The Aggregation problem in employment theory:
The representative individual or indivisible employees models?

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Abstract

Employment theory does not agree with whether variation in employment should be expressed by a change in the hours worked by a representative individual or by a change in the population of employed individuals.

Thus, the present article describes how this distinction brings about serious theoretical consequence by using an OLG model. The crucial factor that separates the property of employment theories is the existence of the intertemporal substitution effect.

Monetary expansion increases the rate of return for money if it is *credible* in the sense of Otaki (2011). It enhances the hours worked in the representative individual model, and thus, aggregate supply causes demand.

In the indivisible employees model, by contrast, such an intertemporal substitution effect does not exist. Monetary expansion directly improves the purchasing power of money, and thereby increases the aggregate demand for goods by the older generation. Thus, demand pulls supply.

Keywords: Representative Individual, Indivisible Labor Supply, Intertemporal Substitution, Credibility of Money, Fiscal Multiplier.

JEL Classifications: E24, E31, E32, E42, J22.

1   Introduction

Independent of whether scholars adopt the neoclassicals or new Keynesian economic models, recent employment theories have rested on the assumption of a representative individual. However, it is important that the variation of the hours worked by a representative individual essentially differs from that of the number of employed indivisible workers. In the present paper, we show that how this distinction can bring about serious theoretical consequences.

For a representative individual, monetary expansion increases the rate of return as long as money is *credible*, and stimulates labor supply. Hence, apart from certain spurious differences, both neoclassicals and new Keynesian economics seek to find the causes of employment variation due to supply-side incentives.

By contrast, no such substitution effect exists in the indivisible employees model.[1] Monetary expansion directly heightens the purchasing power of money even if the money-supply rule obeys that proposed by Lucas (1972) as long as money is *credible*. It also implies that monetary expansion leads to an upturn in aggregate demand, which increases real GDP. In other words, demand causes the corresponding supply as Keynes (1936) points out.

The remainder of this paper is organized as follows. Section 2 constructs alternative models of employment theory, and section 3 concludes.

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[1]Although we can principally separate the adjustment of hours worked from that of employment level (see Fukao and Otaki (1993)), this requires far more complex dynamics, which is not essential to the present discussion. Furthermore, if there is no fixed sunk cost for being employed, it is clear that every firm uniformly offers minimal hours worked because the increasing marginal disutility of labor requires higher wages for compensation. In such a case, the production adjustment by working hours never incurs a difficult problem.

2 The Model

2.1 The Structure of the Model

We consider a standard two-period deterministic OLG model operating in a production economy. In every period, a unit of individual is born. Each individual can work only when they are young. Unit working hour produces unit goods.

Money supply obeys Lucas's (1972) rule; that is,

$$m_t = m_{t-1}x, \tag{1}$$

where m_{t-1} is the nominal money stock per capita that is carried over from the previous period. x is the gross growth rate of money. In this sense, new money is supplied at its own nominal interest rate.

We make the following alternative assumptions about labor supply: (i) in the representative individual model, he or she can choose working hours and there is nounemployment; (ii) each individual faces a discrete choice over whether to work in the indivisible employees model.

2.2 The Representative Individual Model

2.2.1 The Definition of Equilibrium

For simplicity, we assume that a representative individual has the following utility function U_R:

$$U_R \equiv u(c_{1t}, c_{2t+1}) - v(h_t), \quad v', \ v'' > 0, \tag{2}$$

where u is a well-behaved strictly concave and homothetic function. $c_{1t}, \ c_{2t+1}$ denote the consumption level of generation t during the young and old stages of life, respectively. h is the hours worked. Following Lucas (1972), further we assume that leisure and current consumption are mutually normal goods; that is,

$$u_{11} - \frac{p_{t+1}}{p_t x}u_{12} < 0, \tag{3}$$

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where u_{ij} is the second derivative concerning i and j arguments of u. The lifetime budget constraint is

$$c_{1t} + \frac{m_{t-1}}{p_t} \leq h_t, \quad p_{t+1}c_{2t} \leq m_{t-1}x$$

$$\Leftrightarrow \quad c_{1t} + \frac{p_{t+1}}{p_t x}c_{2t+1} \leq h_t. \tag{4}$$

$$\tag{5}$$

The optimality conditions are

$$\frac{u_2}{u_1} = \frac{p_{t+1}}{p_t x}, \tag{6}$$

$$\frac{v'}{u_1} = 1, \tag{7}$$

$$h_t^* = c_{1t}^* + \frac{p_{t+1}}{p_t x}c_{2t+1}^*, \tag{8}$$

where u_i is the partial derivative of the i-th argument of u.

