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Global Warming, Imputed Prices, and Sustainable Development

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Abstract

In this paper, we prove in terms of a simple dynamic model of global warming that sustainable time-paths of the atmospheric accumulations of carbon dioxide are obtained as the standard market equilibrium under the system of proportional carbon taxes, where the carbon taxes are levied at the rate that is proportional to the per capita national income of each country, with the discounted present value of the impact coefficient of global warming as the coefficient of proportion.

Keywords

global warming
imputed price
virtual capital market
sustainable development
proportional carbon taxes

1. Introduction

Global warming involves international and intergenerational equity and justice. Although global warming is largely caused by the emission of carbon dioxide and other greenhouse gases associated with economic activities mostly in developed countries and by the disruption of forests − particularly tropical rain forests − again mostly resulting from the industrial activities of the developed countries, it is the people in developing countries who have to bear the burden. While the current generation may enjoy the fruits of the economic activities that cause global warming, it is the people in all future generations who will have to suffer from a significant increase in atmospheric instability as a consequence.

The problems of global warming are genuinely dynamic. We inherit an excess concentration of atmospheric carbon dioxide from past human activities and the choices we make today concerning the use of fossil fuels and related activities significantly affect all future generations through the phenomenon of global warming that is brought about by the atmospheric accumulations of carbon dioxide due to the combustion of fossil fuels today. Thus, we explicitly have to take into account the changes in the levels of welfare of all future generations caused by the increases in the atmospheric accumulations of carbon dioxide today. The degree of uncertainty involved with these phenomena, however, is too great to be subject to any objective analysis. We take instead a simplistic approach that all future paths are presumed to be known for certain and derive a number of propositions concerning policy and institutional arrangements to bring about the sustainable processes of economic development. The concept of sustainability as introduced in this paper, however, is defined in such a manner that the basic nature of policy choices and institutional arrangements to bring about the sustainable economic development remains valid regardless of the specific pattern of future paths that is presumed to be known for certain at the beginning of the analysis.

The dynamic analysis of global warming in this paper is developed primarily along the lines suggested by a large number of contributions concerning the dynamic theory of optimum capital accumulation and scientific analysis of global warming, as in detail referred to in Uzawa (2003, 2005). We are particularly concerned with the processes of economic development that result in the sustainable time-path of the atmospheric accumulations of carbon dioxide. One of the main conclusions obtained in this paper is the proposition that sustainable processes of the atmospheric accumulations of carbon dioxide are obtained as the standard market equilibrium under the system of proportional carbon taxes, where the carbon taxes are levied at the rate that is proportional to the per capita national income of each country, with the discounted present value of the impact coefficient of global warming as the coefficient of proportion.

Our analysis is focused upon the role of the imputed price of the atmospheric accumulations of carbon dioxide in the processes of sustainable economic development. The imputed price π_t^ν of the atmospheric accumulations of CO_2 for each country ν at a particular time t expresses the extent to which the marginal increase in the atmospheric accumulations of CO_2 at time t induces a marginal decrease in units of market prices in the welfare level of country ν, including those of all future generations. The imputed price π_t^ν is at the sustainable level at time t, if it remains stationary at time t; i. e.,

$$\dot{\pi}_t^\nu = 0 \text{ at time } t,$$

where $\dot{\pi}_t^\nu$ refers to the time derivative with respect to the time of the virtual capital market for the atmospheric accumulations of carbon dioxide at time t. A time-path of the atmospheric accumulations of CO_2 is defined sustainable for country ν, if the imputed price π_t^ν for country ν is at the sustainable level at all time t.

Because the sustainability of the imputed price is defined with respect to the fictitious time of the virtual capital market at time t, the sustainability of the imputed price does not necessarily imply the stationarity of the imputed price, as inadvertently stated in Uzawa (2003, 2005). All the policy and institutional conclusions obtained in Uzawa (2003, 2005), however, remain valid with regard to the concept of the sustainability of time-paths of the atmospheric accumulations of carbon as introduced in this paper.

