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Exact microeconomic foundation for the Phillips curve under complete markets: A Keynesian view

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Abstract

Assuming that the labor productivity varies with the previous employment level, we derive the Phillips curve based on the *standard* dynamic microeconomic foundation. The usage of the term *standard* implies that our theory entirely excludes the assumptions unfamiliar with microeconomics such as price or information stickiness, and money in the utility function.

We find that when labor productivity decreases, disinflation advances. This is because disinflation, ceteris paribus, limits the current goods supply and increases the rate of return on money (the inverse of the inflation rate) in an overlapping generations (OLG) model. In addition, mass unemployment becomes a hazard for the intergenerational skill transformation, and thus, the higher the unemployment is, the lower the labor productivity becomes in the stationary state. Consequently, the negative correlation between inflation and unemployment emerges even in the dynamic general equilibrium in complete markets.

It is also noteworthy that we depend neither on linear approximations nor on numerical methods: the method used to derive the Phillips curve is purely analytical.

1 Introduction

Every work on the Phillips curve presumes some market imperfection. For example, Lucas (1972) assumes that the equilibrium price is a noisy signal disturbed by monetary shocks. Calvo (1983) and Woodford (1996) stochastically confine the opportunity of price realignment. Mankiw and Reis (2002) insist that there exist some substantial diffusive lags that inform about the necessity of price revision. These works imply that if there does not exist some price stickiness assumption or imperfect information (that is, the markets are *complete*), the negative correlation between inflation and unemployment will disappear, and the vertical Phillips curve will reemerge as Friedman (1968) suggests. This also means that money is neutral in the long run, when all adjustments are complete.

Our main concern in this article is to construct the stationary negative correlation between inflation and unemployment in certainty without price frictions. This would not only enable us to interpret the Phillips curve as the long-run trade off relationship like in the original work (Phillips (1958)) but also contribute toward shortening the gulf between macroeconomic and microeconomic theory.

The change in the labor productivity due to the learning effect plays a crucial role in this paper. Although recently, the focus with regard to the Phillips curve has been on the responses of inflation and unemployment to a monetary shock, the effect of a real shock (namely, the change in the labor productivity rate) should also be seriously considered. Hayashi and Prescott (2002) reveal that significant declines in the total factor productivity (TFP) and hours worked were observed in Japan during the 1990s. It is also noteworthy that disinflation was prominent throughout that decade, despite the easy monetary policy.

The decline in the labor productivity, which partially consists of the TFP, results in disinflation in the deterministic overlapping generations (OLG) model of Otaki (2007, 2009), even if we do not make any price stickiness

assumption a priori. When the labor productivity is lowered, ceteris paribus, the current goods supply is reduced, and this increases the rate of return on money (the inverse of the inflation rate). Thus, the decline in the labor productivity is accompanied by disinflation. The same mechanism—intertemporal substitution—is also adopted by Lucas (1972).

We further assume that skills nurtured through production process transmits to the child generation, and that the more employment opportunities are offered to fathers, the more productive children become. It is considered to be based on the enlargement of the educational opportunity by the increase of income, and on, perhaps unintentional, educational effect in a family. To sum up, the current labor productivity is assumed to be the increasing function of the previous employment level.

In stationary states, where the employment level is kept constant, the lower the employment level is, the fewer individuals are productive. Consequently, there emerges the negative correlation between the inflation and unemployment rate. This is our understanding on the long-run Phillips curve or the aggregate supply curve under complete markets.

The paper is organized as follows. Section 2 exhibits the basic model. The Phillips curve is also derived. Section 3 deals with the economic welfare implications of the expansionary fiscal-monetary policy. Section 4 contains brief concluding remarks.

2 Model

2.1 Model Structure

We consider a standard two-period OLG model. There exist continuous individuals $[0,1] \times [0,1]$. They supply labor at their discretion only when they are young. Each firm monopolistically produces the differentiated good that is distributed within $[0,1]$. The monopoly rent is equally distributed among the young regardless of their employment status.

Fiat money is the only store of value. The government finances its expenditure by seigniorage. For simplicity, we assume that the goods purchased by the government bear no additional utility to the individuals.

2.2 Individuals

Each individual maximizes the following lifetime utility function:

$$U = u(C_t^1, C_{t+1}^2) - \delta_t \cdot \alpha, \quad C_j^i \equiv \Big[\int_0^1 c_j^i(z)^{1-\eta^{-1}} dz\Big]^{\frac{1}{1-\eta^{-1}}}, \tag{1}$$

where $c_j^i(z)$ denotes the consumption of good z by the individuals in the ith stage of life during period j. α is the disutility of labor. δ_t is a definition function that takes the values one (when employed) and zero (when unemployed). In addition, we assume that $u(\ \cdot\ )$ is a linear homogenous function.