In addition to these three optimality conditions, there is an independent market equilibrium condition. Here, we consider the condition for the money market equilibrium; that is,

$$m_{t-1}x = p_{t+1}^* c_{2t+1}^*. \tag{9}$$

Furthermore, we assume the *credibility* of money in the sense proposed by Otaki (2011):[2]

$$\frac{dp_{t+1}^*}{dx} = 0. \tag{10}$$

There are five endogenous variables $(c_{1t}^*, c_{2t+1}^*, h_t^*, p_t^*, p_{t+1}^*)$ and five independent equations (6), (7), (8), (9), and (10). Hence, the model is closed, and the solution consists of temporary rational expectation equilibrium.

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[2]The concept of *credibility* of money is a device to select a unique rational expectation equilibrium (REE) among multiple REEs that are generic to the OLG model of the monetary economy. *Credibility* economically means that people rationally believes its intrinsic value is kept intact even if the velocity of monetary acceleration is changed.

2.2.2 Comparative Statics

By combining Equation (8) with (6), we obtain

$$h_t^* = c_{1t}^* + \frac{u_2}{u_1} \frac{m_{t-1} x}{p_{t+1}^*}. \tag{11}$$

From Equations (6) and (10), the right-hand side of Equation (11) is solely a monotonously increasing function of c_{1t}^*, which contains x as an exogenous parameter.

The curves UU and VV in Figure 1 are loci of Equations (7) and (11), respectively. The intersection E_0 is the temporally rational expectation equilibrium that presumes the *credibility* of money.

When monetary expansion occurs (i.e., x increases), the curve VV shifts in the south-east direction because of the growth in future consumption stimulates labor supply and economizes leisure. Thus, the equilibrium moves from point E_0 to point E_1.

From the concavity of u and Equation (6), it is clear that the effective inflation rate (the inverse of the real interest rate) $\frac{p_{t+1}^*}{p_t^* x}$ decreases with an acceleration in monetary growth (i.e., an increase in x).

To summarize up, as long as money is *credible*, easy monetary policy increases the real rate of interest, thereby a representative individual works longer hours in order to enjoy a greater degree of future consumption.

Accordingly, monetary expansion advances the intertemporal substitution from current consumption and leisure to future consumption by increasing the real rate of interest. Consequently, the expansionary effect of monetary policy is entirely based on labor supply incentives not on the expansion of aggregate demand. In this sense, the representative individual model is can be classified as a neoclassical macroeconomic model.

2.2.3 The Time-Independency of the Model

Assume that a representative individual rationally expects that the real effective inflation rate $\frac{p_{t+1}^*}{p_t^* x}$ is kept intact after period $t+1$, since no economic

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environment is changed after period t. This implies that

$$\frac{dx}{x} = \frac{dp^*_{t+j+1}}{p^*_{t+j+1}} - \frac{dp^*_{t+j}}{p^*_{t+j}}, \quad \forall j \geq 1. \tag{12}$$

In addition $m_{t+j-1}x = p^*_{t+j+1}c^*_{2t+j+1}$ and $m_{t+j-1} = p^*_{t+j}c^*_{2t+j}$ holds from the money market equilibrium condition (9). Thus, we obtain

$$\frac{dc^*_{t+j+1}}{c^*_{t+j+1}} - \frac{dc^*_{t+j}}{c^*_{t+j}} = \frac{dx}{x} - \left[\frac{dp^*_{t+j+1}}{p^*_{t+j+1}} - \frac{dp^*_{t+j}}{p^*_{t+j}}\right] = 0. \tag{13}$$

In other words, future consumption c^*_2 becomes time-independent. Hence, from Equations (6), (7), and (8), (c^*_1, h^*) are also time-independent. Consequently, the rational expectation equilibrium characterized by the initial condition x and the expectation formations (10) and (12) is stationary.

2.3 The Indivisible Employees Model

Here, we assume that labor supply is indivisible, and that each individual has the identical utility function U_I:

$$U_I \equiv u(c_{1t}, c_{2t+1}) - \delta_t v, \tag{14}$$

where u is the same consumption utility function as that described in Equation (2). δ_t denotes a definition function that takes value unity when he/she works unit time and becomes zero when he or she does not work. v is the disutility of labor.

According to Otaki (2011), the nominal minimal revenue that individuals decide to work NR is represented as

$$NR = \psi(p_t, \frac{p_{t+1}}{x})f(v), \quad \psi_1 > 0, \quad \psi_2 > 0, \tag{15}$$

where ψ is a linear homogeneous function.

At any interior equilibrium, all individuals are indifferent about whether to work, and thus, we obtain the following difference equation concerning the evolution of price sequence:[3]

$$p^*_{t+j} = \psi(p^*_{t+j}, \frac{p^*_{t+j+1}}{x})f(v) \quad \Rightarrow \quad 1 = \psi(1, \frac{p^*_{t+j+1}}{p^*_{t+j}x})f(v), \quad 0 \leq j. \tag{16}$$

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[3]See Otaki (2011), for the case that the equilibrium is located at the boundaries.