2. Accumulation of Carbon Dioxide in the Atmosphere − A Simple Dynamic Model

We denote by V_t the amount of CO_2 accumulated in the atmosphere at time t. The quantity V_t is measured in actual tons (in weights of carbon content) or in terms of the density of CO_2 in the atmosphere. We also adopt as the origin of the measurement the stable pre-Industrial Revolution level of 600 GtC (approximately corresponding to the density of 280 ppm). The current level of 760 GtC (380 ppm) is expressed as V_t = 160 GtC.

The premises of the dynamic model introduced in this paper are primarily based upon the scientific findings on global warming and related topics, as reported in Dyson and Marland (1979), IPCC (1991, 1996, 2002, 2007), Keeling (1968, 1983) Ramanathan (1985), Stern (2007), Takahashi (1980), Uzawa (199l, 2003, 2005) , among others.

Changes of Carbon Dioxide Accumulations over Time

The atmospheric accumulations of CO_2 change over time due to natural and anthropogenic activities. Let us first consider changes in the atmospheric level of CO_2 due to natural activities.

A certain portion of atmospheric accumulations of CO_2 is absorbed by the oceans (roughly estimated at 50%) and by living land plants as well. In the simple dynamic model postulated in the present paper, the exchange of CO_2 between the atmosphere and the terrestrial bio-sphere is not taken into consideration. It is explicitly discussed in the more general model introduced in Uzawa (2003, 2005).

Roughly 75-90 GtC of carbon are annually exchanged between the atmosphere and the surface oceans. The mechanisms by which atmospheric CO_2 is absorbed into the surface oceans are rather complicated. They depend partly upon the extent to which the surface waters of the oceans are saturated with CO_2, on the one hand, and partly upon the extent to which CO_2 is accumulated in the atmosphere in excess of the equilibrium level, on the other. The studies made by several meteorologists and oceanologists, in particular by [1, 7, 8, 9, 11] suggest that the rate of ocean uptake is closely related to the atmospheric accumulations of CO_2 in excess of the stable, pre-Industrial Revolution level of 280 ppm.

As an approximation of the first order, we assume that the amount of atmospheric CO_2 annually absorbed by the oceans is given by μV_t, where V_t is the atmospheric accumulations of CO_2 measured in actual tons of CO_2, with the pre-Industrial Revolution level of 600 GtC as the origin of measurement, and the rate of absorption μ is a certain constant. The rate of absorption μ is rather difficult to estimate. The studies made by Takahashi and others referred to above suggest that the rate of absorption μ would have a magnitude of 2-4%. Indeed, according to the estimates made by Ramanathan (1985), CO_2 remains airborne 7 to 17 years, and about a half is absorbed by the surface ocean, again resulting in the magnitude of μ = 2-4%.

In the simple dynamic model introduced in this paper, the anthropogenic change in the atmospheric accumulations of CO_2 is exclusively due to the combustion of fossil fuels in connection with industrial, agricultural, and urban activities. We denote by a_t the annual rate of increase in the atmospheric level of CO_2 due to anthropogenic activities. The magnitude of a_t is currently estimated around 6 GtC per annum; that is, a_t = 6 GtC.

The dynamic equation for the atmospheric level of CO_2 is now formally written as

$$\dot{V}_t = a_t - \mu V_t, \qquad (1)$$

where $\dot{V}_t = \frac{dV_t}{dt}$ denotes the rate of change in V_t over time.

The rate of anthropogenic change in the atmospheric accumulations of CO_2, a_t, is primarily determined by the combustion of fossil fuels, to be closely related to the levels of production and consumption activities conducted during the year observed.

There exist n countries in the world. Each country is generically denoted by ν $(\nu = 1, \ldots, n)$. We are concerned with the processes by which various relevant economic variables are determined at each time t. However, for the time being, all variables are denoted without explicit reference to time t.

Specifications for Utility Functions

We assume that the level of the utility u^ν of each country ν depends upon the atmospheric accumulations of CO_2 at time t, V. That is, the utility function for each country ν is expressed in the following manner:

$$u^\nu = u^\nu(c^\nu, V),$$

where $c^\nu = (c_j^\nu)$ is the vector of goods consumed in country ν on the per capita basis and V is the atmospheric accumulations of CO_2 measured in tones of CO_2 accumulated in the atmosphere, both at time t.

