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Sustainability, Endogenous Social Discount Rate, and Proportional Carbon Tax

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Abstract

Whether or not the utility of future generations should be discounted is the most serious problem, specifically when we consider the allocation of intergenerational public bads such as CO_2. This paper analyzes how the social optimum is attained under the constraint of *sustainability* proposed by Pezzey (1997). We find the following equivalent relationship between the optimal policies: the condition that utility should not be discounted in social planning is equivalent to adopting the socially optimal carbon tax rate in a decentralized economy.

1 Introduction

"Sustainability" is used in various contexts. For example, Dasgupta and Heal (1974), Hartwick (1977), and Solow (1986) define sustainability as the maintaining of some constant consumption level over time. However, the welfare economics foundation for such a definition is not necessarily clear.

This article uses a utilitarian definition of sustainability proposed by Pezzey (1997). In other words, we adopt the egalitarian view that each generation can enjoy the same utility level iff an economy is sustainable. Based on this definition, we analyze the optimal tax that is proportionate to the emission of CO_2. Since generations are not concerned with the utility of the generations thereafter, and only a part of the CO_2 emission is absorbed by sea or expelled from the earth, etc., excess emission remains in the future. This is because the proportional carbon tax is desirable to *sustain* the economy.

By employing the method of Negishi (1960), we can also exactly calculate how much weight should be allotted to the utility of each generation in the social welfare function of a planning economy. Namely, the Negishi method makes it possible to clarify the relationship between the carbon tax rate in the market economy and the weight of each generation in the social welfare function for intertemporal social planning.

Usually, researchers presume that the utility of future generations can be discounted. Nevertheless, such discounting possesses are not rigorous in the welfare economic foundation. When used with our definition of sustainability, the Negishi method reveals that conventional discounting in social planning cannot achieve optimal taxation in a market economy, and that if each generation is certified to enjoy the same utility, its utility should be equally weighed.

This paper is organized as follows. Section 2 constructs a model in which the consumption/emission decision is diversified across generations. We also analyze the properties of the model in the stationary state, and calculate the optimal carbon tax rate in a market economy and the weight of each generation in the additive social welfare function for a planning economy. Section 3 contains brief concluding remarks.

2 Model

For simplicity, we assume the dynamics of CO_2 emission as follows:

$$e_t = \alpha e_{t-1} + c_t, \quad 0 < \alpha < 1, \tag{1}$$

where e_t is the stock of CO_2 in period t, c_t is the consumption level, and α denotes the remaining ratio of CO_2 per period. This difference equation implies the assumption that one unit of consumption emits unit of CO_2. Note that this simplifying assumption can be easily relaxed.

Individuals live for one period and their well-behaved identical utility function U_t is

$$U_t \equiv u(c_t, e_t), \quad u_c > 0, u_e < 0, \tag{2}$$

where each subscript represents the partial derivative on the argument. Individuals' budget constraint is

$$e_t = \alpha e_{t-1} + (1+\theta)c_t - \tau_t, \tag{3}$$

where θ denotes the proportional carbon tax rate and τ_t is the transfer from the government.

The government transfers the collected tax to individuals equally. Namely, the budget constraint of the government is

$$\theta c_t = \tau_t. \tag{4}$$

In a market economy, each individual maximizes (2) subject to (3). Hence, the following first-order condition should hold:

$$\frac{u_c(t)}{1+\theta} + u_e(t) = 0, \quad \forall\, t. \tag{5}$$

The dynamics of the market economy is fully described by two difference equations: (1) and (5).

For simplicity, we assume that the economy is initially located at some stationary state (c^*, e^*). Then, it is straightforward from Figure 1 that the optimal tax rate θ^* in the stationary state E^* is $\dfrac{\alpha}{1-\alpha}$.

We now focus on the social planning problem under an egalitarian definition of sustainability[1]. Namely, we have the following:

Definition 1 *An economy is sustainable iff*

$$u(c_t, e_t) \geq \overline{U}, \quad \forall t \tag{6}$$

holds for some given $\overline{U}$. ∎

[1] This definition of sustainability is identical to that in Pezzey (1997), although we do not consider some contradiction between sustainability and "optimality."

¿From this definition, we have the maximization problem of the government as follows:

$$\max_{c_t} u(c_0, e_0), \quad \text{s.t. } e_{-1} = \overline{e}, \quad u(c_t, e_t) \geq \overline{U} \quad \forall\, t. \tag{7}$$

The corresponding Lagrangian $\mathcal{L}$ is

$$\mathcal{L} \equiv u(e_0 - \alpha e_{-1}, e_0) + \sum_{t=1}^{T} \lambda_t \Big[u(e_t - \alpha e_{t-1}, e_t) - \overline{U} \Big]. \tag{8}$$

The first-order condition yields

$$\lambda_t \Big[u_c(t) + u_e(t) \Big] - \alpha \lambda_{t+1} u_c(t+1) = 0. \tag{9}$$

An important property of the stationary state emerges in (9). The stream of Lagrangian multipliers, $\Big\{ \lambda_t^* \Big\}_{t=1}^{T}$, satisfies the following difference equation:

$$\lambda_t^* \Big[1 - [-\frac{u_e(*)}{u_c(*)}] \Big] = \alpha \lambda_{t+1}^* \quad \Leftrightarrow \quad \lambda_{t+1}^* = \frac{\Big[1 - [-\frac{u_e(*)}{u_c(*)}] \Big]}{\alpha} \cdot \lambda_t^*, \quad \lambda_0^* = 1, \tag{10}$$

where $-\frac{u_e(*)}{u_c(*)}$ is the marginal substitution rate between consumption and the stock of CO_2.

