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under  
Perfect and Monopolistic Competitions**

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**Abstract**

This article analyzes the difference in the properties of economic growth theory between perfect and monopolistic competition. Whether or not capital investment is constrained by effective demand is the crucial factor that characterizes economic growth theories in different degrees of competition. Whenever a firm faces a downward sloping demand curve the location of which is determined by the strength of effective demand (i.e., the real GDP), its capital accumulation is inevitably constrained by effective demand. Thus, as far as the business environment is kept unchanged, capital investment is as well. However, when the goods market is perfectly competitive, firms never perceive such a demand constraint, and thus, capital investment autonomously advances independent of the phase of the business cycle.

An important macroeconomic implication of such a difference in the attitude toward capital investment is as follows. When an economy is in perfect competition, capital investment becomes an independent driving force of economic growth as Keynes considers, although it is subject to other independent expenditure (e.g., government expenditure) and falls into a subsidiary component of effective demand otherwise.

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## 1 Introduction

Little is known how the market structure of goods markets affects economic growth in a monetary economy. Although Dutta and Rustichini [1] and Smets [2] built models of investment function under uncertainty and pointed out that the function depends on the level of effective demand, they were not concerned with how such investment related to economic growth as a whole. Otaki [3] developed a general equilibrium growth model under monopolistic competition and also found that there exists no endogenous economic force for sustainable growth in a monetary economy.

In his seminal work, Uzawa [4] analyzed the properties of the investment function under perfect competition in the context of a general equilibrium model. Although his theory excludes the existence of money, he found that the optimal ratio of investment to capital is free from the level of effective demand. The optimal ratio is dependent on the profit rate which is endogenously determined only by relative prices. Such a prominent property of the investment function implies that, in contrast of the monopolistic competition case analyzed by Otaki [3], capital investment enables an economy to sustain its growth. This is because the accumulated past capital investments (i.e., the existing capital stock itself) empowers current investment regardless of the condition of effective demand<sup>1</sup>

The main theoretical issue addressed in this article is to check the validity of Uzawa's [4] assertions that capital investment stimulates future investment expansion under perfect competition and that economic growth is sustainable even in a monetary economy. The result is as follows. Since the effective demand principle works because of the indeterminacy of the

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<sup>1</sup>It is quite ambiguous why more capitals enhance investment in Uzawa [4]. He attributes this property to the existence of *managerial resources*, which are rewarded by whole earned quasi-rents within the firm. However, it seems difficult to identify the substance of *managerial resources*. Otaki [5] instead introduces the concept of the *dexterity* of labor forces, which provides physical capital with positive externalities such as process innovations. A firm is regarded as an ingenious device for the internalization of such externalities, and thus, since there is no limit to sale under perfect competition, a firm attains sustainable growth together with the accumulation of *dexterity*.

equilibrium price sequence (for more detail Otaki [6]), an economy is not necessarily able to attain a GDP that guarantees full resource utilization even though goods markets are competitive. However, since capital investment becomes an autonomously expanding independent expenditure of effective demand, the suspending power to economic growth becomes stronger compared with the case of monopolistic competition as analyzed in Otaki [3]. Consequently, the government deficits necessary for attaining full resource utilization grow only at a constant rate, which is equalized to the GDP growth rate, although such a rate is accelerated together with capital accumulation in the monopolistic competition case (see Theorem 2 in Otaki [3]).

The remainder of this paper is organized as follows. In Section 2, we construct and analyze a two-period overlapping-generations monetary growth model with *impure altruism*<sup>2</sup>. Section 3 contains concluding remarks.

## 2 The Model

### 2.1 Structure of the Model

There are two strata in this economy: employers and employees. Each employee holds *impure altruism*; he is not only concerned with his own lifetime utility but also his descendant's young-time utility. The inheritances he leaves are the seeds of the dexterity of future employees via the education of current incumbent employees (we assume that it takes a considerable length of time for such education to be effective). There is no disutility of labor in this stratum.

Each employee provides his unit of labor when he is young at his discretion. The disutility is denoted as  $\alpha$ . His lifetime utility which comes from the consumption stream  $(c_{1t}, c_{2t+1})$  is a Cobb-Douglas function (note that such a function is common with employers). Thus, the lifetime utility  $U$  is defined as

$$U \equiv [c_{1t}]^{1-s} [c_{2t}]^s - \delta_t \cdot \alpha, \quad 0 < s < 1, \quad (1)$$

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<sup>2</sup>See, for example, Acemoglu [7] on the details of *impure altruism*.

where  $\delta_t$  is a definition function the value of which takes unity when employed and zero when unemployed. Without loss of generality to simplify the calculation, we assume that the economy is located at the full employment equilibrium (the full-employment level is fixed to unity.).

