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A Dynamic Equilibrium Model for Relationship-Lending

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Abstract

In this paper, we develop a dynamic equilibrium model of the banking industry that takes into account for firm and bank heterogeneity and the market for new and existing loans with a full description for the demand and supply of bank loans. The consequences of various shocks to the economy are considered through the equilibrium analysis. The model predicts that as the degree of interbank competition becomes higher, the number of loan relations for each firm becomes smaller. Our model is also consistent with several empirical regularities regarding the loan structure.

Key words: Relationship-lending; Dynamic model; Stationary equilibrium
JEL Classification: G21, G32, D53, C78

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1 Introduction

The dominant presence of bank financing in the capital and debt structures of firms has stimulated various theoretical and empirical studies about the role of banks as a financing source. In the literature, one distinctive feature associated with bank loans is characterized as "relationship-lending"; that is, bank financing accompanied with some over-time informational activity (Freixas and Rochet (2008)).[1] Existing theoretical studies on relationship-lending have provided interesting implications regarding, for example, the dynamics of loan prices (Rajan (1992)) and the implications of better credit information about borrowers (Padilla and Pagano (1997), Hauswald and Marquez (2003)). The empirical literature has also examined the implications of relationship-lending (James (1987); Peterson and Rajan (1994); Berger and Udell (1995, 2006)).

The main mechanism and several implications originating from relationship-lending are relatively well understood in the literature. Yet the question remains as to how different degrees of competitiveness in the banking industry affect the firms' loan structure in the context of relationship-lending. Since traditional banking institutions have recently become more competitive and subject to replacement by transaction-based institutions, it is important to be able to answer this question. The motivation of this paper is to develop a theoretical relationship-lending model to study this question.

To illustrate the main mechanism of our model, consider an economy where a firm has a set of young venture projects and established projects, both of which are financed by separate banks. When the firm faces an opportunity to upgrade one of its venture projects to an established project (e.g., expanding their sales area or augmenting a new manufacturing line for a specific product), it may need additional financing and expert advice. Presumably, the bank holding an incumbent loan relation with this firm (for the original venture project) is in an advantageous position to provide such financing and advice. We interpret this as relationship-lending. Rajan (1992) demonstrates that banks might even be willing to offer a discounted interest rate for the venture project so that they can establish a loan relationship that eventually gives them some expertise. After obtaining this expertise, banks can charge a higher interest rate. In contrast to Rajan (1992), we develop a model in which the banks' rent in the later stage of the relationship is determined as an equilibrium object. We do this by modeling the demand and supply for the start-up loan as the solutions to the dynamic optimization problems of firms and banks. The loan spread between the start-up loan for a venture project and the follow-up loan for an established project governs the demand and supply, and in equilibrium, the loan spread is endogenously determined to clear the bank loan market. This is the basic mechanism incorporated in this paper. For this dynamic model, we employ an analytically tractable dynamic programming problem with discrete state variables developed in Klette and Kortum (2004). One important difference is that we have two sides corresponding to the demand and supply of the loan. In order to close the model, we also employ a technique that is similar to Mortensen and Pissarides (1994)

[1] Ex-ante screening and/or ex-post monitoring are the most typical informational activities and "over-time" implies that these activities are implemented at some point(s) over their relationships.

in order to characterize a stationary equilibrium in a dynamic search framework.

Compared to existing models, our model has two distinctive features. First, our model contains the individual optimization problems for the demand and supply of bank loans. Most existing papers generally abstract from the demand side of bank loans by assuming that demand is inelastic (Boot and Thakor (2000)). Thus these models cannot analyze demand-side repercussions caused by various economic shocks. As we will see later, this channel generates interesting results. Second, our model employs a dynamic equilibrium framework that complements the multi-stage game theoretic framework used in existing studies. This approach allows us to endogenously determine the effects of several economic shocks to loan prices. Firms and banks choose their optimal project and loan portfolios by considering the costs and benefits associated with their choices. Our dynamic equilibrium framework explicitly captures this mechanism. Through the equilibrium analysis, we can study the individual behaviors of firms and banks, as well as analyze the implications of these behaviors at the industry-level.

In this paper, we also construct the model to be consistent with several empirical regularities associated with the structure of bank loans. A number of existing empirical studies (Miyakawa (2008b)) have documented that firms tend to diversify their loans across banks and that loan relationships respond to various covariates (e.g., firm size, firm profitability, and bank size) in a systematic way. As far as we know, there is no theoretical model that comprehensively considers these features. Our paper provides a potential underlying mechanism for such multiple loan structures.

Following the literature on relationship-lending, our model features an intangible capital that accumulates between firms and banks over time. This modeling strategy reflects the traditional view that banks have the ability to search for better projects within the firms' pool of uncertified projects (Greenwood and Jovanovic (1990))[2]. It also reflects a relatively new perspective that banks that have close and sustained relations with a particular firm has better visibility to screen the firm's projects (Hauswald and Marquez (2003)).[3]

This paper is structured as follows. Section 2 surveys the related literature. Sections 3 and 4 presents the model and describes the equilibrium. Section 5 implements comparative statics to study the implications of the model. Section 6 simulates the model and demonstrates some quantitative comparative statics. Section 7 concludes and presents future research questions.

[2] In banking literature, this screening ability is recognized as an important feature of banks along with a monitoring ability. The former corresponds to an ex-ante activity and the latter is a ex-post activity.

[3] There is an accumulation of empirical studies on this relationship-banking view. See Peterson and Rajan (1994); Berger and Udell (1995); Degryse and Cayseele (2000).

2 Related Literature

The role of banks has been discussed in two classic papers. First, Greenwood and Jovanovic (1990) takes the view that banks have an ex-ante screening ability. The main idea is that since banks have a large pool of clients and investment opportunities, they have an advantage over firms in searching for profitable projects. Second, Diamond (1984, 1996) points to the banks' ex-post monitoring capabilities and argues that delegating this monitoring to the banks helps avoid the duplication of monitoring costs among investors.[4] Our model is based on the former view and abstracts from the one associated with ex-post monitoring.

In the context of relationship-lending, there are two closely related studies.[5] First, Rajan (1992) models a bank's dynamic loan provision. In his model, each bank needs to pay out once-and-for-all sunk cost in monitoring for providing a loan to a firm. The existence of this sunk cost gives rise to a standard switching cost since the party incurs a duplicated monitoring cost when they switch banks.[6] If we associate lower monopoly rents with a low degree of interbank competition, the model in Rajan (1992) has the implication that higher inter-bank competition leads to smaller numbers of loan relations. Our model shares the persistent effect related to switching costs and our main result follows the perspective of his model. In our setup, the existing loan relation allows banks and firms to implement a better project. This has a similar effect to the once-and-for-all sunk costs. Note that the main contribution of his paper is to model the ex-post monopoly of information and the resulting dynamic price pattern, which starts at a discount and increases later in the relationship. This result is similar to conventional switching cost models. In contrast, we are mainly interested in the resulting loan structure corresponding to several shocks. Moreover, unlike his two-period game theoretic framework, our model employs a dynamic equilibrium setup which allows us to endogenously determine the loan price. Second, Hauswald and Marquez (2003) model the emergence of relationship banking based on the perspective of incumbent lenders' accumulated ability to screen projects, which is also employed in our model. The main contribution of their paper is to show that the effects of better private information, obtained through relationship-lending, has the opposite effect of better public information. Intuitively, if the importance of private information is quite high, the ex-post monopoly problem considered in Rajan (1992) becomes severe while better public information alleviates such a problem. Our model takes a similar view to their model about banks' informational activity. The main difference is that we allow for multiple loan relationships, which is one of the key issues that we want to consider.

[4] Several papers (e.g., Boyd and Prescott (1990)) discuss the potential contracting mechanism allowing this delegation to work. In this paper, we abstract from this contracting problem.