Remark: To simplify the exposition, our discussion is carried out as if the size of population of each country ν is one. However, when we discuss the policy and institutional implications, the standard case of numerous individuals in each country will be explicitly brought out.

For each country ν, we assume that utility function $u^\nu(c^\nu, V)$ is strongly separable with respect to c^ν and V in the sense originally introduced in Goldman and Uzawa (1964); that is,

$$u^\nu(c^\nu, V) = \varphi^\nu(V)\, u^\nu(c^\nu).$$

The function $\varphi^\nu(V)$ expresses the extent to which people in country ν feel against the effect of global warming, which is referred to as the impact index of global warming for country ν. We assume that the impact index function $\varphi^\nu(V)$ satisfies the following conditions:

$$\varphi^\nu(V) > 0,\ \ \varphi^\nu{}'(V) < 0,\ \ \varphi^\nu{}''(V) < 0 \ \text{ for all } V.$$

The relative rate of the marginal change in the impact index $\varphi^\nu(V)$ due to the marginal increase in the atmospheric accumulation of CO_2 is given by

$$\tau^\nu(V) = -\frac{\varphi^\nu{}'(V)}{\varphi^\nu(V)},$$

which is referred to as the impact coefficient of global warming for country ν. In what follows, we assume that the impact coefficient of global warming $\tau^\nu(V)$ is identical for all countries. That is,

$$\tau^\nu(V) = \tau(V) \text{ for all } \nu.$$

The impact coefficient $\tau(V)$ satisfies the following conditions:

$$\tau(V) > 0, \quad \tau'(V) - [\tau(V)]^2 > 0.$$

The impact index function $\varphi(V)$ of the following form is often postulated:

$$\varphi(V) = (\hat{V} - V)^\beta, \quad 0 \leqq V < \hat{V}, \tag{2}$$

where $\hat{V}$ is the critical level of the atmospheric accumulation of CO_2 and β is the sensitivity parameter, $0 < \beta < 1$. If the level of atmospheric accumulations of CO_2 were higher than the critical level $\hat{V}$, irrevocable damage would be inflicted on the global environment. In Uzawa (199l), where the concept of the impact coefficient of global warming is originally introduced, the critical level $\hat{V}$ is assumed to be twice that prevailing before the Industrial Revolution, i. e., $\hat{V}$=560 GtC.

For the impact index function $\varphi(V)$ of the form (2), the impact coefficient $\tau(V)$ is given by

$$\tau(V) = \frac{\beta}{\hat{V} - V}.$$

We assume that the neoclassical assumptions are satisfied for utility functions $u^\nu(c^\nu)$:

(U1') $u^\nu(c^\nu)$ is defined, positive, continuous, and continuously twice-differentiable for all $c^\nu \geqq 0$.

(U2') Marginal utilities are positive for the consumption of goods $c^\nu = (c_j^\nu)$:

$$u_{c^\nu}^\nu(c^\nu) > 0 \ \text{ for all } \ c^\nu \geqq 0.$$

(U3') $u^\nu(c^\nu)$ is strictly quasi-concave with respect to $c^\nu \geqq 0$; that is, for any pair of vectors of consumption, c_0^ν, c_1^ν, such that $u^\nu(c_0^\nu) = u^\nu(c_1^\nu)$ and $c_0^\nu \neq c_1^\nu$,

$$u^\nu[(1-\lambda)c_0^\nu + \lambda c_1^\nu] < (1-\lambda)u^\nu(c_0^\nu) + \lambda u^\nu(c_1^\nu) \ \text{ for all } 0 < \lambda < 1.$$

(U4') $u^\nu(c^\nu)$ is homogeneous of order one with respect to c^ν:

$$u^\nu(\lambda c^\nu) = \lambda u^\nu(c^\nu) \ \text{ for all } \lambda \geqq 0, \ c^\nu \geqq 0.$$

The Euler identity will play a crucial role in the following analysis.

$$u^\nu(c^\nu) = u_{c^\nu}^\nu(c^\nu)c^\nu \ \text{ for all } c^\nu \geqq 0.$$

Specifications for Production Possibility Sets

The conditions concerning the production of goods in each country ν are specified by the production possibility set T^ν.

