to work.[5]

Under these conditions, for an arbitrarily given money supply, the current price level flexibly adjusts the purchasing power of money to attain the full-employment equilibrium. Thus, one-to-one correspondence emerges between the current price level and the quantity of money. Namely, the quantity theory of money holds and money becomes neutral under the two assumptions above.

Consequently, money is intrinsically non-neutral and affects the employment and output level without any price friction as Keynes [5] tacitly considers.[6] The quantity theory of money is upheld by the extraneous belief

[3]Most new Keynesian economists believe that a price-stickiness assumption is unavoidable for obtaining the non-neutrality of money. For example, see Calvo [2] and Mankiw and Reis [8]. Galí [4] provides a summary of the new Keynesian economics from this perspective.

[4]Lucas [7] specifies the equilibrium price function as $p = m\phi(z)$ to support the quantity theory of money under perfect information. This assumption corresponds to the extraneous belief in this article.

[5]The method of injection of money also leads to a different consequence. We shall discuss this problem in Section 4.

[6]According to Keynes [5], "The division of economics between the theory of value and distribution on the one hand and theory of money on the other hand is, I think, a false division. The right dichotomy is, I suggest, between the theory of the individual industry

that money is only a measure of value and possesses no substantial value.

The contents of the paper are follows. Section 2 exhibits the basic model and indicates the non-neutrality of money under perfect competition. Section 3 provides a sufficient condition for supporting the quantity theory of money. It also discusses the difference in the Keynesian and monetarist views on money. Section 4 explains how the methods of injecting money affect the conclusion. Section 5 provides concluding remarks.

2 The basic model

2.1 Optimization problems of economic agents

2.1.1 Individuals

We consider a standard two-period OLG model with money and one perishable good under certainty. Individuals are born with continuum density between $[0, 1]$ in each period, and live for two periods: the youth and the old age. They can supply unit labor at their discretion when they are young. The disutility is denoted as α. The lifetime utility function of each individual U is

$$U(c_{1t}, c_{2t+1}, \delta_t) \equiv u(c_{1t}, c_{2t+1}) - \delta_t \cdot \alpha, \tag{1}$$

where c_{1t} and c_{2t+1} are the current and future consumption of generation t respectively. δ_t is a definition function that takes the value of unity when the individual works and zero when he/she does not work.

$u(\ \cdot\ )$ is the well behaved homothetic function that represents the lifetime utility derived from consumption.[7] Although the separability between the consumption stream and leisure seems restrictive, the assumption can be justified by Diewert's discussion as follows [W.E. Diewert "Cost of living indexes and exact index numbers" Discussion Paper 09-06, Department of Economics, University of British Columbia, August 6, 2009]: "Although

or firm and of the rewards and the distribution between different uses of a *given* quantity of resources on the one hand, and the theory of output and employment *as a whole* on the other hand."

[7]Shephard [11] proves that iff the utility function is homothetic, the true unit cost of living becomes the function of prices independent of the consumption quantity.

this (homothetic) assumption is generally not justified when we consider the consumer's overall cost of living index, it can be justified in the context of a *subaggregate* if we assume that the consumer has a separable subaggregator function, $f(q)$, which is linearly homogenous. In this case, q is no longer interpreted as the entire consumption vector, but refers only to a subaggregate such as "food" or "clothing" or some more narrowly defined aggregate." From the economic perspective, Diewert suggests that aggregation should be performed among similar goods. In this sense, in (1) we assume that consumption and leisure have quite different properties as compared to the current and future consumption.

The budget constraint that each individual faces is

$$p_t c_{1t} + M_t \leq \delta_t W_t, \quad p_{t+1} c_{2t+1} \leq M_t, \tag{2}$$

where p_t is the price of the good; W_t is the nominal wage; and M_t is the nominal money demand of generation t to prepare for future consumption. The profits of the firm can also become the income. However, we assume that all the markets (goods, money, and labor) are in perfect competition. Hence, we can neglect the profits as a source of income.