[5] Also, there are many empirical papers that attempt to establish evidence for relationship lending (James (1987); Peterson and Rajan (1994); Berger and Udell (1995, 2006)). Considering the characteristics of this paper, we omit surveying these empirical papers. For a comprehensive survey, see Freixas and Rochet (2008).

[6] This informational activity modeled in Rajan (1992) is associated with the ex-ante cost. Although this is called as a monitoring cost in his paper, we should categorize it as one variant of the ex-ante screening model.

3 Model

In this section, we construct a dynamic equilibrium model for relationship-lending. In our model, we endogenously determine (i) the number of banks from which each firm borrows and (ii) the status of each bank loan relationship between firms and banks.

Several important issues discussed in the existing corporate finance and banking literature are omitted. First, we treat each loan contract as a given object and do not discuss a general optimal contract from which such loan contract emerges.[7] Second, we treat the bank loan as an exclusive financing channel and ignore other financing options (e.g., equity or any other hybrid securities).[8] Third, there is no information asymmetry between firms and banks. The only friction we consider is that each firm and bank needs to incur search costs to initiate their loan relationships and can upgrade the project to a more profitable one only if they continue the incumbent loan relation. This over-time informational activity, originally introduced by Rajan (1992) and extended in Hauswald and Marquez (2003), is the mechanism we employ.

3.1 Environment

Consider an economy consisting of a continuum of risk-neutral firms and banks. Each firm is represented by a set of projects. We assume that each firm does not originally have any financial resources to implement a project and needs to use bank loans to finance each project. For tractability, we also assume that each project needs to be financed by a different bank. This setup implies that a firm's project portfolio choice is equivalent to its borrowing structure.

There are two types of projects: uncertified venture-projects (U-P) and certified established-project (C-P), where (C-P) yields a higher revenue than (U-P). With some positive probability, (U-P) can be upgraded to a (C-P) in the future. We assume that this economy has a fixed unit mass of projects in total. The relative aggregate shares of (C-P) and (U-P) in this economy is endogenously determined. One interpretation of this assumption is that firms are competing for limited business opportunities. In order to implement a project, each firm needs to snatch another firm's current client.

We also assume that in order to implement one (C-P), each firm needs to start from one (U-P) and then use a loan from the bank that financed this particular (U-P). Thus, firms need to take another loan from their incumbent bank to upgrade the project. This corresponds to our notion of relationship-lending which requires time to exhibit a better project screening ability.

[7] Various frictions might be able to justify the debt contract as claimed in incomplete contract literature (Hart and Moore (1998)). Miyakawa (2008a) discusses this issue from a complete contract context.

[8] Coexistence of debt and equity in a firm's capital structure is one important issue as discussed in, for example, Holmstrom and Tirole (1997).

3.1.1 Technology

The initiation of a project requires inputs from both the firm and bank as well as a loan from the bank. This can be interpreted as the provisions of project idea from the firm side and screening/monitoring capital from the bank side.[9]

Each (U-P) provides a deterministic outcome $\pi_u > 0$ and each (C-P) yields a strictly larger deterministic outcome $\pi_c > \pi_u$ over the next instantaneous moment. In this sense, the difference $(\pi_c - \pi_u)$ summarizes the value of the certification.

We assume that each firm incurs a cost to find a new (U-P). A new (U-P) arrives at Poisson rate $\Lambda(e, n_c)$. The variable e represents the firm's search cost, a choice variable, and n_c denotes the number of (C-P) held by the firm. Intuitively, a firm with a larger number of (C-P) and/or exerts a larger search cost is more likely to find an additional (U-P) over the next instantaneous moment. We assume $\Lambda(e, n_c)$ is homogenous of degree one in e and n_c, and $\Lambda(0, 0) = 0$.[10]

3.1.2 Bank Loan

As with firms, each bank is represented by its loan portfolio. For tractability, we also assume that each bank can offer at most one loan relation to each firm. There are two types of loans: Start-up loan for an uncertified project (U-L) and follow-up loan for a certified project (C-L). In this paper, we interpret (C-L) as relationship-lending.

To implement one (U-P), it suffices for a firm to use a start-up loan (U-L). For simplicity, we assume that (U-L)'s face interest factor is an exogenous number $R \equiv 1 + r < \pi_u$. Since the project outcome is deterministic, R can be interpreted as a risk-free interest rate factor.[11] To implement (C-P), however, each firm needs to use a follow-up loan (C-L) from its incumbent bank (i.e., the same bank provided (U-L) to the (U-P)), which has a spread $\overline{\gamma} > 0$ over the risk-free interest factor. This spread is endogenously determined in equilibrium later. This setup also implies that each bank cannot directly provide (C-L) but needs to start from (U-L).

In order to find a new opportunity to provide (U-L), the bank needs to exert some search cost. A new (U-L) arrives at Poisson rate $\Phi(d, m_f)$. The variable d denotes the bank's search effort, which is a bank's choice variable, and m_f denotes the number of (C-L) held by the bank. We assume $\Phi(d, m_f)$ is homogenous degree one in d and m_f, and $\Phi(0, 0) = 0$.

[9] In this sense, we are assuming the Leontief production function which has the firm's idea and bank's screening/monitoring capital as its inputs.

[10] This specification is widely used in the search literature. Klette and Kortum (2004) also employs this assumption.

[11] We abstract from the discount on the interest rate associated with (U-L), which is considered in Rajan (1992), and focus on the spread between (U-L) and (C-L).

3.1.3 Evolution of Match

Once a firm and bank match, the project implemented by this match gives a stream of profits according to the following stochastic process: (i) One (U-P) match upgrades to one (C-P) match with an exogenous Poisson rate $\theta \geq 0$, (ii) one (U-P) drops from the project/loan portfolio with a Poisson rate $\sigma \geq 0$ which is treated as given in an individual firm/bank problem but is endogenously determined in equilibrium, and (iii) one (C-P) match degrades into one (U-P) match with an exogenous Poisson rate $\delta \geq 0$. We can interpret (iii) as a break-up of the long-term loan relationship.

Note that θ, δ, and σ are common to both firms and banks. Thus, we can interpret these three parameters as Poisson rates for each match. Once they are matched through a (U-P), these three parameters govern the dynamics of the match.

3.1.4 Firm's Project Portfolio

Each firm's current project portfolio consists of n_c (C-P) with project size 1 and n_u (U-P) with project size 1, both of which have already been held by the firm at the beginning of a period. Since (n_c, n_u) are predetermined, they are treated as state variables in the firm's dynamic optimization problem. One interpretation of this timing assumption is that the menu of projects has been established via past business activities and each firm needs to choose the search effort for an additional uncertified project for its future project portfolio. We treat a firm in state $n_c = 0$ and $n_u = 0$ as having permanently exited from the economy.

3.1.5 Bank's Loan Portfolio

The structure of each bank's loan portfolio takes a similar structure to the firm's project portfolio. It consists of the financing of m_c (C-P)'s with loan size 1 and m_u (U-P)'s with loan size 1, both of which are state variables. We treat a bank in state $m_c = 0$ and $m_u = 0$ as having permanently exited from the economy.

3.1.6 Market Structure

Without loss of generality, we assume that banks with lending capacities post offers for new contracts that involve the following rates of interest: (i) R when the project is uncertified, and (ii) $R+\overline{\gamma}$ when it becomes certified. Perfect competition determines the spread $\overline{\gamma}$ so as to equate the demand and supply of the loan for new uncertified projects as described later.[12]

[12] Precisely speaking, the rate of interest $R+\overline{\gamma}$ determines how to split the surplus from (C-P) between the firm and bank. Since our model contains only one degree of freedom associated with the loan price as we will see later, we fix R and endogenously determine $\overline{\gamma}$.

