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On the function of money in the search model

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Abstract

We check whether or not money can circulate as a purely transaction medium in the search model. We obtain the following result: unless the role of a store of value is added to the function of money, money does not circulate and the economy degenerates into a barter economy.

1 Introduction

This paper explores the function of money in the search model. There exist two main functions of money: transaction demand for clearing and demand for store of value. Kiyotaki and Wright (1991), which is the seminal work on the search money model, insist that money can circulate solely by the motive of transaction even if it does not operate as a store of value.

However, if money is perishable while differentiated goods are durable, some advantage exists on the side of goods. Namely, the visiting opportunities of exchange in a barter economy are much frequent than those in a monetary economy. This leads us to the following hypothesis: unless money possesses both of the above properties, it is unable to circulate. Based on Kiyotaki and Wright (1991), we ascertain the validity of this hypothesis.

This paper is organized as follows. In Section 2, we construct a simple model based on Kiyotaki and Wright (1991), and exhibit that money never circulates only as a transaction medium. Section 3 proves that money does not circulate without being a store of value. Section 4 contains brief concluding remarks.

2 Model and its properties

2.1 Structure of the model

Our model depends entirely on Kiyotaki and Wright (1991). The individuals are classified into three: producers, commodity traders, and money traders. A producer possesses nothing and is searching for the opportunity of production. A commodity trader has already finished production and is searching for a counterpart for exchange. The counterpart is admissible regardless of whether he/she is a commodity trader or a money trader. To become a money trader, it is necessary to first become a commodity trader. A money trader seeks a commodity trader for consumption. Money is assumed to be

accepted with probability one by any commodity trader, whereas there never exists such a guarantee in barter trade.

2.2 Assumptions

We now state the assumptions of the model.

1. A unit of good produced by each producer is differentiated in the interval $z \in [0, 1]$. The good z is more preferable when z approaches 0. Namely, the utility derived from the consumption of a unit good z, $u(z)$, is a decreasing function of z. This implies that individuals are uniformly distributed around a circle of diameter 2 and that the preferable goods are enumerated in the clockwise direction.

2. Money is perishable. Namely, a money trader can stay in its position only within period $\epsilon \approx 0$ and becomes a producer thereafter. This means that money serves only as a transaction medium and not as a store of value.

3. The opportunity of production follows the Poisson process with mean α. Further, the opportunity of exchange follows the Poisson process with mean β.

4. $F(C)$ is the cumulative distribution function of the production cost C. If C is located above x, the producer waits for the next chance.

2.3 Analysis of the model

Let us denote the expected lifetime utility of a producer, commodity trader, and money trader as V_p, V_c, and V_m, respectively.

It is easy to show that

$$rV_p = \alpha \int_0^x [V_c - V_p - C] dF(C) \tag{1}$$

and

$$rV_c = \beta[1-m]\{\int_0^x \{u(z) - [V_c - V_p]\}dz\} + \beta my[V_m - V_c]. \tag{2}$$

m is the ratio of money traders to total traders, which is exogenously given. y is the upper bound of the good that the money trader admits to exchange.

The problem is in the derivation in V_m. The transition from a money trader to a producer is classified into four cases:

1. To match with a commodity trader and exchange money for a good,
2. To match with a not-preferred commodity trader and get nothing,
3. To match with a money trader and get nothing,
4. To match with no one.

Summing these four cases, we obtain

$$V_m = e^{-r\epsilon}\Big\{\beta\epsilon\{[1-m]\int_0^y u(z)dz + V_p\} + [1-\beta\epsilon]V_p\Big\} + o(\epsilon). \tag{3}$$

The first term in the RHS of (3) corresponds to Cases 1, 2, and 3, and the second term corresponds to Case 4. Subtracting $e^{-r\epsilon}V_m$ from both sides of (3), and rearranging the terms, we have

$$\frac{[1-e^{-r\epsilon}]}{\epsilon}V_m = e^{-r\epsilon}\Big\{\beta[1-m][\int_0^y u(z)dz] + [\frac{V_p - V_m}{\epsilon}]\Big\} + o(\epsilon).$$