An individual maximizes (1) on $(c_{1t}, c_{2t+1}, M_t, \delta_t)$ subject to (2). Since U is homothetic, the true cost of living function Ψ exists such that

$$\Psi(p_t, p_{t+1}, u) = f(u)\psi(p_t, p_{t+1}), \tag{3}$$

where $\psi(\ \cdot\ )$ is a linear homogenous function. It also increases with p_t and p_{t+1}. We can calculate the nominal reservation wage W_t^R by using the true cost of living function (3) as

$$W_t^R = f(\alpha)\psi(p_t, p_{t+1}). \tag{4}$$

In addition, the aggregate current consumption function of the young generation, C_t, becomes

$$C = c(\frac{p_{t+1}}{p_t})[w_t l_t], \quad w_t \equiv \frac{W_t}{p_t}, \tag{5}$$

where l_t is the employment level that is located within the interval $(0, 1)$.

2.1.2 Firms

Next, we proceed to the optimization problem of a representative firm. The only production factor is labor. For simplicity, the representative firm faces the constant return production function:

$$y_t^s = l_t, \tag{6}$$

where y_t^s denotes the output level. Since the firm acts as a price taker, the profits become zero in the equilibrium. Hence, using (4), we obtain the following important difference equation. Namely,

$$p_t^* = W_t^R \quad \Rightarrow \quad p_t^* = f(\alpha)\psi(p_t^*, p_{t+1}^*) \quad \Leftrightarrow \quad 1 = f(\alpha)\psi(1, \frac{p_{t+1}^*}{p_t^*}). \tag{7}$$

Thus, we obtain

Lemma 1 *If equilibrium employment is located within* $(0,1)$*, that is, the equilibrium price is determined by the marginal cost, the equilibrium price sequence,* $\{p_{t+j}^*\}_{j=0}^{+\infty}$*, is determined independently of the sequence of the quantity of money* $\{M_{t+j}\}_{j=0}^{+\infty}$*. Furthermore, the equilibrium inflation rate,* $\rho^K \equiv \dfrac{p_{t+j+1}^*}{p_{t+j}^*}$*, is constant over time.*

2.1.3 The government

Finally, we must specify the money-supply rule. New money is injected through the government expenditure G_t.[8] Thereafter, money is supplied to keep the real cash balance $\{\dfrac{M_{t+j}}{p_{t+j}}\}_{j=1}^{\infty}$ equal to the initial level $m \equiv \dfrac{M_t}{p_t}$. Therefore, using Lemma 1, the real government expenditure g_{t+j} is expressed by

$$g_{t+j} \equiv \frac{G_{t+j}}{p_{t+j}} = \begin{cases} \dfrac{M_t - M_{t-1}}{p_t}, & \text{if } \ j = 0, \\ (1 - \dfrac{1}{\rho^K})m, & \text{if } \ j \geq 1. \end{cases} \tag{8}$$

2.2 Market equilibrium

There are three kinds of markets in the model: goods market, labor market, and money market. Following Walras' Law, we confine our attention to

[8] For simplicity, all goods that the government purchases are assumed to be wasted.

the former two markets. When the labor market is in interior equilibrium, $0 < l_t < 1$, the equilibrium nominal wage is equal to the nominal reservation wage W_t^R.

The aggregate demand for the good y_t^d is defined by

$$y_t^d \equiv C_t + g_t + \frac{M_{t-1}}{P_t}. \tag{9}$$

Substituting (5), (6), (8), and the zero-profit condition of the firm into (9), and using Lemma 1, we obtain

$$y_t^d = c(\rho^K) y_t^s + m.$$

Since $y^* = y^d = y^s$, the equilibrium condition of the goods market is

$$y^* = c(\rho^K) y^* + m. \tag{10}$$

Consequently, we obtain

Theorem 1 *If m is sufficiently small, there is an interior equilibrium in the sense that some individuals are unemployed and*

$$0 < y^* < 1$$

holds.