Those factors of production that are essentially needed in the production of goods are generically denoted by ℓ ($\ell = 1, \cdots, L$). The endowments of factors of production available in each country ν are expressed by a vector $K^\nu = (K_\ell^\nu)$.

In the dynamic situation, the endowments of factors of production available in each country ν at each time are endogenously determined in the sense that they are the outcomes of the choices made in the past by the members of the economy. In this paper, however, we assume that the endowments of factors of production are regarded as given, focusing our attention upon the atmospheric accumulations of CO_2 and their implications upon the natural and human environments.

The conditions concerning the production of goods in each country ν are specified by the production possibility set T^ν that summarizes the technological possibilities and organizational arrangements for country ν, with the quantities of factors of production available in country ν as given. The main conclusions of this paper, however, remain valid under the general circumstances where the quantities of factors of production change over time.

In country ν, the minimum quantities of factors of production that are needed to produce goods by the vector of production $x^\nu = (x_j^\nu)$ with the CO_2 emission at the level a^ν are specified by a vector-valued function $f^\nu(x^\nu, a^\nu) = (f_\ell^\nu(x^\nu, a^\nu))$. The production possibility set T^ν is then given by

$$T^\nu = \left\{(x^\nu, a^\nu):\ (x^\nu, a^\nu) \geqq 0,\ \ f^\nu(x^\nu, a^\nu) \leqq K^\nu\right\}.$$

We assume that marginal rates of substitution between the production of goods and the emission of CO_2 are smooth and diminishing, there always exist trade-offs between the production of goods and the emission of CO_2, and the conditions of constant returns to scale prevail. That is, we assume that

(T1) $f^\nu(x^\nu, a^\nu)$ are defined, positive, continuous, and continuously twice-differentiable for all $(x^\nu, a^\nu) \geqq 0$.

(T2) $f_{x^\nu}^\nu(x^\nu, a^\nu) > 0,\ \ f_{a^\nu}^\nu(x^\nu, a^\nu) \leqq 0$ for all $(x^\nu, a^\nu) \geqq 0$.

(T3) $f^\nu(x^\nu, a^\nu)$ are strictly quasi-convex with respect to (x^ν, a^ν) for all $(x^\nu, a^\nu) \geqq 0$.

(T4) $f^\nu(x^\nu, a^\nu)$ are homogeneous of order one with respect to (x^ν, a^ν); that is,

$$f^\nu(\lambda x^\nu, \lambda a^\nu) = \lambda f^\nu(x^\nu, a^\nu) \ \text{ for all } \lambda \geqq 0,\ (x^\nu, a^\nu) \geqq 0.$$

Postulates (T1-3), as specified above, imply that the production possibility set T^ν is a closed, convex set of vectors (x^ν, a^ν).

Market Equilibrium and Global Warming

We consider the situation where carbon taxes at the rate of θ^ν are levied upon the emission of CO_2 in each country ν.

Market equilibrium for the world economy is obtained if we find the prices of goods p at which total demand is equal to total supply:

$$\sum_\nu c^\nu = \sum_\nu x^\nu .$$

Total CO_2 emissions, a, are given by

$$a = \sum_\nu a^\nu .$$

Supply and demand conditions are specified in the following manner.

(i) Supply conditions in each country ν are obtained by maximizing net profits

$$px^\nu - \theta^\nu a^\nu$$

over $(x^\nu, a^\nu) \in T^\nu$; that is, subject to the constraints that

$$f^\nu(x^\nu, a^\nu) \leqq K^\nu .$$

(ii) Demand conditions in each country ν are obtained by maximizing utility function

$$u^\nu(c^\nu, V) = \varphi(V) u^\nu(c^\nu)$$

subject to budget constraints:

$$pc^\nu = y^\nu , \ \ c^\nu \geqq 0,$$

where y^ν is per capita national income of country ν:

The optimum levels of production and CO_2 emissions in country ν are determined by the following marginality conditions:

$$p = r^\nu f^\nu_{x^\nu}(x^\nu, a^\nu) \tag{3}$$

$$\theta^\nu = r^\nu[-f^\nu_{a^\nu}(x^\nu, a^\nu)], \tag{4}$$

where r^ν is the vector of rentals prices of factors of production in country ν.