Then, the indirect utility function H of u is represented as

$$H(P_t, P_{t+1}, \delta_t W_t + \Pi_t) = \frac{[\delta_t W_t + \Pi_t]}{h(P_t, P_{t+1})}, \quad P_t \equiv \Big[\int_0^1 p_t(z)^{1-\eta} dz\Big]^{\frac{1}{1-\eta}}, \tag{2}$$

where W_t is the nominal wage and Π_t is the aggregate nominal monopoly rent. We must note that h is also linear homogenous.

Using (2), we can calculate the nominal reservation wage W_t^R as

$$W_t^R = \alpha \cdot h(P_t, P_{t+1}). \tag{3}$$

Since our main concern is the imperfect employment equilibrium where some individuals are always unemployed and possess no bargaining power, W_t^R becomes the equilibrium nominal wage. However, the nominal wage might exceed the reservation wage when the individuals possess collective bargaining power. Nevertheless, as long as the nominal wage negotiation is efficient in the sense of McDonald and Solow (1981), there is no need to discuss any modification. See Otaki (2009) for the precise proof.

It is also noteworthy that the following aggregate consumption function of the younger generation C_t^1 is obtained from the linear homogeneity of u:

$$C_t^1 \equiv c(\rho)[\frac{W_t^R L_t}{P_t} + \frac{\Pi_t}{P_t}] \equiv c(\rho)\frac{Y_t}{P_t}, \tag{4}$$

where $\rho(\equiv P_{t+1}/P_t)$ denotes the gross inflation rate. L_t is the aggregate employment level.

2.3 Firms

From (1), each firm faces the following identical aggregate demand function $D_t(z)$:

$$D_t(z) = \Big(\frac{p_t(z)}{P_t}\Big)^{-\eta}[\frac{W_t^R L_t}{P_t} + \frac{\Pi_t}{P_t}], \tag{5}$$

where L_t is the aggregate employment level. Furthermore, we assume that each firm z has the following identical production function $y_t^s(z)$:

$$y_t^s(z) = \gamma(L_{t-1})L_t(z), \quad \gamma' > 0. \tag{6}$$

γ is the function of the productivity of labor, which plays a key role in our comparative statics. (6) implies that there is a socially significant learning effect in the productivity of labor. Namely, when the more individuals who belong to the previous generation are employed and more skills for production are socially accumulated, the easier they are transmitted to the current generation.This is considered to be an overlapping-generations model version of Arrow's (1962) *learning by doing*.

The profit maximization problem leads us to the following optimal pricing rule:

$$p_t(z) = \frac{W_t^R}{\gamma(L_{t-1})[1-\eta^{-1}]} = \frac{\alpha \cdot h(P_t, P_{t+1})}{\gamma(L_{t-1})[1-\eta^{-1}]}. \tag{7}$$

Aggregating (7) on z, we obtain the following important difference equation:

$$P_t = \frac{\alpha \cdot h(P_t, P_{t+1})}{\gamma(L_{t-1})[1-\eta^{-1}]}. \tag{8}$$

From the linear homogeneity of h, Eq.(8) determines the equilibrium inflation rate ρ independent of the nominal money supply. Differentiating (8) logarithmically and using Roy's identity, we obtain

$$(1-c)\frac{d\rho}{\rho} = \frac{L\gamma'}{\gamma}\frac{dL}{L} \equiv \nu\frac{dL}{L}, \quad 0 < c < 1, \tag{9}$$

where ν is the elasticity of the labor productivity to the employment level. The left-hand side of Eq.(9) indicates the number of future goods that can be substituted for present goods when inflation occurs.

From Eq.(9), it is clear that the inflation rate ρ is determined so as to equalize the additional present aggregate supply $\nu\dfrac{dL}{L}$ to the additional demand for present goods $(1-c)\dfrac{d\rho}{\rho}$. Namely, an improvement in the labor productivity implies a potential increase in the current consumption level; this advances inflation and reduces the rate of return on money to decrease savings.