Proof. Since $u(\ \cdot\ )$ is homothetic, $0 < c(\rho^K) < 1$ holds. By Lemma 1, the change of nominal money supply M does not affect the price of the good p. Hence, we can choose a sufficiently small m such that Eq.(10) has a solution within $(0, 1)$. ∎

Figure 1 illustrates Theorem 1. When the prices are determined by the marginal cost, the Hicks-Samuelson 45° line analysis is justified under perfect competition and rational expectations without any price stickiness. If the expansionary monetary-fiscal policy is implemented, AD line shifts upward, and the employment and output increase. Thus, money is non-neutral. The fiscal multiplier is $\dfrac{1}{1 - c(\rho^K)}$, as shown by elementary macroeconomics.[9]

[9] Kiyotaki and Wright [6] provide an alternative and persuasive theory of money. They criticize OLG model method because it cannot endure the *"rate of return dominance."* Nevertheless, the search model is not necessarily suitable for analyzing the neutrality of money because it lacks the path through which money is injected into the economy.

We must note that the induced effective demand theory, which is summarized by Eq.(10), corresponds to the long-run stationary equilibrium without any price friction. It implies that the friction or stickiness concerning prices is not a necessary condition for the non-neutrality of money, as most new Keynesian economists implicitly consider. Furthermore, the property of the basic model clearly differs from the new Keynesian economics, as in money is non-neutral even in the long run and that the theory extended in Keynes [5] can be interpreted as the economics of the stationary state.

3 On the quantity theory of money

The previous section proves that money is intrinsically non-neutral, and an expansionary fiscal-monetary policy stimulates the employment and output. This section deals with a sufficient condition for sustaining the quantity theory of money in the basic model.

3.1 Two different beliefs on the value of money

The basic model supports the Keynesian view that imperfect unemployment equilibrium emerges from the lack of effective demand without any friction on prices. Eq.(7) plays a crucial role in this assertion. This equation implies that individuals believe that money has an intrinsic value in that young individuals are ready to accept all forms of additional money at the prevailing price of the good. Here, we define the *credibility of money* as

Definition 1 *We say that money is credible when newly issued money can be exchanged for a unit of the good at the prevailing price.*

Even if money is *credible*, the value of money is determined by its own future (rational) expectation. Eq.(7) also implies that if individuals expect money to become more valuable in the future, it is soon transmitted to its current value appreciation (deflation) and vice-versa. Such fragility of the base of the *credibility of money* is rooted in the fact that money does not provide any utility by itself. These properties of money resemble those of the fiat money we actually use. To sum up, when prices are determined by marginal costs,

the value of money is determined not by its quantity but by its *credibility*. This is considered to be the Keynesian view on money.

In other words, the fact that the price of the good is insensitive or sticky to a monetary shock does not indicate the significant existence of *menu cost* but the high *credibility of money*.

On the other hand, monetarist regards money only as a measure of value; hence, individuals believe that the increase in the quantity of money brings about a proportional price increase, and has no effect on the employment and output level under rational expectations.[10]

In comparison to the Keynesian view, which considers that people believe in the intrinsic value of money, monetarist entirely lacks the view of *credibility of money*. Friedman and Schwartz [3] consider that money can be circulated solely on the basis of the "*confidence of others will*."

Although the two concepts, the *credibility of money* and the *confidence of others will*, appear to resemble each other, the situation where the '*confidence of others will'* becomes indispensable to sustain the monetary economy, by itself, reveals that the *credibility of money* is entirely lost and that the role of money has become quite restrictive. This is because estimating the will of numerous and anonymous others is far more difficult than assuming each individual simply believes in the intrinsic value of money. Furthermore, even if the *confidence* exists, there is another problem. Namely, how much money do young individuals require in exchange for a unit of goods when the *credibility* collapses? Thus, once the *credibility of money* is lost, money ceases to have *absolute* substance and is reduced to the *relative* measure of value. It is plausible for each individual to expect prices are determined by the quantity of money in such cases.

Such a phenomenon occurs in the following two polar cases. In the first case, the economy is located at the full-employment equilibrium. In this case, any additional money does not produce any output. Accordingly, prices proportionately increase with money. Keynes [5] calls such an inflation

[10] According to Friedman and Schwartz [3], "Money is a veil. The 'real' forces are the capacities of the people, their industry and ingenuity, the resources they command, their mode of economic and political organization, and the like."

as *true inflation*.