The government newly issues fiat money to finance its fiscal expenditures which is, for simplicity, bear no additional utility in the private sectors. It is also assumed that the government pays real dole  $d$  in proportion to his dexterity  $L_t$ , which is null since our reference point is the full-employment equilibrium. The arbitrage condition within the labor market requires

$$p_t d = W_t^R L_{t-1}, \quad (2)$$

where  $W_t^R$  is the nominal reservation wage, which is endogenously determined as below.

The budget constraint of the government becomes

$$\begin{aligned} M_t - M_{t-1} = p_t G_t &\Leftrightarrow \frac{M_t}{p_t L_{t-1}} - \frac{M_{t-1}}{p_t L_{t-1}} = \frac{G_t}{p_t L_{t-1}} \\ &\Leftrightarrow g_t = m_t - \frac{m_{t-1}}{[1 + \pi_t][1 + \theta]}, \end{aligned} \quad (3)$$

where

$$m_t \equiv \frac{M_t}{p_t L_{t-1}}, \quad g_t \equiv \frac{G_t}{p_t L_{t-1}}, \quad \pi_t \equiv \frac{p_{t+1}}{p_t} - 1, \quad \theta \equiv \frac{L_{t-1}}{L_{t-2}} - 1.$$

$p_t$  is the current price of the good produced in the economy.  $g_t$  is the real government expenditure per efficient unit of the labor force.  $L_t$  is the labor force per employee measured by the efficiency unit.  $\theta$  denotes the degree of progress in dexterity nurtured by the employer's capital investment.

## 2.2 Agents' Maximization Problems

### 2.2.1 Employers

Since an employer is assumed to be *impure altruistic*, his marginal substitution rate between his own future come and his descendant's income is fixed to unity. Hence, the consumption/saving decision is independent of whether savings are put into capital stock or money. Furthermore, since the marginal substitution rate between current and future consumption is equalized to the

gross inflation rate  $1 + \pi_t$ , which is common in both capital investment and money hoarding decisions, the optimal capital investment decision problem becomes equivalent to the maximization problem on the discounted net cash flow obtained from capital. Accordingly, the optimal economic behavior of an employer can be expressed by the following equations<sup>3</sup>.

$$S^{ER} = s[rL_{t-1} - W_t^R L_{t-1}], \quad (4)$$

$$\phi'(\theta^*) = [1 + \pi_t][r - \phi(\theta^*)], \quad (5)$$

where  $S^{ER}$  is the aggregate savings of employer stratum.  $r$  is the rate of return from skilled labor force.  $\phi$  denotes the average adjustment cost for educating and nurturing dexterity, which is defined as

$$\Phi(L_t, L_{t-1}) \equiv \phi(\theta)L_{t-1}, \quad \phi' \phi'' > 0,$$

where  $\Phi$  is the total adjustment cost.

### 2.2.2 Employees

Since the lifetime utility function of consumption is the Cobb-Douglas form, the aggregate savings of employees  $S^{EE}$  is

$$S^{EE} = s \cdot W_t^R L_{t-1}. \quad (6)$$

In addition, the indirect lifetime utility  $IU_t$  becomes

$$IU_t = \frac{W_t^R L_{t-1}}{[p_t]^{1-s}[p_{t+1}]^s} \Rightarrow W_t^R L_{t-1} = \alpha[p_t]^{1-s}[p_{t+1}]^s. \quad (7)$$

Combining (7) with (2), we obtain the following fundamental equation concerning the dynamic motion of the equilibrium price sequence.

$$p_t d = \alpha[p_t]^{1-s}[p_{t+1}]^s \Rightarrow 1 + \pi^* = \left[\frac{d}{\alpha}\right]^{\frac{1}{s}}. \quad (8)$$

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<sup>3</sup>Although, for simplicity, we henceforth assume that the nominal wage is equal to the nominal reservation wage, it is natural to consider the nominal wage to be determined through a bargaining process since labor forces are regarded as quasi-fixed production factors. Even though we introduce such a negotiation process into the model, the obtained results intact. For details, see Otaki [8].