Using the method of Negishi (1960), we can prove that the maximal problem (8) with the initial condition, $e_{-1} = e^*$, is equivalent to the maximization problem of the following social welfare function:

$$\max_{e_t} \sum_{t=0}^{T} \lambda_t^* u(e_t - \alpha e_{t-1}, e_t), \quad e_{-1} = e^*. \tag{11}$$

It is clear from (10) that utility discounting is permitted only when

$$1 - \alpha = \frac{1}{1 + \theta^*} < -\frac{u_e(*)}{u_c(*)}.$$

Whenever such planning becomes incentive compatible for every generation, in other words the same allocation is also attained by a market economy, (5) should hold. Consequently, the necessary and sufficient condition for permitting discounting programming is

$$\theta^* > \theta, \tag{12}$$

where θ is the existing carbon tax rate.

Such a steady state is illustrated by point E_S in Figure 1. It implies that utility-discounting social programming yields excess emission of CO_2 from the egalitarian viewpoint. To sum up the discussion, we have the following theorem:

Theorem 1 *As long as the economy is sustainable, the weight of each generation's utility in the social welfare function should be equally allotted. Utility discounting results in the effective carbon tax rate being lower than the optimum.* ∎

Finally, we discuss the case where the time horizon for planning is infinite. From Theorem 1, the equal weight is not confined to unity. As such, we set $\frac{1}{T}$ for the weight and take the limit $T \to +\infty$. Then, we have the correct social welfare function for the infinite horizon case:

$$\lim_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T} u(c_t, e_t). \tag{13}$$

Thus, the divergence problem for the sum of utility can be avoided even if the social discount rate is unity.

3 Concluding Remarks

This paper analyzed the theoretical relationship between the social discount rate in a planning economy and the tax rate of CO_2 emission in a market economy. If the social planner discounts the utility of future generations, it corresponds to lowering the carbon tax rate below the optimum in a market economy. Under the egalitarian viewpoint of sustainability, it is desirable to pay the correct price of CO_2, which is

$$\alpha + \alpha^2 + \cdots = \frac{\alpha}{1-\alpha}.$$

This implies that the planner should not discount the utility of any generation in the centralized economy, and that the optimal tax rate must be equal to the total remaining ratio $\frac{\alpha}{1-\alpha}$ in a decentralized economy.

References

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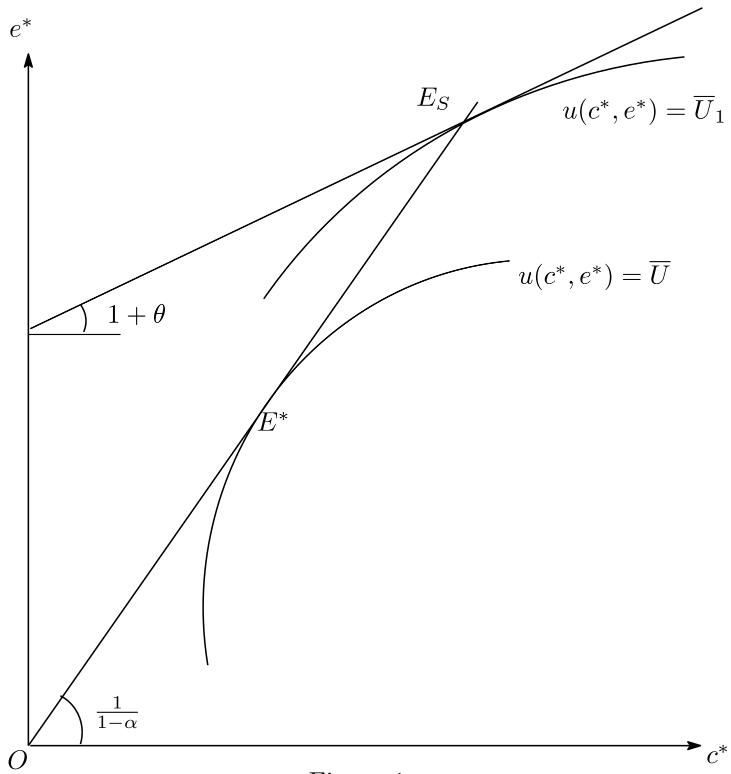

Figure 1