The second is the polar case of hyperinflation in which money utterly loses *credibility* and is used only as a measure of value.

Note that the seminal empirical work of Cagan [1] concerning the quantity theory of money confines the data to the period of hyperinflation in six European countries immediately after World War I and II.[11] According to Cagan [1], "Even a substantial fall in real income, which generally has not occurred in hyperinflations, would be small compared with the typical rise in prices. Relations between monetary factors can be studied, therefore, in what almost amounts to isolation from the real sector of the economy."

However, there are two persuasive reasons that the *credibility of money* is highly damaged soon after the World Wars. First, the potential production capacity of the economy was at its lowest. In addition, governments were forced to monetize huge amounts of debt issued for military expenditure. Such aspects of hyperinflation are similar to those of *true inflation*.

The second reason, which is more important than the first, concerns the incentive of labor supply. When individuals hold the extraneous belief that prices increase in proportion to the quantity of money, the rate of increase of nominal money supply is equal to the equilibrium inflation rate. Once the inflation rate is higher than some threshold,[12] the equilibrium nominal reservation wages begin to exceed the price of the current good because the true cost of living index ψ becomes extremely high owing to the acceleration of inflation. Consequently, individuals begin to lose their incentive to work. The *credibility of money* is entirely lost in this polar case. Contrary to Cagan [1], hyperinflation can be regarded as the pathology of the monetary economy.

On the basis of the above discussion, in the next subsection, we shall show how the basic model is transformed into a model that justifies the quantity theory of money.

[11] Greece is an exception. The data used for Greece belong to the World War II period.

[12] The critical value of the inflation rate is ρ^K defined by Eq.(7). We shall comprehensively discuss this problem in the next subsection.

3.2 Rational extraneous belief and the monetary policy

To transform the basic model into the monetarist model, we need to assume the following.

Assumption 1 *Every individual believes that money is not credible and holds an extraneous belief that the price of a good is proportional to the quantity of money. That is, each individual considers that the equilibrium price function takes the following form:*

$$p_{t+j} = \kappa^{-1} M_{t+j}, \quad \forall\ j. \tag{11}$$

Under Assumption 1, we can prove the following theorem:

Theorem 2 *There is a rational extraneous belief equilibrium under full employment. That is, there exist* p^{f*}, κ^{f*}, *and* $\mu^{f*} \equiv \dfrac{M_{t+j+1}}{M_{t+j}}$ *that satisfy Eq.(11) and* $y^* = 1$ *for an arbitrarily given* M_t.

Proof. To attain the full employment equilibrium, the price of the good must exceed the equilibrium reservation wage. From Eq.(7), the equilibrium price p_t^{f*} should satisfy

$$p_t^{f*} > W_t^R \quad \Rightarrow \quad p_t^{f*} > f(\alpha)\psi(p_t^{f*}, p_{t+1}^{f*}) \quad \Leftrightarrow \quad 1 > f(\alpha)\psi(1, \frac{p_{t+1}^{f*}}{p_t^{f*}}). \tag{12}$$

Since each individual agrees with Eq.(11), substituting it into (12), we obtain

$$1 > f(\alpha)\psi(1, \mu^{f*}). \tag{13}$$

Because ψ is a continuous and increasing function on μ, taking Eq.(7) into consideration, μ^{f*} must satisfy

$$\rho^K > \mu^{f*}. \tag{14}$$

By the continuity of ψ, it is certain that there exists μ^{f*} that satisfies (14).

Condition (14) assures full employment. Next, we determine (p^{f*}, κ^{f*}) in order to be consistent with Eq.(11) and note that $y^* = 1$. Then, using Eqs.(10) and (11), we obtain

$$1 = c(\mu^{f*}) + \kappa^{f*} \quad \Rightarrow \quad \kappa^{f*} = 1 - c(\mu^{f*}). \tag{15}$$

By Eqs.(11) and (15), we finally determine the equilibrium price function as

$$p_t^{f*} = \frac{M_{t+j}}{\kappa^{f*}}. \tag{16}$$

This equation completes the proof. ∎

Next, we shall deal with the case of hyperinflation. To avoid the unboundedness of the current equilibrium price, we make the following assumption:

Assumption 2 *There are some individuals whose disutility of labor is zero. Their Lebesgue measure is* ϵ, $0 < \epsilon \ll 1$.[13]

Under this assumption, we obtain the following theorem concerning hyperinflation.