### 2.3 Market Equilibrium

We have two markets in the model: the goods market and the money market. By Walras's law, we can concentrate the equilibrium condition for the goods market. By adding up (4) and (6), the saving function of the economy as a whole  $S_t$  is

$$S_t \equiv s \cdot r L_{t-1}. \quad (9)$$

To avoid unessential non-linearity in the investment function, we assume that the average adjustment cost function  $\phi$  is a power function. That is,

$$\phi(\theta) \equiv \theta^\beta, \quad \beta > 1.$$

Then, from the optimality condition for the optimal capital investment (5), the investment function  $I_t$  is derived as

$$I_t \equiv \phi(\theta^*) L_{t-1} = \frac{\phi'(\theta^*)}{\beta} L_{t-1} = \frac{r[1 + \pi^*]}{\beta[2 + \pi^*]} L_{t-1}, \quad (10)$$

where  $1 + \pi^*$  is the equilibrium inflation rate in (8).

Furthermore, we assume that, in contrast to from Otaki [3], the growth rate of the fiscal deficit in terms of the labor forces is set to zero and  $m = m_t = m_{t-1}$  holds. The government budget constraint (3) is transformed into

$$g = \left[ 1 - \frac{1}{[1 + \pi^*][1 + \theta^*]} \right] m. \quad (11)$$

Equations (9), (10) and (11) lead us to the following equilibrium condition for the good markets normalized by the existing labor force in terms of efficiency units  $L_{t-1}$  as

$$\begin{aligned} s \cdot r &= \frac{r[1 + \pi^*]}{\beta[2 + \pi^*]} + g + \frac{1}{[1 + \pi^*][1 + \theta^*]} m \\ \Rightarrow \quad s \cdot r &= \frac{r[1 + \pi^*]}{\beta[2 + \pi^*]} + m, \end{aligned} \quad (12)$$

where the third term of the right-hand side of the above equation in (12) is the expenditure of the old generation in terms of the efficiency units of labor force. As far as the real cash balance in terms of efficiency units of

the labor force  $m$  is determined so that both sides of (12) are equalized, the economy can sustain full capacity utilization<sup>4</sup>.

Since it is apparent that the monetary growth rate under the full capacity utilization equilibrium is equal to that of nominal GDP  $[1 + \pi^*][1 + \theta^*]$ , we finally obtain the following theorem.

**Theorem 1** *The growth of the monetary economy under perfect competition is sustainable in the sense that the ratio of public debts to nominal GDP is kept constant over time.*

The theory contrasts sharply with the properties of the monetary growth model under monopolistic competition in Otaki [3], in which the public debts-nominal GDP ratio ( $\frac{G_t}{Y_t}$ ) is explosive. The decisive economic reason that determines whether such a ratio is explosive or not is whether employers are subject to the effective demand constraint. In the perfect competition case, every employer can expand his firm without any constraint in the long run. This implies that the aggregate capital investment is autonomously expanded, and hence it consists of an endogenous force of economic growth as constant effective demand stimulus.

On the other hand, capital investment is constrained by effective demand in the case of monopolistic competition. Thus, other exogenous expansionary shocks such as acceleration of fiscal expenditure are indispensable to sustaining economic growth because capital investment does not have the power to create new additional demand per se. Accordingly, fiscal deficits and the public debts-nominal GDP ratio ( $\frac{G_t}{Y_t}$ ) becomes explosive as proved by Otaki [3].

### 3 Concluding Remarks

This paper analyzed how market competitiveness relates to the sustainability of economic growth. The obtained result is as follows. Because there is

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<sup>4</sup>As Otaki [9] argues, if individuals rationally believe that the future purchasing power of money is unaffected by a change in the current nominal money supply  $M_t$  (i.e., money is *credible*), the current price  $p_t$  also becomes insensitive to  $M_t$  (see (8)), and thus, the government can control the real cash balance.

no demand constraint whenever the market is competitive, capital investment creates additional effective demand in the future by itself. This fact implies that an economy steadily grows without unsustainable help from its government.

In turn, as Otaki [3] shows, if goods provided in the economy are differentiated even narrowly and markets become less competitive, every employer perceives that he faces a downward-sloping demand function the location of which is determined by effective demand. Therefore, capital investment is subject to effective demand, and thus, it loses the driving force for economic growth. The progress of labor productivity by capital investment needs an explosive fiscal expenditure to maintain the full resource utilization equilibrium.

In this sense, competitiveness plays a key role in sustaining a stable fiscal balance with moderate economic growth.

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