Because the technologies are assumed to be subject to constant returns to scale, (T4), we have that

$$p x^\nu - \theta^\nu a^\nu = r^\nu [f^\nu_{x^\nu}(x^\nu, a^\nu) x^\nu + f^\nu_{a^\nu}(x^\nu, a^\nu) a^\nu] = r^\nu f^\nu(x^\nu, a^\nu) = r^\nu K^\nu .$$

That is, the net evaluation of output is equal to the sum of the rentals payments to all factors of production.

When all factors of production are owned by the individual members of the country v, then national income y^v of country v is equal to the sum of the rentals payments $r^v K^v$ and the tax payments $\theta^v a^v$ made by the producers for the emission of CO_2 in country v ; that is,

$$y^v = r^v K^v + \theta^v a^v = (p x^v - \theta^v a^v) + \theta^v a^v = p x^v .$$

That is, national income y^v of country v is equal to the value of outputs in units of market prices, px^v.

The optimum vector of consumption c^v is obtained by solving the following marginality conditions:

$$\varphi(V) u^v_{c^v}(c^v) = \lambda^v p,$$

where λ^v is the Lagrange unknown associated with the budgetary constraints.

3. Deferential Equilibrium

One of the basic problems we face in the dynamic analysis of global warming is to see how the carbon tax rate θ^ν in each country ν is determined. Our analysis will be carried out in terms of the concept of deferential equilibrium, originally introduced in [14].

Deferential equilibrium is obtained if, when each country chooses the levels of economic activities today, it takes into account the negative impact upon the future levels of its own utilities brought about by the CO_2 emissions of that country today.

Consider the situation in which a combination (x^ν, a^ν) of production vector x^ν and CO_2 emissions a^ν is chosen in country ν. Suppose CO_2 emissions a^ν in country ν are increased by a marginal amount. This would induce a marginal increase in the aggregate amount of CO_2 emissions in the world, effecting a marginal increase in the atmospheric level of CO_2. The resulting marginal increase in the degree of global warming would induce a marginal decrease in country ν's utility. Deferential equilibrium is obtained if this marginal decrease in country ν's future utility due to the marginal increase in CO_2 emissions in country ν today is taken into consideration in determining the levels of consumption, production, and CO_2 emissions today.

The marginal decrease in country ν's utility due to the marginal increase in CO_2 emissions in country ν is given by the partial derivative, with minus sign, of utility function $u^\nu(c^\nu, V)$ of country ν with respect to atmospheric accumulations of CO_2, V; that is,

$$-\frac{\partial u^\nu}{\partial V} = -\varphi'(V)u^\nu(c^\nu) = \tau(V)\varphi(V)u^\nu(c^\nu),$$

where $\tau(V)$ is the impact coefficient of global warming.

We assume that future utilities of country ν are discounted at the social rate of discount δ that is exogenously given. We also assume that the rate of discount δ is a positive constant and is identical for all countries in the world. As atmospheric carbon dioxide is annually absorbed at the rate μ, the imputed price ψ^ν of the atmospheric accumulations of CO_2 of each country ν is given by the discounted present value of the marginal decrease in country ν's utility due to the marginal increase in CO_2 emissions in country ν; that is,

$$\psi^{\nu} = \frac{1}{\delta+\mu}[-\frac{\partial u^{\nu}}{\partial V}] = \frac{1}{\delta+\mu}\tau(V)\varphi(V)u^{\nu}(c^{\nu}). \tag{5}$$

Deferential equilibrium is obtained if this marginal decrease in country ν's future utility due to the marginal increase in CO_2 emissions today in country ν is balanced with the marginal increase in country ν's utility due to the resulting marginal increase in the level of production.