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Hence, the equilibrium real interest rate $r^* \equiv \dfrac{p^*_{t+j}x}{p^*_{t+j+1}}$ is independent of x and takes a constant value.

The equilibrium condition for the money market is

$$s(r^*)y^*_{t+j} = \frac{m_{t+j-1}x}{p^*_{t+j+1}} \equiv \tilde{m}^*_{t+j}, \quad 0 \le j, \tag{17}$$

where $s(\ \cdot\ )$ is the marginal propensity to save. Assuming the *credibility* of money (i.e., $\dfrac{dp^*_{t+1}}{dx} = 0$), according to Equation (16), $\dfrac{dp^*_t}{p^*_t} = -\dfrac{dx}{x}$ holds.

That is, an increase in the monetary growth rate x increases the current value of money and empowers the purchasing power of older individuals as long as money is considered a *credible* asset. As such, monetary expansion stimulates the economy through the multiplier effect developed by Otaki (2007).

It is facile to show the time-independency of the equilibrium. Assume people rationally believe that the price level p^*_{t+j+1} grows proportionately with the monetary expansion rate x, that is,

$$\frac{dp^*_{t+j+1}}{p^*_{t+j+1}} = \frac{dx}{x}, \quad 1 \le j \tag{18}$$

then the equilibrium real cash balance $\tilde{m}^*$ becomes time-independent. So is the real equilibrium GDP y^*.

In addition, as we precisely aforementioned, the individual employees model belongs with Keynes' (1936) type model in the sense that a monetary expansion stimulates the aggregate demand unlike the representative individual model that crucially relies on the supply-side intertemporal substitution incentives.

3   Concluding Remarks

This article analyzed how the aggregation problem affects employment theory and drew two main conclusions.

First, owing to the intertemporal substitution between goods and leisure, a change in working hours in the representative individual model is supply-side oriented even if money is *credible* and non-neutral. Acceleration in monetary growth increases the real interest rate of money, and thus, intertemporal substitution occurs from leisure and current consumption to future consumption.

Second, the individual employees model possesses the demand-driven property proposed by Keynes (1936). Although the real rate of interest on money is endogenously fixed, whenever money is *credible*, it becomes to be highly valued and this increases the purchasing power of the older generation by the acceleration of monetary growth. As such, effective demand expands and real GDP increases because of the multiplier effect.

To summarize, Keyes' (1936) economics can be characterized by two attributes: *credibility* of fiat money of which its intrinsic value is basically indeterminate and the specificity of labor as a commodity, namely, the indivisibility of labor.

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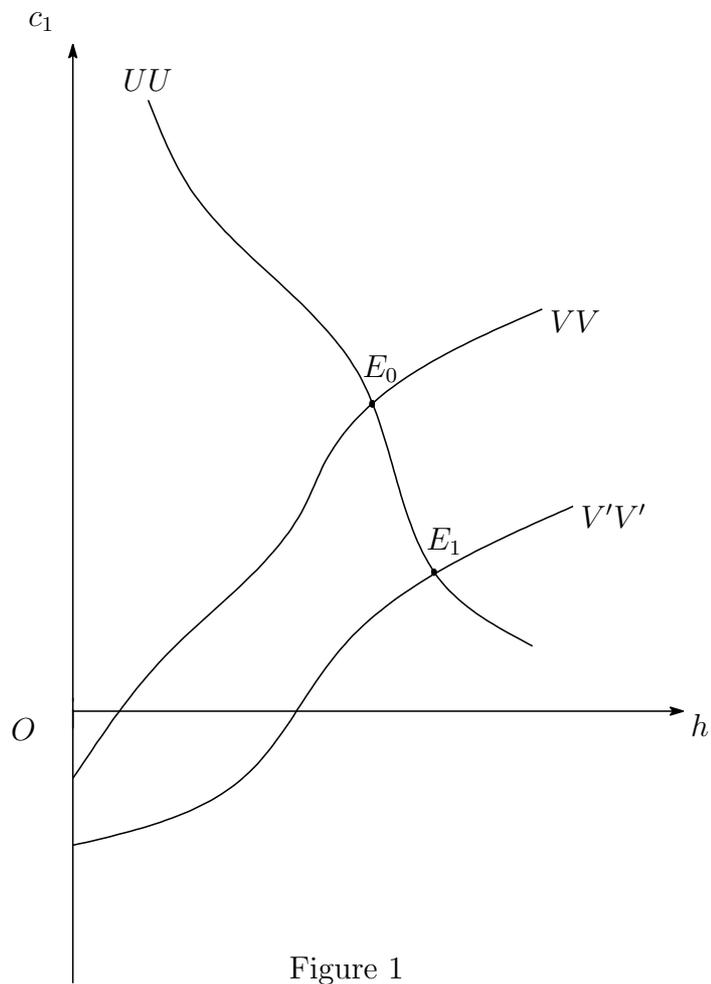
Figure 1

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Figure 1: The determination of (c_1^*, h^*)