3.2 Individual Optimization Problems

Each firm maximizes its expected discounted life-time profit $V(n_c, n_u)$ by choosing optimally the search effort e. Here, from the assumption that Λ is homogeneous of degree one in (e, n_c), we can obtain the following convenient expression (1) in which we assume that each firm's Poisson rate $\lambda\left(\frac{e}{n_c}\right) \equiv \Lambda\left(\frac{e}{n_c}, 1\right)$ is strictly increasing and concave in its search intensity $\frac{e}{n_c}$.

$$\Lambda(e, n_c) = n_c \lambda\left(\frac{e}{n_c}\right) \tag{1}$$

By using this expression, the dynamic optimization problem for each firm can be constructed. For given $(\pi_c, \pi_u, R, \theta, \sigma, \delta, \overline{\gamma})$, each firm with (n_c, n_u) solves the following dynamic programming problem.

$$rV(n_c, n_u) = \max_e \left\{ \begin{array}{l} [\pi_c - (R + \overline{\gamma})]\, n_c + [\pi_u - R]\, n_u - e \\ +\theta n_u [V(n_c + 1, n_u - 1) - V(n_c, n_u)] \\ +\sigma n_u [V(n_c, n_u - 1) - V(n_c, n_u)] \\ +\delta n_c [V(n_c - 1, n_u + 1) - V(n_c, n_u)] \\ +n_c \lambda\left(\frac{e}{n_c}\right) [V(n_c, n_u + 1) - V(n_c, n_u)] \end{array} \right\} \tag{F-P}$$

Similarly, the dynamic optimization problem for each bank's value $W(m_c, m_u)$ can be constructed. Let $\phi\left(\frac{d}{m_c}\right) \equiv \Phi\left(\frac{d}{m_c}, 1\right)$ where each bank's Poisson rate ϕ is assumed to be strictly increasing and concave in its search intensity $\frac{d}{m_c}$. For given $(\pi_c, \pi_u, R, \theta, \sigma, \delta, \overline{\gamma})$, each bank with (m_c, m_u) solves the following dynamic programming problem.

$$rW(m_c, m_u) = \max_d \left\{ \begin{array}{l} \overline{\gamma} m_c - d \\ +\theta m_u [W(m_c + 1, m_u - 1) - W(m_c, m_u)] \\ +\sigma m_u [W(m_c, m_u - 1) - W(m_c, m_u)] \\ +\delta m_c [W(m_c - 1, m_u + 1) - W(m_c, m_u)] \\ +m_c \phi\left(\frac{d}{m_c}\right) [W(m_c, m_u + 1) - W(m_c, m_u)] \end{array} \right\} \tag{B-P}$$

Reflecting our consideration about exit, a firm exits when $V(0,0) = 0$ and a bank exits when $W(0,0) = 0$. These two problems intend to model the choices of project and loan portfolios, part of which require time to establish. The value of creating a new project is determined by R, $\overline{\gamma}$, and the rates of creation and destruction (θ, σ, δ). Banks make profits from establishing new relations under $\overline{\gamma} > 0$.

3.3 Entry

Assume that there is an unbounded mass of potential entrant firms. Each firm must invest at a rate of $F^f > 0$ in return for a Poisson rate 1 of entering to this economy with a single (U-P). Also, we assume that there is an unbounded mass of potential entrant banks, which could also be interpreted as an alternative investment technology of existing banks, and each entry bank must invest at the rate of $F^b > 0$ in return for a Poisson rate 1 of entering to this economy with a single (U-L). As assumed above, each entry firm needs to displace an existing (U-P) in order to implement its own (U-P). This reflects our assumption that the economy only has a limited number of clients for the products made by the project.

3.4 Aggregate Flow

We are assuming that there is a unit mass of projects in total (i.e., the total number of (C-P) and (U-P) is constant), and each project is taken care of by a single firm and financed by a single bank.

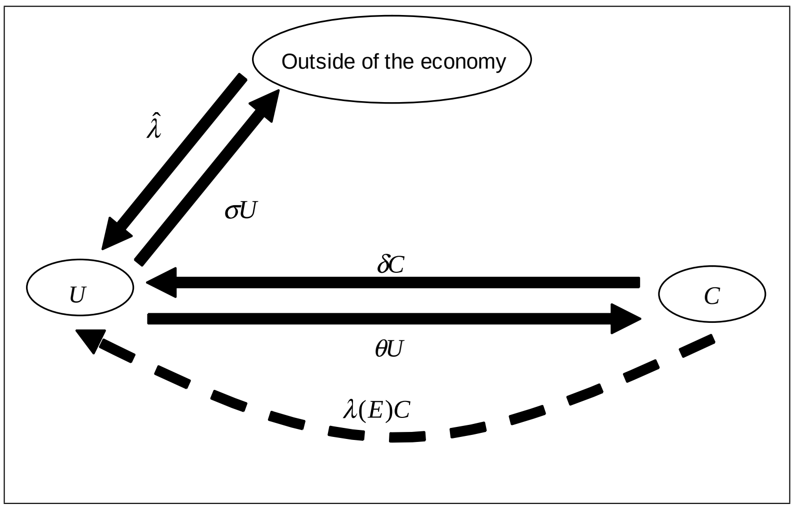

Figure-1: Inflow and Outflow

Let U and C denote the masses of (U-P) and (C-P). This assumption implies that $U + C = 1$, which reflects our assumption about the limited number of clients/project opportunities in this economy. Let $\widehat{\lambda}$ denote the flow of new firms. Figure-1 illustrates the inflow to and outflow from the system. Note that the dashed arrow labeled $\lambda(E)C$ represents the inflow to U provided by the incumbent firms. In the next section, we define the equilibrium of this economy and characterize it.

4 Equilibrium Analysis

4.1 Definition of Stationary Equilibrium

In this section, we establish the equilibrium of this economy and characterize it. First, we define the stationary equilibrium of this economy.

Definition 1 *A stationary equilibrium of this economy involves constant values for (1) the policy* $\left(\lambda(E), \widehat{\lambda}, \phi(D), \widehat{\phi}\right)$, *(2) the allocation* (U, C), *and (3) the price* $\overline{\gamma}$ *such that (i) potential entry firms and banks break even in expectation, (ii) incumbent firms and banks maximize their values, and (iii) the loan market clears.*

In order to characterize the equilibrium, we need to solve the individual optimization problems, derive the break even conditions for entry banks and firms, establish the steady state condition of the economy and the market clearing condition for the loan market.

4.2 Equilibrium Conditions

4.2.1 Solutions for Individual Problems

The solutions for these two problems take convenient linear forms in terms of the state variables. This makes it possible for us to analytically implement some comparative statics. The following lemma summarizes the solutions for the individual problems for each firm and bank.

Lemma 2 *(i) The solution for (F-P) is given by: (a)* $V = An_c + Bn_u$ *and (b)* $e = En_c$ *where* (A, B, E) *are determined by the following conditions*

$$\begin{cases} B = \frac{1}{\lambda'(E)} \\ (r+\delta)A - \delta B - \lambda(E)B - \{\pi_c - (R+\overline{\gamma})\} + E = 0 \\ -\theta A + (r+\theta+\sigma)B - (\pi_u - R) = 0 \end{cases} \tag{FOPT}$$

(ii) The solution for (B-P) is given by: (c) $W = Gm_c + Hm_u$ *and (d)* $d = Dm_c$ *where* (G, H, D) *are determined by the following conditions*

$$\begin{cases} H = \frac{1}{\phi'(D)} \\ (r+\delta)G - \delta H - \phi(D)H - \overline{\gamma} + D = 0 \\ -\theta G + (r+\theta+\sigma)H = 0 \end{cases} \tag{BOPT}$$

Proof. See the appendix. ■

As important variables used for the later discussion, $E \equiv \frac{e}{n_c}$ and $D \equiv \frac{d}{m_c}$ are defined. Note that both of them do not depend on the state variables from the first conditions in (FOPT) and (BOPT).