Letting $\epsilon \to 0$, we obtain

$$rV_m = \beta[1-m]\int_0^y u(z)dz + \lim_{\epsilon\to 0}[\frac{V_p - V_m}{\epsilon}]. \tag{4}$$

In addition to (1), (2), and (4), the value-matching conditions require

$$V_c = V_p + u(x), \quad V_m = V_p. \tag{5}$$

The second equation is necessary for (4) to retain the economic meaning. The reason why $V_m = V_p$ is required is that the rate of return from being a money trader becomes negatively infinite if it differs, because the jump

in the value functions occurs with probability one within any small interval. Economically, it implies that the loss caused by the perished money is kept invariant while the gain from the trade becomes infinitesimally small, when the relevant interval approaches to zero.

This model possesses five equations and five endogenous variables, (V_p, V_c, V_m, x, y). As such, it is closed. Nevertheless, it contains a contradiction for circulating money. We thus have the following theorem.

Theorem 1 . *In the model, $V_c > V_m$, and money never circulates under Assumption 2.* ∎

Proof. From (5), we have

$$V_c = V_p + u(x) > V_p = V_m.$$

Inequality $V_c > V_m$ implies that no commodity trader wishes to become a money trader. Accordingly, money never circulates. ∎

In this case, the search money model degenerates into a Diamond (1982) model without externality.

3 Money as a store of value

In this section, we replace Assumption 2, and assume that money is perpetually storable. Then, the transition from a money trader to a producer is classified into four cases:

1. To match with a preferred commodity trader, exchange money for a good, and then become a producer,

2. To match with a not-preferred commodity trader, and continue being a money trader,

3. To match with a money trader, get nothing, and continue being a money trader,

4. To match with no one, and continue being a money trader.

Consequently, we have the following equation:

$$V_m = e^{-r\epsilon}\Big\{\beta\epsilon\{[1-m]\int_0^y u(z)dz+yV_p\}+\{[1-\beta\epsilon]+\beta\epsilon\{m+[1-m][1-y]\}\}V_m\Big\}+o(\epsilon).$$

The first term in the RHS corresponds to Case 1, and the second term corresponds to Cases 2, 3, and 4. Rearranging the terms and letting $\epsilon \to 0$, we obtain

$$rV_m = \beta[1-m]\Big\{\int_0^y \{u(z) - [V_m - V_p]\}dz\Big\}. \tag{6}$$

(6) is the equation that Kiyotaki and Wright (1991) actually use. Interchanging the value-matching condition from $V_m = V_p$ to $V_m = V_p + u(y)$ and applying their Theorem 2, we can ascertain that the equilibrium is uniquely determined. Thus, we have the following theorem.

Theorem 2 . *When money serves as not only a transaction medium but also as a store of value, it surely circulates.* ∎

4 Concluding remarks

We have shown that money never circulates as a purely transaction medium in the search model. For sustaining the monetary economy, money should also be a store of value.

However, if both roles are required in the search model, the overlapping generations (OLG) model seems to be more tractable[1], because money plays the same roles: the young generation receives money as a store of value and the old generation uses it as a transaction medium.

[1]For example, although whether or not the neutrality of money holds cannot be easily checked by the search model, the OLG model can do so. See Lucas (1972) and Otaki (2007).

References

[1] Diamond, Peter. A. Aggregate-demand management in search equilibrium. Journal of Political Economy 1982; 90; 881-894.

[2] Kiyotaki, N. and Wright, R. A contribution to the pure theory of money. Journal of Economic Theory 1991; 53; 215-235.

[3] Lucas, Robert E. Jr. Expectations and the neutrality of money. Journal of Economic Theory 1972; 4; 103-124.

[4] Otaki, Masayuki. The dynamically extended Keynesian cross and the welfare-improving fiscal policy. Economics Letters 2007; 96; 23-29.

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