Theorem 3 *There is a rational extraneous belief equilibrium where the employment and output level is at its lowest* ϵ. *That is, there exist* p^{h*}, κ^{h*}, *and* μ^{h*} *that satisfy Eq.(11) and*

$$y^* = l^* = \epsilon. \tag{17}$$

Furthermore, the equilibrium inflation rate ρ^{h*} *is the highest in comparison with the economies described by Theorems 1 and 2.*

Proof. From Eq.(7), the following inequality is the necessary and sufficient condition that the employment and output level at its lowest ϵ:

$$p_t^{h*} < W_t^R \quad \Leftrightarrow \quad 1 < f(\alpha)\psi(1, \frac{p_{t+1}}{p_t}) = f(\alpha)\psi(1, \mu^{h*}) \quad \Leftrightarrow \quad \rho^K < \rho^{h*}. \tag{18}$$

By the continuity of ψ, there exists μ^{h*} that satisfies (18).

Next, we prove the existence of κ^{h*}. Using Eq.(10),

$$\epsilon = c(\mu^{h*})\epsilon + \kappa^{h*} \quad \Rightarrow \quad \kappa^{h*} = (1 - c(\mu^{h*}))\epsilon.$$

Finally, by Eq.(11), we obtain the equilibrium price function as

$$p_{t+j}^{h*} = \frac{M_{t+j}}{\kappa^{h*}}. \tag{19}$$

By Eqs.(14) and (18), the equilibrium inflation rate is shown as

$$\rho^{f*} < \rho^K < \rho^{h*}. \tag{20}$$

This completes the proof. ∎

[13]Note that this additional assumption does not affect the validity of Theorems 1 and 2.

4 On the injection methods of money

In the previous section, we assume that money is supplied through the government expenditure and is equally distributed to each individual. However, it differs from the rule in Lucas [7]. Lucas [7] assumes that new money is injected into the economy as interest on the existing money. In this section, we consider how such a difference in money supply rule affects the conclusions in Theorems 1, 2, and 3.

Let us denote the gross rate of interest of money during period t as x_t. Hence, the money supply rule obeys

$$M_{t+1} = x_{t+1} M_t. \tag{21}$$

In addition, we assume that Assumption 2 holds, and all individuals expect the equilibrium price as in Eq.(11).

Then, the budget constraint of each employed individual becomes

$$p_t c_{1t} + M_t \leq W_t, \quad p_{t+1} c_{2t} \leq x_{t+1} M_t \quad \Rightarrow \quad c_{1t} + \frac{p_{t+1}}{p_t x_{t+1}} c_{2t} \leq w_t. \tag{22}$$

Since from Eq.(11),

$$p_t = \kappa^{-1} M_t, \quad p_{t+1} = \kappa^{-1} x_{t+1} M_t. \tag{23}$$

Substituting (23) into (22), we obtain

$$c_{1t} + c_{2_t} \leq w_t = y_t. \tag{24}$$

Thus, the value of x is irrelevant to an individual's consumption-leisure decision. It implies that money is neutral to any equilibrium employment and output level, $[\epsilon, 1]$. Consequently, we obtain

Theorem 4 *When every individual holds the extraneous belief of Eq.(11), and money supply obeys rule (21), money is neutral to any equilibrium employment and output level y^*, $0 < y^* \leq 1$, in the sense that x and M do not affect y^* and κ^*.*

Proof. First, we show that the equilibrium output level y^* is independent of M and x. Since the effective inflation rate is unity in this case, by Eq.(7),

y^* is classified into the following three cases:

$$y^* = \begin{cases} \epsilon, & \text{if } 1 > \rho^K, \\ y^*, & \text{if } 1 = \rho^K, \\ 1, & \text{if } 1 < \rho^K. \end{cases} \tag{25}$$

$y^* = y^*$ in Eq.(25) is used as the following sense. Namely, when $1 = \rho^*$, the utility derived from consumption is equal to the disutility of labor for the majority, $1 - \epsilon$, and they are indifferent whether work or not. Thus, the equilibrium output level becomes indeterminate within $(\epsilon, 1)$. It implies that when individuals decide to work, the equilibrium output increases along their intention. Thus we can ascertain that y^* is independent of M and x.