If the level of CO_2 emissions in country ν is a^{ν}, then the net level of country ν's utility is given by

$$\varphi(V)\,u^{\nu}(c^{\nu}) - \psi^{\nu} a^{\nu}.$$

The choice of the levels of consumption, production, and CO_2 emissions for each country ν under the deferential behavioristic postulates then may be viewed as the optimum solution to the following maximum problem:

Find the combination $(c^{\nu}, x^{\nu}, a^{\nu})$ of consumption vector c^{ν}, production vector x^{ν}, and CO_2 emissions a^{ν} that maximizes the net level of country ν's utility

$$\varphi^{\nu}(V)u^{\nu}(c^{\nu}) - \psi^{\nu} a^{\nu}$$

subject to the constraints that

$$pc^{\nu} = px^{\nu},\ c^{\nu} \geqq 0 \tag{6}$$

$$f^{\nu}(x^{\nu}, a^{\nu}) \leqq K^{\nu}. \tag{7}$$

The maximum problem is solved in terms of the Lagrangian form:

$$L^{\nu}(c^{\nu}, x^{\nu}, a^{\nu}; \lambda^{\nu}, \lambda^{\nu} r^{\nu})$$
$$= \varphi^{\nu}(V)\,u^{\nu}(c^{\nu}) - \psi^{\nu} a^{\nu} + \lambda^{\nu}(px^{\nu} - pc^{\nu}) + \lambda^{\nu} r^{\nu}[K^{\nu} - f^{\nu}(x^{\nu}, a^{\nu})],$$

where V and ψ^{ν} are given. The variables λ^{ν}, $\lambda^{\nu} r^{\nu}$ are the Lagrangian unknowns associated with constraints (6) and (7), respectively.

The optimum conditions are:

$$\varphi^v(V)\,u^v_{c^v}(c^v) = \lambda^v p$$

$$\lambda^v p = \lambda^v r^v f^v_{x^v}(x^v, a^v)$$

$$\psi^v = \lambda^v r^v [-f^v_{a^v}(x^v, a^v)].$$

When expressed in units of market prices, the optimum conditions are

$$\alpha^v \varphi^v(V)\,u^v_{c^v}(c^v) = p \tag{8}$$

$$p = r^v f^v_{x^v}(x^v, a^v) \tag{9}$$

$$\theta^v = r^v [-f^v_{a^v}(x^v, a^v)], \tag{10}$$

where α^v is the coefficient to convert values in units of the utility to those in units of market prices

$$\alpha^v = \frac{1}{\lambda^v} > 0,$$

and θ^v is the imputed price of atmospheric accumulations of CO_2 in units of market prices

$$\theta^v = \alpha^v \psi^v,$$

which, by substitution into (5), implies

$$\theta^v = \frac{1}{\delta + \mu}\tau(V)\alpha^v \varphi^v(V)\,u^v(c^v). \tag{11}$$

By multiplying both sides of relations (8) by c^v,

$$\alpha^v \varphi^v(V)\,u^v_{c^v}(c^v)c^v = pc^v.$$

By making use of the Euler identity and noting the budget constraints (6), we obtain

$$\alpha^v \varphi^v(V)\,u^v(c^v) = y^v, \tag{12}$$

where y^v is per capita national income of country v :

$$y^v = px^v.$$

In view of (11) and (12), we obtain

$$\theta^v = \frac{\tau(V)}{\delta + \mu} y^v. \tag{13}$$

Marginality conditions (9) and (10) imply that net profits

$$p\,x^v - \theta^v a^v$$

are maximized over the technological possibility set $(x^v, a^v) \in T^v$, where carbon tax rate θ^v is given by (13).

Hence, the conditions for deferential equilibrium of global warming are identical with those for the standard market equilibrium when carbon taxes with rate θ^ν given by (13) are levied; that is,

(i) For each country ν, the consumption vector c^ν maximizes country ν's utility function

$$u^\nu(c^\nu, V) = \varphi^\nu(V)\, u^\nu(c^\nu)$$

subject to the budgetary constraints

$$pc^\nu = y^\nu,\ \ c^\nu \geqq 0,$$

where y^ν is per capita national income of country ν, and the level of atmospheric accumulations of CO_2, V, is assumed to be given.

(ii) For each country ν, the combination (x^ν, a^ν) of production vector x^ν and CO_2 emissions a^ν maximizes net profits

$$px^\nu - \theta^\nu a^\nu$$

over the technological possibility set $(x^\nu, a^\nu) \in T^\nu$.