4.2.2 Break-Even Condition for Potential Entrants

If there is positive entry into this economy in equilibrium, $V(0,1) = F^f$ needs to hold so as to make each entrant break-even in expectation, which implies $B = F^f$. Also, if there is positive entry into this economy in equilibrium, $W(0,1) = F^b$ needs to hold, which implies that $H = F^b$.

4.2.3 Steady State and Market Clearing Conditions

First, the change in U can be expressed as follows.

$$\dot{U} = \min\left(\widehat{\lambda} + \lambda(E)C, \widehat{\phi} + \phi(D)C\right) + \delta C - \sigma U - \theta U$$

The first term on the right-hand side is the aggregate new inflow to the pool of (U-P)-(U-L) matches. Given the technological assumption that requires inputs from both the firm and bank, it is similar to a Leontief production function. The second term accounts for the inflow to the pool of uncertified projects as a result of degraded certified projects. The third term corresponds to the outflow from the pool of uncertified projects that drops out of the economy. The fourth term corresponds to the outflow from the pool of uncertified projects as a result of these projects being upgraded to certified projects.

Also, the change in C can be summarized as follows. The first and second terms of the right-hand side correspond to the inflow to C and outflow from C.

$$\dot{C} = \theta U - \delta C$$

We restrict our analysis to the steady state of this economy which is achieved when $\dot{U} = 0$ and $\dot{C} = 0$. From these two steady state conditions along with the assumption that the total mass of projects is fixed, we can derive the steady state levels of the two masses, $U_{ss} = \delta/(\delta+\theta)$ and $C_{ss} = \theta/(\delta+\theta)$. Then, the following condition suffices to clear the loan market.

$$\widehat{\lambda} + \lambda(E)\tfrac{\theta}{\delta+\theta} = \widehat{\phi} + \phi(D)\tfrac{\theta}{\delta+\theta}$$

The condition guarantees that the size of the inflow to (U-P) in the firm's side and the size of the inflow to (U-L) in the bank's side are equal.[13] This market clearing condition can also be summarized by the following equation (MKT) which confirms the equilibrium relationship between $\widehat{\lambda}$ and $\widehat{\phi}$. Since D and E are predetermined under the break-even conditions for potential entrants, this market clearing condition works as one equilibrium condition for our model in which $\left(\overline{\gamma}, \widehat{\lambda}, \widehat{\phi}\right)$ are endogenously determined.

$$\widehat{\lambda} = \widehat{\phi} + Z \quad \text{where} \quad Z \equiv \tfrac{\theta}{\delta+\theta}\left\{\phi(D) - \lambda(E)\right\} \tag{MKT}$$

Second, by using the steady state conditions and equation (MKT), we can obtain a relationship between the rate of destruction σ, which determines the total surplus made

[13] From the construction of the steady state, this also guarantees that the size of the outflow from (U-P) and the size of the outflow from (U-L) are equal.

by each match, and $\left(\widehat{\lambda}, \widehat{\phi}\right)$, where $\widehat{\phi}$ denotes the flow of new banks to U.

$$\begin{cases} \sigma = (1+\theta/\delta)\,\widehat{\lambda} + \lambda(E)\theta/\delta \\ \sigma = (1+\theta/\delta)\,\widehat{\phi} + \phi(D)\theta/\delta \end{cases} \tag{STAT}$$

Then, the break-even condition for entry firms with the conditions in (FOPT) and the first equation in (STAT) establishes a negative relationship between $\overline{\gamma}$, which determines the split of surplus from each project, and $\widehat{\lambda}$.[14] This is another equilibrium condition for our model.

$$\begin{cases} \overline{\gamma} = -\frac{(r+\delta)(1+\theta/\delta)F^f}{\theta}\widehat{\lambda} - \frac{\lambda(E)r\theta/\delta+(r+\delta+\theta)r}{\theta}F^f + \frac{(\pi_u - R)}{\theta} + (\pi_c - R) - E \\ \text{where } E \text{ is determined by } F^f = 1/\lambda'(E) \end{cases} \tag{FIRM}$$

As the spread of (C-L) $\overline{\gamma}$ increases, each entry firm decreases its Poisson rate $\widehat{\lambda}$ for (U-P). This is because the relative profitability of searching becomes lower. The equation (FIRM) characterizes such a response to the change in loan spread $\overline{\gamma}$.

Similarly, the break-even condition for entry banks with the conditions in (BOPT) and the second equation in (STAT) establishes a positive relationship between $\overline{\gamma}$ and $\widehat{\phi}$ and this gives the last equilibrium condition. As the spread $\overline{\gamma}$ increases, each entry bank increases its search intensity $\widehat{\phi}$ for (U-L). The equation (BANK) characterizes such a response of the bank side to the change in loan spread $\overline{\gamma}$.

$$\begin{cases} \overline{\gamma} = \frac{(r+\delta)(1+\theta/\delta)F^b}{\theta}\widehat{\phi} + \frac{\phi(D)r\theta/\delta+(r+\delta+\theta)r}{\theta}F^b + D \\ \text{where } D \text{ is determined by } F^b = 1/\phi'(D) \end{cases} \tag{BANK}$$

To summarize, the market clearing condition and the two break-even conditions for entry firms and banks determine the stationary equilibrium of this economy which is characterized by $\left(\overline{\gamma}, \widehat{\lambda}, \widehat{\phi}\right)$.

4.3 Existence and Uniqueness

We now provide existence and uniqueness properties for this stationary equilibrium.

Proposition 3 *If $\pi_u + \theta\pi_c > XF^b + YF^f + (1+\theta)R$ and $Z \geq 0$ where $X \equiv \theta r\frac{\phi(D)}{\delta} + r(r+\delta+\theta)$ and $Y \equiv \theta r\frac{\lambda(E)+Z}{\delta} + (r+Z)(r+\delta+\theta)$, there exists a unique stationary equilibrium with $\widehat{\lambda} > 0$ and $\widehat{\phi} > 0$.*

[14] In order to derive this equation, we plug in the first equation in (STAT) into (FOPT) to eliminate σ, plug in $F^f = B$ into (FOPT) to eliminate B, and use the last two equations of (FOPT) to eliminate A. The level of E is determined by the first equation of (FOPT) and $F^f = B$.

Proof. See the appendix. ■

The equilibrium of this economy, summarized by $\left(\overline{\gamma}, \widehat{\lambda}, \widehat{\phi}\right)$, can be determined as in Figure-2. The proposition confirms that if the profitability of the projects are high enough compared to the fixed entry costs and the risk-free interest factor, the economy has positive inflows of firms and banks as a unique equilibrium.[15]

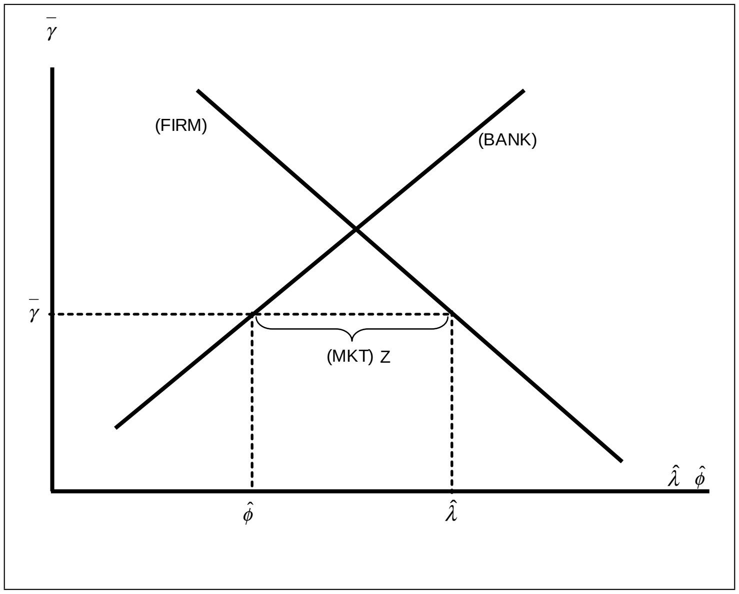

Figure-2: Equilibrium

We need to remark that the stationarity of the equilibrium hinges on the free entry (with cost) assumption. As with Klette and Kortum (2004), we cannot have a stationary equilibrium if there is no entry to this economy. If the economy has no entry, the number of firms and banks staying in the economy decreases while the number of projects and loans held by firms and banks increases on average. This is because a portion of incumbent matches are broken with an exogenous Poisson rate and some firms and banks exit. In this paper, we limit our interest to the case where the stationary equilibrium is an equilibrium.