2.4 Government

In this model, the role of the government is very simple. It issues new money, $M_t - M_{t-1}$, to finance wasteful expenditure G_t. Avoiding any diversion from the stationary equilibrium, we assume that the expenditure is controlled so as to keep the real money stock $m \equiv \dfrac{M_t}{P_t}$, constant over time. Hence, the budget constraint of the government can be written as

$$g \equiv \frac{G_{t+j}}{P_{t+j}} = \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} = m - \frac{m}{\rho}. \tag{10}$$

2.5 Market Equilibrium

2.5.1 The Aggregate Demand Function

Since our attention is confined to the imperfect employment equilibrium in stationary state, the labor markets are in equilibrium whenever $W_t = W_t^R$ and $L_t = L_{t-1}$. The real aggregate demand y is defined as

$$y = c(\rho)y + g + \frac{m}{\rho} = c(\rho)y + m, \quad y \equiv \frac{Y}{P}, \tag{11}$$

where the third term of Eq.(11) is the consumption demand of the old individuals. Because every equilibrium is stationary in the sense that it is independent of time t, we abbreviate the subscripts. This is the dynamically extended multiplier process developed by Otaki (2007, 2009). Solving

Eq.(11), we obtain

$$y = \frac{m}{1 - c(\rho(L))}. \tag{12}$$

(12) is the aggregate demand function of our model. It can be easily seen that an expansionary fiscal-monetary policy increases the real GDP as long as the inflation rate is kept constant.

2.5.2 The Aggregate Supply Function: The Phillips Curve

We have already established the negative correlation between inflation and unemployment in 2.3. Phillips curve is implicitly defined by Eq. (9). Consequently, the Phillips curve is obtained as illustrated by Curve AS in Figure 1.

2.5.3 Market Equilibrium

The good market equilibrium is expressed by the solution of (8) and (12). It is illustrated by Figure 2. Although the slope of the aggregate demand function AD in (L, ρ) plain is ambiguous, a sufficient stability condition of the stationary state E is that Curve AD cuts AS from the downward. It implies that the intertemporal substitution rate on consumption is positive but not too high. From empirical analyses, (For example, see Hansen and Singleton (1983) and Campbell and Mankiw (1989)) it does not seem so restrictive assumption, and hence, we assume that the stationary state is stable.

The fiscal-monetary policy can only control the location of the aggregate demand curve (12). Namely, fiscal-monetary expansion results in a rightward parallel shift in the aggregate demand curve. This implies that a discretionary expansion in the fiscal-monetary policy can increase the real GDP y together with accelerating inflation.

3 The Welfare Implication of Fiscal-Monetary Policy

Since the indirect lifetime utility is represented by Eq.(2), considering that the labor supply is never in surplus, the social welfare SW is defined as

$$SW(L) \equiv \frac{\Pi_t}{h(P_t, P_{t+1})} = \frac{\alpha\eta^{-1}h(\ \cdot\ )}{[1-\eta^{-1}]h(\ \cdot\ )}L = \frac{\alpha\eta^{-1}}{1-\eta^{-1}}L. \tag{13}$$

Let us analyze the welfare implication of our macroeconomic policy in the imperfect employment equilibrium. The result can be easily obtained from Figure 2 and Eq.(13). From Figure 2, it is apparent that the real money supply m increases the real GDP y and employment level L. Accordingly, Eq.(13) reveals that the social welfare is always enhanced by an expansionary fiscal-monetary policy.

4 Concluding Remarks

We have developed the microeconomic foundation for the Phillips curve under complete markets. The obtained results are as follows.

First, the Phillips curve can be derived not from a monetary shock but from an endogenous structure of the economy per se. Whenever the labor productivity increases, inflation rises. This is because the current consumption needs to be stimulated in accordance with the potential increase in the production capacity. Hence, the rate of return on money decreases, and the inflation rate increases. Thus, a positive correlation between the labor productivity and inflation emerges.

On the other hand, the lower labor productivity is reproduced by itself. Namely, the mass unemployment of the previous generation deprives the formal and informal educational opportunity from the current generation. Thereby, the lower employment level causes the lower labor productivity. Thus, the stationary negative correlation between the inflation and unemployment emerges even under complete markets.

Second, as long as the stationary state is stable, an expansionary policy can raise the real GDP and improve the social welfare. This is because the expansionary policy increases the real GDP by the multiplier process and improves the labor productivity through the reduction of the unemployment level.