Differential equilibrium for the world economy then is obtained if we find the prices of goods p at which total demand is equal to total supply:

$$\sum_\nu c^\nu = \sum_\nu x^\nu$$

and total CO_2 emissions a are given by

$$a = \sum_\nu a^\nu .$$

The preceding discussion may be summarized by the following proposition.

Proposition 1. *Deferential equilibrium corresponds precisely to the competitive market equilibrium under the system of proportional carbon taxes, where, in each country* ν, *the carbon taxes are levied at the rate* θ^ν *that is proportional to the per capita national income* y^ν *of each country* ν, *with the discounted present value* $\frac{\tau(V)}{\delta + \mu}$ *of the impact coefficient of global warming* $\tau(V)$ *as the coefficient of proportion; that is,*

$$\theta^{\nu}=\frac{\tau(V)}{\delta+\mu}y^{\nu},$$

where $\tau(V)$ *is the impact coefficient of global warming,* δ *is the social rate of discount, and* μ *is the rate at which atmospheric* CO_2 *is annually absorbed by the oceans.*

4. Imputed Price and Sustainable Development

In the dynamic analysis of global warming, a crucial role is played by the concept of the imputed price of the atmospheric accumulations of CO_2. The imputed price ψ_t^{ν} of the atmospheric accumulations of CO_2 in units of utility for country ν at time t expresses the extent to which the marginal increase in the atmospheric accumulations of CO_2 at time t induces the marginal decrease in the welfare level of country ν, including those of all future generations. The calculation of the magnitude of the welfare gains may be made along the lines similar to those discussed in the previous sections.

Suppose ψ_t^{ν} is the price in units of utility charged to the emission of CO_2 in country ν at time t. The resulting pattern of consumption, production, and the level of CO_2 emissions in country ν are obtained as the optimum solution for the following maximum problem:

Find the combination $(c^{\nu}, x^{\nu}, a^{\nu})$ of consumption vector c^{ν}, production vector x^{ν}, and CO_2 emissions a^{ν} that maximizes the net level of country ν's utility

$$\varphi^{\nu}(V)u^{\nu}(c^{\nu})-\psi^{\nu}a^{\nu}$$

subject to the constraints that

$$pc^{\nu}=px^{\nu},\ c^{\nu}\geqq 0$$

$$f^{\nu}(x^{\nu}, a^{\nu})\leqq K^{\nu}.$$

The maximum problem is solved exactly in the same manner as for the case of deferential equilibrium. The marginal decrease m_t^ν in the welfare level of country ν at time t induced by the marginal increase in the atmospheric accumulations V_t of CO_2 at time t is given by

$$m_t^\nu = -\frac{\partial}{\partial V_t}[\varphi^\nu(V_t)\, u^\nu(c_t^\nu)] = \tau(V_t)\varphi^\nu(V_t)\, u^\nu(c_t^\nu), \tag{14}$$

which, in units of market prices, may be expressed as

$$\alpha_t^\nu m_t^\nu = \tau(V_t)\alpha_t^\nu\varphi^\nu(V_t)\, u^\nu(c_t^\nu) = \tau(V_t) y_t^\nu, \tag{15}$$

where y^ν is per capita national income of country ν at time t.

Let us denote by π_t^ν the imputed price of the atmospheric accumulations of CO_2 in units of market prices for country ν at time t; that is, $\pi_t^\nu = \alpha_t^\nu \psi_t^\nu$. Then, by noting (15), we have from the definition of the imputed price that

$$\pi_t^\nu = \int_t^\infty \alpha_\tau^\nu m_\tau^\nu e^{-(\delta+\mu)(\tau-t)} d\tau = \int_t^\infty \tau(V_\tau) y_\tau^\nu e^{-(\delta+\mu)(\tau-t)} d\tau. \tag{16}$$

By differentiating both sides of (16) with respect to time t, we obtain the following differential equation:

$$\dot{\pi}_t^\nu = (\delta+\mu)\pi_t^\nu - \tau(V_t) y_t^\nu. \tag{17}$$

Differential equation (17) is nothing but the Euler-Lagrange differential equation in the calculus of variations or the Ramsey-Keynes equation in the theory of optimum economic growth.