5 Comparative Statics

In this section, we use the stationary equilibrium established in the previous section to analyze the consequence of several shocks to this economy. In particular, how do different degrees of interbank competition affect the firms' loan structure? Note that the firm's

[15] In this paper, we assume $Z > 0$ and this does not affect the qualitative implication of this model (unless noted otherwise). Intuitively, this implies that the incumbent banks' Poisson rate for finding a new (U-L) is larger than the incumbent firm's Poisson rate for finding a new (U-P). Considering that existing banks are likely to have a larger number of loan relations than firms, this assumption is plausible.

loan structure can also be interpreted as firm size since the loan structure represents the firm's project portfolio. This is a direct consequence of our assumption that each project is financed by different banks. Thus, the primal interest is analyzing the effect of higher inter-bank competition on the loan structure or firm size distribution. This distribution can be summarized by $\left(\lambda(E), \widehat{\lambda}\right)$.

To illustrate how these two parameters summarize the loan structure or firm size distribution, suppose $\lambda(E)$ stays constant and $\widehat{\lambda}$ increases as a response to some specific shock. From the first equation in (STAT), this implies that σ increases. As a result, the measure of incumbent firms with a large number of loan relations decreases since the incumbent firms now face a larger Poisson rate of losing one of the (U-P)'s. This also leads to a smaller number of (C-P)'s for the incumbent firms. Hence, the firms' loan structure, on average, now consists of a smaller number of loan relationships and is, thus, more concentrated. This is a simple illustration of our comparative statics. In the next subsections, we will change several parameters in the model and see what happens to $\left(\lambda(E), \widehat{\lambda}\right)$ and $\left(\phi(D), \widehat{\phi}\right)$.

5.1 Interbank Competition

First, we study the impact of different degrees of interbank competition. Recall that the parameter F^b denotes the fixed entry cost of each potential entry bank to the banking industry. As this entry cost becomes lower, there will be more entry of banks, and thus, higher competition. The next proposition summarizes the key result of this paper.

Proposition 4 *As the cost of entry to the banking industry F^b decreases (increases), (i) the spread $\overline{\gamma}$ decreases (increases), (ii) the incumbent firms' search intensity $\lambda(E)$ is constant, (iii) the incumbent banks employ lower (higher) search intensities $\phi(D)$, (iv) the flows of entering firms and banks $\left(\widehat{\lambda}, \widehat{\phi}\right)$ increase (decrease), and (v) the overall effect on σ is positive (negative).*

Proof. See the appendix. ■

We have already established that $\lambda(E)$ is determined independently of F^b. The proposition confirms that the equilibrium level of $\widehat{\lambda}$ increases as F^b decreases. This implies that the loan structure of each incumbent firms become more concentrated and is associated with a smaller number of loan relations on average. To see this point more clearly, suppose F^b decreases. Each incumbent firm keeps its search intensity constant while the entry flow increases. This decisively leads to higher σ.[16] As a result of this higher σ, each incumbent firm's (U-P) and (C-P) decrease.

From the bank's side, as the banking industry becomes more competitive, the incumbent banks choose lower search intensity for additional (U-L) while the flow of entry

[16] This result also implies that the impact of a higher $\widehat{\phi}$ dominates the impact of a lower $\phi(D)$.

increases. The large entry flow decreases the value associated with each (U-P), which induces the incumbent banks to reduce the search intensity. The loan spread becomes lower (e.g., Japanese bank loan market in 1980s) as demonstrated in Figure-3.

To summarize, higher competition in the banking industry induces entry not only into the banking industry but also into the firm industry. The latter makes the firms' portfolio/borrowing structure more concentrated and reduces the number of loan relations on average. One empirical implication of this model is that higher competition in the banking market (via lower entry barriers) is accompanied by an increase in the number of firms and a reduction in the number of loan relations per firm. This can partially explain a recent phenomenon in the Japanese bank loan market. Deregulation over the past two decades in the bank loan market leads to higher competition between banks, which induces young and small firms to enter the economy as predicted in our model. This effect would not arise without considering endogenous entry of firms.

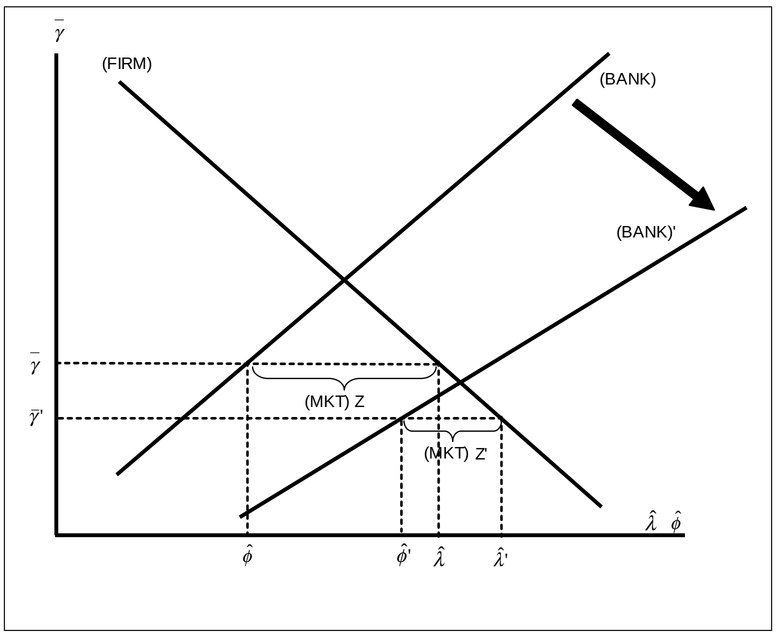

Figure-3: Interbank Competition

5.2 Firm's Profitability

In this section, we see how changes in firm profitability affect the loan structure. In the model, (π_c, π_u) and the search cost are the parameters of firm profitability. For the sake of simplicity, we fix the costs and see how the equilibrium loan structure responds to different levels of π_c and π_u. The following proposition summarizes the result of this comparative static.

Proposition 5 *As π_c and/or π_u become larger (smaller), (i) the spread $\overline{\gamma}$ increases (decreases), (ii) the incumbent firms and banks keep search intensities constant, (iii) the flows of entering firms $\widehat{\lambda}$ and banks $\widehat{\phi}$ increase (decrease), and (iv) the overall effect on*

σ is positive (negative).