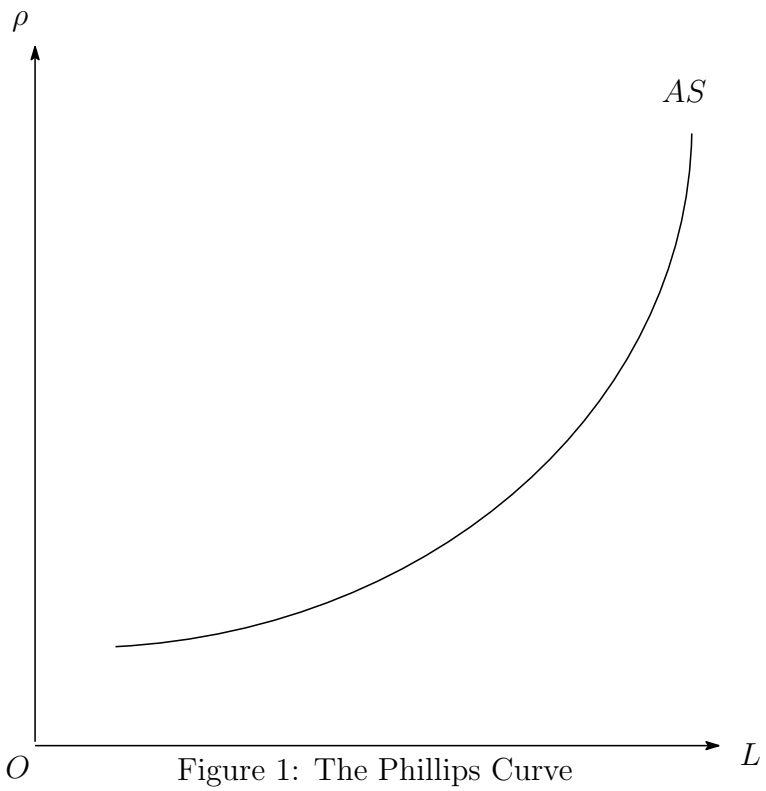

Figure 1: The Phillips Curve

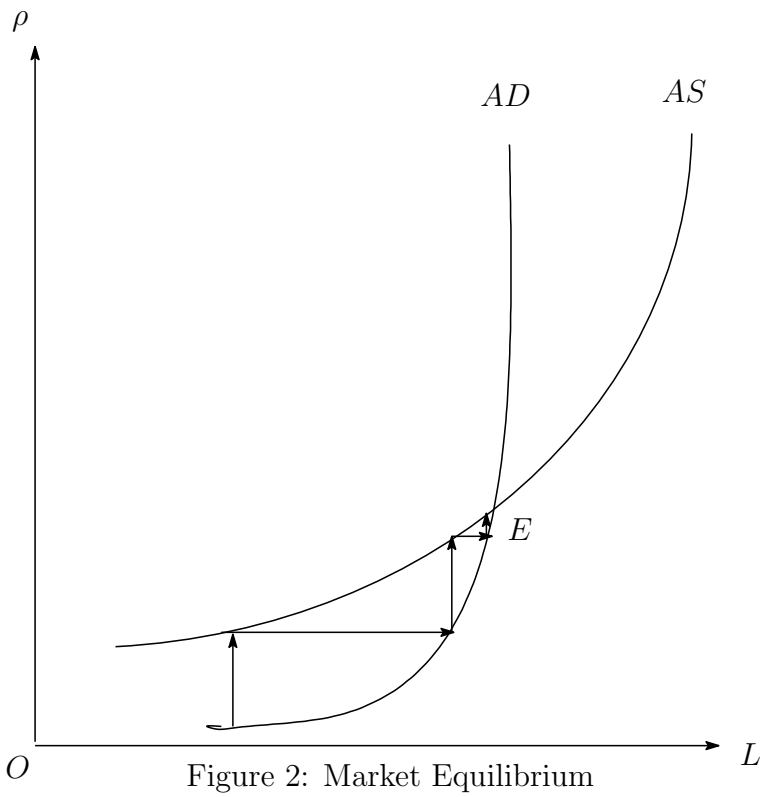

Figure 2: Market Equilibrium

References

[1] Arrow, K. J. (1962). The economic implications of learning by doing.*Review of Economic Studies*, 155-173.

[2] Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 383-398.

[3] Campbell, J. Y. and Mankiw, N. G. (1989). Consumption, income, and interest rates: reinterpreting the time series evidence. *NBER Macroeconomics Annual* 4, 185-216.

[4] Friedman, M. (1968). The role of monetary policy. *American Economic Review*, 1-17.

[5] Hansen, L. P. and Singleton, K. J. (1983). Stochastic consumption, eisk aversion, and the temporal behavior of asset returns. *Journal of Political Economy*, 249-265.

[6] Hayashi, F. and Prescott, E. C. (2002). The 1990s in Japan: a lost decade. *Review of Economic Dynamics*, 206-235.

[7] Lucas, R.E. Jr. (1972). Expectations and the neutrality of money. *Journal of Economic Theory*, 103-124.

[8] Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve. *Quarterly Journal of Economics*, 1295-1328.

[9] McDonald, I. M. and Solow, R. M. (1981). Wage bargaining and employment. *American Economic Review*, 896-908.

[10] Otaki, M. (2007). The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, 23-29.

[11] Otaki, M. (2009). A welfare economics foundation for the full-employment policy. *Economics Letters*, 1-3.

[12] Phillips, A. W. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861-1957. *Economica, New Series*, 283-299.

[13] Woodford, M. (1996). Control of the public debt: A requirement for price stability? *NBER Working Papers*: 5684.