To clarify the intrinsic meaning of the imputed price of the atmospheric accumulations of CO_2, we introduce a virtual capital market at time t that is perfectly competitive and the atmospheric accumulations of CO_2 are transacted as an asset. The imputed price π_t^ν at time t is identified with the market price. Consider the situation in which the unit of such as asset is held for a short time period $[t,\ t+\Delta t]$, ($\Delta t > 0$). The gains obtained by holding such an "asset" are composed of "capital gains" $\Delta\pi_t^\nu = \pi_{t+\Delta t}^\nu - \pi_t^\nu$ and "earnings" $\tau(V_t) y_t^\nu \Delta t$, minus "depreciation charges" $\mu\pi_t^\nu \Delta t$; that is,

$$\Delta\pi_t^\nu + \tau(V_t) y_t^\nu \Delta t - \mu\pi_t^\nu \Delta t.$$

On other hand, the cost of holding such an "asset" for the time period $[t,\ t+\Delta t]$ is "interest" payment $\delta\pi_t^v\Delta t$, where the social rate of discount is identified with the " market rate of interest". Hence, on the virtual capital market, these two amounts become equal; that is,

$$\Delta\pi_t^v + \tau(V_t)y_t^v\Delta t - \mu\pi_t^v\Delta t = \delta\pi_t^v\Delta t,$$

which, by dividing both sides by Δt and taking the limit as $\Delta t \to 0$, yields differential equation (17).

It may be noted that the time derivative in the differential equation (17) refers to the fictitious time of the virtual capital market at time t and remains valid only at time t. For any future time τ ($\tau > t$), equation (17) has to be modified by taking into account the changing circumstances that are relevant in the determination of market equilibrium.

We define that the imputed price π_t^v of the atmospheric accumulations of CO_2 for country v is at the sustainable level at time t, if $\dot{\pi}_t^v = 0$ at time t. That is,

$$\pi_t^v = \theta_t^v, \quad \theta_t^v = \frac{\tau(V_t)}{\delta + \mu} y_t^v \quad \text{at time } t.$$

A time-path (V_t) of the atmospheric accumulations of CO_2 is defined sustainable for country v, if the imputed price π_t^v for country v is at the sustainable level at all time t. It is defined sustainable, if it is sustainable for all countries in the world.

The discussion above may be summarized as the following proposition:

Proposition 2. *Sustainable time-paths* (V_t) *of the atmospheric accumulations of* CO_2 *are obtained as the competitive market equilibrium under the system of proportional carbon taxes, where, in each country* v, *the carbon taxes are levied at the rate* θ^v *that is proportional to the per capita national income* y^v *of each country* v, *with the discounted present value* $\frac{\tau(V)}{\delta + \mu}$ *of the impact coefficient of global warming* $\tau(V)$ *as the coefficient of proportion; that is,*

$$\theta^v = \frac{\tau(V)}{\delta + \mu} y^v,$$

where $\tau(V)$ *is the impact coefficient of global warming,* δ *is the social rate of discount, and*

μ is the rate at which atmospheric CO_2 is annually absorbed by the oceans.

5. A Hypothetical Case

A hypothetical case of the incidences of the proportional carbon taxes with the coefficient of proportion 0.01 is presented in terms of the statistical data of 2005 (in US$). The data include all radiative forcing agents (RFA).

Country	National Income per capita	RFA Increase per Capita (Ct)	Imputed Price per Ct	Carbon Taxes per capita
United States	42,000	5.90	420	2,500
Canada	34,000	6.20	340	2,100
United Kingdom	32,000	3.00	320	950
France	31,000	2.20	310	680
Germany	31,000	3.20	310	980
Italy	28,000	2.20	280	600
Netherlands	35,000	3.60	350	1,200
Sweden	32,000	1.90	320	610
Denmark	34,000	3.20	340	1,100
Indonesia	3,100	1.70	30	50
Japan	31,000	2.70	310	840
Korea	21,000	2.60	210	560
Malaysia	11,000	1.90	110	210
Philippines	3,200	0.30	30	8
Singapore	40,000	3.20	400	1,300
Thailand	6,900	1.20	70	80
India	2,200	0.30	20	7
China	4,100	1.10	40	40
Australia	33,000	7.10	330	2,300
New Zealand	23,000	3.50	230	790

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