Proof. (i), (ii), and (iii): A larger (smaller) π_c and/or π_u shift the equation (FIRM) to the right (left). Considering that E and D are independent of (π_c, π_u) and the equation (BANK) does not shift even if π_c and/or π_u change, such a shift in the equation (FIRM) only generates a change in $\overline{\gamma}$ and $\left(\widehat{\lambda}, \widehat{\phi}\right)$, as illustrated in Figure-4. (iv): Recall the equation (STAT). The results follow immediately. ■

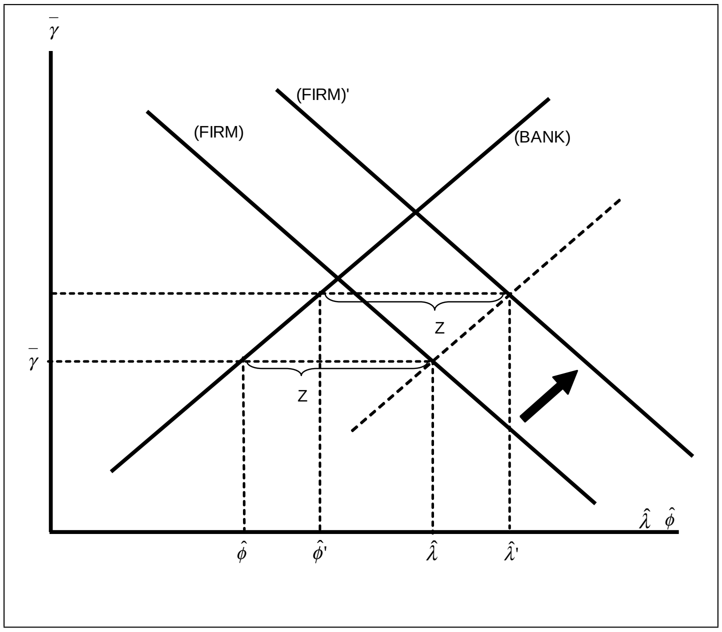

Figure-4: Firm's Profitability

The proposition confirms that the equilibrium levels of $\widehat{\lambda}$ and $\widehat{\phi}$ are increasing in firms profitability. This implies that the loan structure of each incumbent firm become more concentrated under a higher π_c and/or π_u through the same mechanism demonstrated in the previous proposition.

From the firm's side, higher profitability induces a larger flow of entrants and these entering firms take part of (U-P)'s originally held by incumbent firms. As a consequence, the loan structure of incumbent firms becomes more concentrated and leads to a smaller number of loan relations on average.

From the bank's side, higher firm profitability does not directly improve payoffs since banks can only use straight debt. Nonetheless, each bank's value improves due to a higher equilibrium level of $\overline{\gamma}$. This induces an increase in the flow of new entry banks, thus accommodating the increased demand for (U-L)s from firms.

To summarize, an increase in firms' profitability induces entry in both the firm and bank industries. The additional entry of firms makes the firms' portfolio/borrowing structure more concentrated and leads to a smaller number of loan relations. A positive correlation between firms' profitability and the concentration of loan structure are established in several empirical studies (Degryse and Ongena (2001); Harhoff and Korting (1998); Foglia et al. (1998); Gordon and Schmid (2000); Machauer and Weber (1999)).[17] Our model successfully replicates these empirical findings.

5.3 Search Technology for Incumbent Banks

Consider a technological improvement that makes it easier for incumbent banks to find a new client. In the setup of the base-line model, this technological progress can be summarized by a change in the shape of the function $\phi(\cdot)$.

Suppose $\phi(D)$ shifts up while $\phi'(D)$ is held constant. Note that so long as $\phi'(D)$ for each level of D is unchanged, the equilibrium level of D does not change. The higher $\phi(D)$ with a constant D results in the shift of the equation (BANK) to the left and Z increases.[18] We can confirm that improvements in bank's search technology leads to a smaller $\widehat{\phi}$. As a result, the incumbent banks hold a larger number of loans and this leads to a larger number of loan relations on average.

This feature can be interpreted as an illustration of a recent trend in which financial institutions have become larger as a result of progress in information technology. As the incumbent banks obtain more efficient search technologies relative to potential entrants, the size of incumbent banks becomes larger.

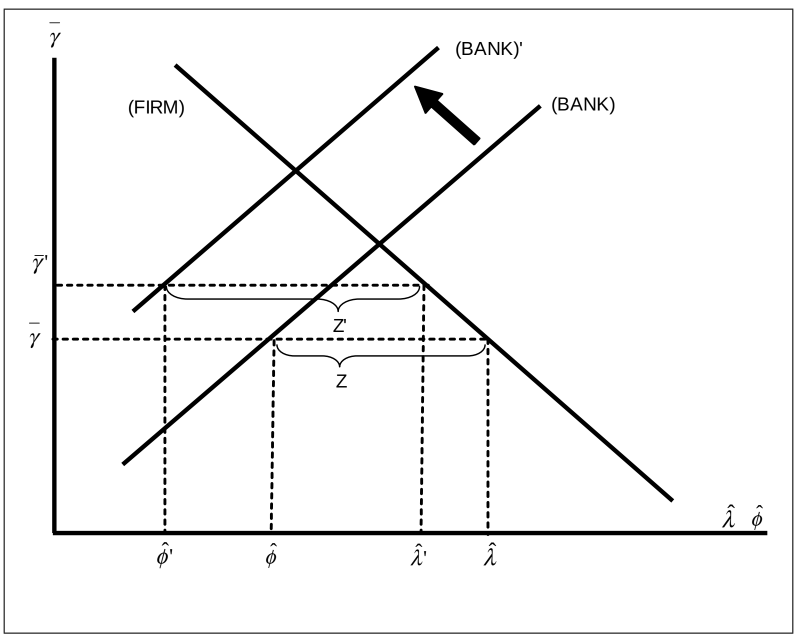

Figure-5: Technological Progress in Bank's Searching Technology

[17] Weinstein and Yafeh (1998) claims the opposite result.

[18] This can be verified by (BANK). On the right-hand side of (BANK), the terms associated with $\phi(D)$ can be summarized as $\phi(D)\, r/\delta$, which implies (BANK) shifts to the left as $\phi(D)$ shifts up.

5.4 Empirical Regularities

Is the model consistent with empirical regularities? First, our model is designed to be consistent with the empirical fact that large firm size (asset and/or debt) leads to a larger number of loan relations (Ogawa et al. (2007), Miyakawa (2008b)). Second, we have confirmed that, in equilibrium, higher firm profitability leads to more concentrated loan relations (Degryse and Ongena (2001)). Third, loan relations are modeled to exhibit persistence (Miyakawa (2008b), Tachibanaki and Taki (1991)) in this paper.

6 Quantitative Discussion

6.1 Simulated Loan Structure/Firm Size Distribution

In the previous section, we have established the existence of a stationary equilibrium. Unfortunately, it is difficult to analytically characterize the loan structure or firm size distribution over the two state variables under the current setup. In this section, we simulate this firm size distribution and consider quantitative comparative statics.[19][20]

		$F^b = 0.6$		$F^b = 0.3$	
		(C-P)	(U-P)	(C-P)	(U-P)
(i)	Number of Firms	12998		15374	
(ii)	Mean	1.0488	1.4807	1.0268	1.2662
(iii)	Standard Deviation	0.2237	0.6395	0.1692	0.5056
(iii)'	(Coefficient of Variation)	(0.2133)	(0.4319)	(0.1648)	(0.3993)
(iv)	Skewness	4.6626	1.2295	6.6340	1.8037
(v)	Kurtosis	22.2274	1.6174	47.1659	2.8216

Table-1: Quantitative Comparative Statics

Table-1 summarizes the summary statistics of the simulated distribution under two levels of entry cost to the banking industry ($F^b = 0.6$ and 0.3).[21] As our analytical result suggests, when the entry cost to the banking industry decreases from 0.6 to 0.3 (i.e., higher inter-bank competition), (i) there are more firms in this economy, (ii) the average number of (C-P)'s and (U-P)'s held by each firm decrease. Moreover, we can see that (iii) the dispersion of the distribution becomes smaller, (iv) the distribution becomes more skewed, and (v) the distribution exhibits higher kurtosis.

[19] The simulation algorithm is summarized in the appendix.

[20] Bank distribution over the two state variables can be obtained through a similar simulation algorithm. We omit this object and focus on the firm distribution in this paper.