Finally, we show that κ^* possesses the same properties as y^*. Eq.(24) can be reinterpreted as the goods market equilibrium condition. Let us denote the optimal consumption decision as (c_1^*, c_2^*). Furthermore, note that these values are independent of equilibrium prices (p_1^*, p_2^*) because the equilibrium output y^* is determined by Eq.(25). Then, by Eqs.(11) and (24),

$$c_1^*(y^*) + c_2^*(y^*) = y^* \;\Rightarrow\; \frac{M}{p^*} = c_2^*(y^*) \;\Rightarrow\; \kappa^* = c_2^*(y^*). \tag{26}$$

This completes the proof. ∎

A monetarist may find Theorem 4 to be very effective at the first glance. It implies that the quantity theory of money is upheld even in the normal economy at least mathematically. Nevertheless, we must note that some unusual phenomenon shall be observed in this economy. That is, even if the economy possesses idling resources, and the marginal cost is constant, additional money only raises the price level. In other words, the *credibility of money* is entirely lost in the economy.

Eq.(11) in Assumption 1 and Eq.(21) are crucial factors. The newly issued money subject to Eq.(21) refers to a kind of denomination -the change of the unit of money- and hence, it is possible for individuals to lose *the credibility of money*. As a result, individuals hold an extraneous belief that prices increase proportionately with the quantity of money. Such a method of injecting money, that is, *continuous denomination*, which reduces money from the *absolute* substance to the *relative* measure of value, is scarcely

adopted in fact. Therefore, the relevance of Theorem 4 is much lower than that of Theorem 1.

5 Concluding remarks

We have analyzed the mechanism of aggregate price determination, which closely relates to the problem of the neutrality of money. The results obtained are as follows.

First, when the economy stays within imperfect employment equilibrium, the price of the good is determined by its marginal cost, independent of the quantity of money. It conversely implies that imperfect employment equilibrium emerges from the lack of effective demand (or money).

The stickiness of the aggregate price, which the new Keynesian economists emphasize, may not indicate the substantial cost of changing the price, but the high *credibility of money*. We have succeeded in proving the aggregate price stability by introducing the concept of *credibility of money* by using a model in which prices can change flexibly in accordance with exogenous shocks.

Second, we have also succeeded in transforming the basic Keynesian model into a monetarist model in which the quantity theory of money is upheld and money is insignificant.

The transformation requires two additional conditions to the basic model. One is the extraneous belief on the equilibrium aggregate price level. Namely, all individuals believe that the aggregate price level changes proportionately with the quantity of money. The other is the qualification on the rate of increase of money supply.

Under such an extraneous belief, the inflation rate becomes equal to the rate of increase of money supply. Accordingly, if the rate of increase of money supply is sufficiently low, nominal reservation wages will be lower than the current price of the good. Hence, full-employment equilibrium is attained. Since newly issued money cannot bear any output, the extraneous belief becomes self-fulfilling. Keynes [5] calls this case *true inflation*.

The other polar case is hyperinflation. When the rate of increase of

money supply (the inflation rate) is high enough, nominal reservation wages exceed the current price of the good. In such a case, massive unemployment emerges and the production level falls to its lowest. Thus, the quantity theory of money holds.

To sum up, the quantity theory of money is valid in the two polar cases where money loses its intrinsic value and only operates as a relative measure of value. Although the money supply rule that new money is added as interest on the outstanding money strengthens the monetarist's proposition, such a rule is rarely adopted in reality.

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Figure 1