[21] The summary statistics are conditional on positive project numbers. For example, the mean of (C-P) is the average of the (C-P)s number among the firms with at least one (C-P).

6.2 How to Bring the Model to Data?

As detailed in the previous section, our model describes the initiation of loan relations and the evolution of each match in a stationary equilibrium. The model further implies a distribution of the duration of loan relations between firms and banks. For example, if the benefits of relationship-lending considered in our model are significant, matches with longer relations are expected to break up less often. Thus, the parameters governing the evolution of each match lead to a specific distribution for the duration of loan relations.

One possibility towards sophisticated quantitative analyses is to match the distribution of the duration of loan relations generated by our model with the data. For example, Miyakawa (2008b) constructs a unique data set that contains the duration of long-term loan (i.e., original maturities are greater than 1 year) relations between all Japanese listed firms and all Japanese banks. By using such data sets, we can structurally estimate the model's parameters, for example through the simulated method of moments, and use the estimated model to quantitatively evaluate various shocks to the economy. We leave this to our future research.

7 Concluding Remarks

In this paper, we develop a dynamic equilibrium model for relationship-lending that is consistent with empirical regularities established in the literature. In our model, the sustained loan relation makes it possible to implement relatively profitable projects through the provision of relationship-lending. The loan spread associated with such relationship-lending, which determines how to allocate the surplus between firms and banks, is endogenously determined so as to clear the bank loan market. The novel feature of our paper is that it employs a dynamic equilibrium model to study the implications of different degrees of competition for bank loan structures or firm size distribution. This is in contrast to the existing literature which uses multi-period game theoretic models to analyze this issue.

The model predicts that higher interbank competition leads to a more concentrated loan structure and smaller number of loan relations for each firm. It also implies that (i) an increase in firm profits leads to a more concentrated loan structure with a smaller number of loan relations for each firm, and (ii) improvements in the bank's client search technology (e.g. progress in informational technology) leads to a larger number of loan relations for each incumbent bank.

This paper motivates several questions for future research. First, as mentioned in the last section, we plan to structurally estimate the model. This allows us to examine the impact of financial shocks (e.g., sudden break-down of incumbent banks) to the economy quantitatively. When a bank becomes insolvent or bankrupt, governments often take great interest in ensuring the survival of firms that previously had loan relations with these banks. This policy sometimes takes the form of providing liquidity to these firms and/or financial markets. However, aside from the usual systemic risk considerations for the economy, it is not clear how we can justify such a policy intervention. Our model can potentially provide one plausible framework to answer this question. We conjecture that the termination of an incumbent relation leads to some loss of economic value if there exists a relationship-capital, as modeled in this paper. The quantification of such a loss could be informative for policy discussions. Second, a potential extension of our model is to incorporate some matching frictions. Our current model essentially assumes that firms and banks can match in the market instantaneously. Considering that the bank loan market has been considered not to necessarily fit the Walrasian market, it is fruitful to consider this friction and study its implications. Third, the assumption that each project for a firm needs to be financed by different banks should be relaxed. The empirical literature shows that the actual loan relations are highly asymmetric. This asymmetry has not been taken into account in our current model. By relaxing these assumptions, we can analyze the implications of various economic shocks (e.g., competitiveness in bank loan market) more precisely. Fourth, one drawback of our model is that for given parameters (θ, δ), we have no variation of the aggregate share of (C-P) and (U-P) in the stationary equilibrium. Endogenizing such share can help us to understand the impact of several economic shocks on the share of (C-P) and (U-P). One possibility to accomplish this extension is to introduce an effort choice for upgrading (U-P)'s to (C-P)'s.

8 Appendix:

8.1 A. Proof

Proof. (Lemma-2): (i) Guess the functional form for the value function and policy function for the firm's problem as (a) $V = An_c + Bn_u$ and (b) $e = En_c$. Then, the Bellman equation (F-P) takes the following form:

$$rAn_c + rBn_u = \left\{ \begin{array}{l} \left[\pi_c - (R + \overline{\gamma})\right] n_c + \left[\pi_u - R\right] n_u - En_c \\ +\theta n_u \left[A\left(n_c + 1\right) + B\left(n_u - 1\right) - An_c - Bn_u\right] \\ +\sigma n_u \left[An_c + B\left(n_u - 1\right) - An_c - Bn_u\right] \\ +\delta n_c \left[A\left(n_c - 1\right) + B\left(n_u + 1\right) - An_c - Bn_u\right] \\ +n_c \lambda \left(\frac{e}{n_c}\right) \left[An_c + B\left(n_u + 1\right) - An_c - Bn_u\right] \end{array} \right\}$$

First, this Bellman equation can be rewritten as

$$\begin{array}{r} \left[(r + \delta) A - \delta B - \lambda (E) B - \{\pi_c - (R + \overline{\gamma})\} + E\right] n_c \\ + \left[-\theta A + (r + \theta + \sigma) B - (\pi_c - R)\right] n_u = 0 \end{array}$$

In order to hold this equation for all the possible cases of (n_c, n_u), we need

$$\begin{array}{ll} (r + \delta) A - \delta B - \lambda (E) B - \{\pi_c - (R + \overline{\gamma})\} + E = 0 & \cdots (1 - 1) \\ -\theta A + (r + \theta + \sigma) B - (\pi_c - R) = 0 & \cdots (1 - 2) \end{array}$$

Furthermore, from the first-order-condition with respect to e, we have

$$0 = -1 + \lambda' (E) B \quad \cdots (1 - 3)$$

$(1 - 1)$ to $(1 - 3)$ are the three conditions characterizing (A, B, E).

(ii) Guess the functional form for the value function and policy function for the bank's problem as (a) $W = Gm_c + Hm_u$ and (b)$d = Dm_c$. Then, the Bellman equation (B-P) takes the following form:

$$rGm_c + rHm_u = \left\{ \begin{array}{l} \overline{\gamma} m_c - Dm_c \\ +\theta m_u \left[G\left(m_c + 1\right) + H\left(m_u - 1\right) - Gm_c - Hm_u\right] \\ +\sigma m_u \left[Gm_c + H\left(m_u - 1\right) - Gm_c - Hm_u\right] \\ +\delta m_c \left[G\left(m_c - 1\right) + H\left(m_u + 1\right) - Gm_c - Hm_u\right] \\ +m_c \phi \left(\frac{d}{m_c}\right) \left[Gm_c + H\left(m_u + 1\right) - Gm_c - Hm_u\right] \end{array} \right\}$$

First, this Bellman equation can be rewritten as

$$\begin{array}{r} \left[(r + \delta) G - \delta H - \phi (D) H - \overline{\gamma} + D\right] m_c \\ + \left[-\theta G + (r + \theta + \sigma) H\right] m_u = 0 \end{array}$$

In order to hold this equation for all the possible cases of (n_c, n_u), we need

$$\begin{array}{ll} (r+\delta)\,G - \delta H - \phi\,(D)\,H - \overline{\gamma} + D = 0 & \cdots(1-1)' \\ -\theta G + (r+\theta+\sigma)\,H = 0 & \cdots(1-2)' \end{array}$$

Furthermore, from the first-order-condition with respect to d, we have

$$0 = -1 + \phi'\,(D)\,H \quad \cdots(1-3)'$$

$(1-1)\prime$ to $(1-3)\prime$ are the three conditions characterizing (G, H, D). ■

Proof. (Proposition-3): The equations (FIRM), (BANK), and (MKT) imply the monotonic relations between (i) $\overline{\gamma} \Leftrightarrow \widehat{\lambda}$, (ii) $\overline{\gamma} \Leftrightarrow \widehat{\phi}$, and (iii) $\widehat{\lambda} \Leftrightarrow \widehat{\phi}$, which guarantee the uniqueness of the equilibrium. Suppose the equations (FIRM) and (BANK) are expressed as $\overline{\gamma} = -M_1\widehat{\lambda} + M_2$ and $\overline{\gamma} = N_1\widehat{\phi} + N_2$. Recall that the equilibrium $\widehat{\phi}$ can be determined from the three equations (FIRM), (BANK), and (MKT) $\widehat{\lambda} = \widehat{\phi} + Z$ as follows.

$$\widehat{\phi} = \tfrac{M_2 - M_1 Z - N_2}{M_1 + N_1}$$

Sine the denominator is always positive, $M_2 + N_1 Z - N_2 \geq 0$, which can be summarized as the condition in the proposition, is sufficient to obtain $\widehat{\phi} \geq 0$. The equation (BANK) implies that $\overline{\gamma} \geq 0$ as far as $\widehat{\phi} \geq 0$. Moreover, if Z is positive, $\widehat{\lambda} \geq 0$ as far as $\widehat{\phi} \geq 0$. ■

Proof. (Proposition-4): (i) and (iv) First, the equation (BANK) shifts to right and becomes flatter as F^b decreases. This can be verified from the positive correlation between D and F^b and the equation (BANK).

$$\overline{\gamma} = \tfrac{(r+\delta)(1+\theta/\delta)F^b}{\theta}\widehat{\phi} + \tfrac{\phi(D)r\theta/\delta + (r+\delta+\theta)r}{\theta}F^b + D$$

Second, $Z \equiv \frac{\theta}{\delta+\theta}\{\phi(D) - \lambda(E)\}$ becomes smaller as F^b decreases. Then, we can verify that the size of horizontal shift of the equation (BANK) is always greater than the size of decline in Z. From the equation (BANK), we can obtain the following relationship between $\widehat{\phi}$ and F^b for a fixed $\overline{\gamma}$.

$$d\widehat{\phi} = \left\{ -\tfrac{\widehat{\phi}}{F^b} + \tfrac{r\theta\phi'(D)\phi''(D)}{\delta(r+\delta)(1+\theta/\delta)} - \tfrac{\phi(D)r\theta/\delta + (r+\delta+\theta)r}{(r+\delta)(1+\theta/\delta)F^b} + \tfrac{\theta\phi''(D)}{(r+\delta)(1+\theta/\delta)F^b} \right\} dF^b \qquad \text{(BHOR)}$$

We used the result that $dD = -\phi''(D)dF^b$, which is derived from the first condition in (BOPT) and the break-even condition for the potential entry banks. We can also obtain the following relationship between Z and F^b from the definition of Z.

$$dZ = -\tfrac{\theta}{\delta+\theta}\phi'(D)\phi''(D)dF^b \qquad \text{(ZDEC)}$$

Note that the second and the fourth terms on the right-hand side of the equation (BHOR) can be summarized as follows.

$$\tfrac{rF^b\phi'(D)+\delta}{(r+\delta)F^b\phi'(D)}\tfrac{\theta}{\delta+\theta}\phi'(D)\phi''(D)dF^b$$

Here, $F^b\phi'(D) = 1$, which follows from the first condition in (BOPT) and the break-even condition for the potential entry banks, gives the following result.

$$\frac{rF^b\phi'(D)+\delta}{(r+\delta)F^b\phi'(D)} = 1$$

This implies that the second and the fourth terms in the right-hand side of the equation (BHOR) has the same magnitude as the right-hand side of the equation (ZDEC), which confirms that the rightward horizontal shift of the equation (BANK) corresponding to the decline in F^b is always greater than the decline in Z corresponding to the same decline in F^b in terms of the absolute values (signs are different). Further considering the equation (FIRM) does not shift even if F^b changes, the new equilibrium level of $\widehat{\lambda}$ increase as demonstrated in Figure-3. This is accompanied with a lower level of new $\overline{\gamma}$. (ii) and (iii) We have already established that D decreases (increases) but E stays constant when F^b decreases (increases). (v) Recall the equation (STAT). The result immediately follows. ■

8.2 B. Simulation Algorithm

We use the following algorithm to obtain the stationary firm distribution over its state space.

Step-1: Set R, π_c, π_u, θ, δ, F^f, F^b, $\Lambda(e, n_c)$, and $\Phi(d, m_c)$.[22][23] Step-2: Solve for the optimal level of E and D. Step-3: Compute $\lambda(E)$ and $\phi(D)$. Step-4: Compute the equilibrium levels of $\widehat{\lambda}$, $\widehat{\phi}$, and $\overline{\gamma}$ from the equations (FIRM), (BANK), and (MKT). Compute σ from the equation (STAT). Step-5: Set I firms indexed by $i = 1, \cdots, I$ with one (C-P) and one (U-P) for each.[24] Step-6: Draw $\left\{z_i^j\right\}_{j=1}^4$ for each firm independently from a unit uniform distribution. Step-7: Assign the individual dynamics by the following rule:

(1) Consider the segments in Figure-6 and follow the rule described below.

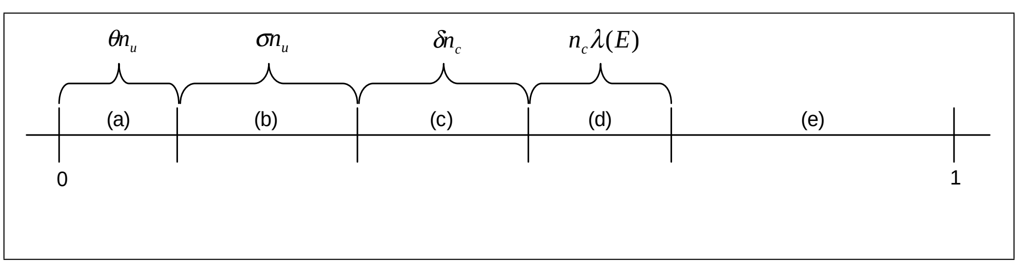

Figure-6: Segmentation for Individual Dynamics

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[22] For this exercise, we use $\Lambda(e, n_c) = Je^a n_c^{1-a}$ and $\Phi(d, m_f) = Kd^b m_f^{1-b}$ where $(a, b) \in (0, 1) \times (0, 1)$ and $J, K > 0$.

[23] As a baseline case, we use $R = 1 + r = 1.04$, $\pi_c = 1.5$, $\pi_u = 1.1$, $\theta = 0.05$, $\delta = 0.45$, $F^f = 0.3$, $F^b = 0.3$ or 0.6, $a = 0.6$, $b = 0.5$, $J = 1$, and $K = 1$.

[24] Let $(n_c(i), n_u(i))$ denote the state variables for firm-i. We use $I = 10000$ in this exercise.

(2) If z_i^1 falls (a), revise the state to $(n_c(i)+1, n_u(i)-1)$. (3) If z_i^1 falls (b), (i) revise the state to $(n_c(i), n_u(i)-1)$, and (ii) add one entry firm with one (U-P) if $z_i^2 \in \left[0, \widehat{\lambda}/\left\{\widehat{\lambda}+\lambda(E)C+\delta C\right\}\right)$ or (ii)' choose one incumbent firm to replace (U-P) based on $\{n_c(i)\}$ and z_i^3 otherwise. (4) If z_i^1 falls (c), revise the state to $(n_c(i)-1, n_u(i)+1)$. (5) If z_i^1 falls (d), (i) revise the state to $(n_c(i), n_u(i)+1)$, and (ii) choose one incumbent firm losing (U-P) based on $\{n_u(i)\}$ and z_i^4. (6) If z_i^1 falls (e), stay at the same state $(n_c(i), n_u(i))$.

Step-8: Drop firm-i if $n_c(i)=0$ and $n_u(i)=0$. Step-9: Repeat Step-7 and -8 until the distribution converges. In order to confirm the convergence, check if the difference between the revised and pre-revised means, variances, and skewness of the (C-P)'s and (U-P)'s distributions becomes smaller than threshold values.

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