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Risk Management in Corporate Financial Policy

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This work shows how risk management interacts with the firm's financial structure, its investment and dividend policies. Financially constrained firms simultaneously determine their optimal capital structure and how to fund investments by deciding the level of cash retained and the level of risk. A firm's cash flow rate and leverage affect its risk taking. Firms with low interest coverage take greater risks if they have investment opportunities. The profile of the investment matters to a firm's risk strategy: Large, lumpy investments induce more aggressive behavior than small, incremental investments. Although it is common for firms with investment opportunities to have lower leverage ratios, it is possible that a firm with investments decide to have high levels of debt, as long as leverage adds more to the probability of investing than to the probability of defaulting. When investors do not observe the firm's cash flow rate and whether the firm has growth options, risk and dividend choices reveal information about the value of the firm. Both debt and equity financing relax financial constraints and accelerate investment, but firms do not freely choose to fund with more debt or equity. Debt financing can lead to overhang after investment, and equity financing incurs high issuance costs. When the firm decides to fund investment with additional equity, it always follows a low risk strategy, because a riskier strategy to accelerate investment becomes unnecessary.

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Abstract

This work shows how risk management interacts with the firm's financial structure, its investment and dividend policies. Financially constrained firms simultaneously determine their optimal capital structure and how to fund investments by deciding the level of cash retained and the level of risk. A firm's cash flow rate and leverage affect its risk taking. Firms with low interest coverage take greater risks if they have investment opportunities. The profile of the investment matters to a firm's risk strategy: Large, lumpy investments induce more aggressive behavior than small, incremental investments. Although it is common for firms with investment opportunities to have lower leverage ratios, it is possible that a firm with investments decide to have high levels of debt, as long as leverage adds more to the probability of investing than to the probability of defaulting. When investors do not observe the firm's cash flow rate and whether the firm has growth options, risk and dividend choices reveal information about the value of the firm. Both debt and equity financing relax financial constraints and accelerate investment, but firms do not freely choose to fund with more debt or equity. Debt financing can lead to overhang after investment, and equity financing incurs high issuance costs. When the firm decides to fund investment with additional equity, it always follows a low risk strategy, because a riskier strategy to accelerate investment becomes unnecessary.

1 Introduction

Risk management is one of the most important activities in corporate finance. Yet, despite the growing attention it has received in the last fifteen years, risk management is still at an embryonic stage. This is partly due to the fact that risk, being everywhere, interacts with the various financial policies of the firm in ways that are not easy to decipher. There have been important advances explaining firms' motivations to control risks. In most cases, the reasons highlight either capital market imperfections [see, for example, Smith and Stulz (1985), and Froot *et al* (1993)]; incomplete information [Ross (1997)] or managerial agency problems [Tufano (1998)]. Usually, these reasons relate financial risk management to a particular policy of the firm, such as investment policy, capital structure policy or cash management, and then evaluate how each policy improves on the value of the firm. The reality is that all these policies work together. Financial policy is not really separable from investment policy. And since cash sourced inside the firm is cheaper than cash sourced outside, liquidity management is related both to financial policy and to dividend policy. A better understanding of the role of risk management requires that the firm's capital structure as well as its investment and dividend policies must be considered in conjunction with risk management.

To analyze how risk management interacts with the various policies of the firm, we use a dynamic model of a firm that generates cash from existing assets and which has growth opportunities. Cash flows follow a random walk with a drift, so the firm hedges cash flow shocks that persist over time. This allows us to measure the effects of risk management on the value of the firm and its claims, and not just on the next instant's cash flows. The dynamic model allows us to incorporate revisions in some control variables. For example, investment gives the firm the opportunity to revise its capital structure.

The firm is financed with equity and debt. In the basic model, after an initial contribution, equityholders are assumed to have no additional money to invest in the firm. The cash flows that originate from operating the assets are used to make payments to the claimholders. With no possibility of resorting to current equityholders' deep pockets, it is possible that the firm may run out of cash. When that happens, the firm falls into financial distress, and is assumed to be liquidated. Liquidation imposes costs to equityholders and debtholders, since both lose everything. Costly bankruptcy creates incentives to retain cash in the firm.

Risk management lets equityholders change the risk profile of the firm's cash flows. Changing the risk of the cash flows alters the expected costs of bankruptcy. Yet it also changes the timing of investing, since investment is funded in part with accumulated cash balances.

We find that risk management varies with the circumstances: At times the firm follows a low-risk strategy; at other times it is deliberately more aggressive. Risk management depends critically on a number of factors, particularly the firm's rate of operating cash flows, the amount of cash balances available, leverage, dividends, the costs of financial distress and the type of investment.

Firms with low interest coverage ratios, resulting from either high leverage or a low growth rate of cash flows, are firms with low levels of cash accruing to equityholders. The marginal benefit of a low risk strategy is therefore small, and equityholders might seek more risk. For such equityholders, the trade off between accelerating investment and the loss from liquidation of a firm that generates low cash flow favors the adoption of a riskier strategy.

On the other hand, equityholders of firms with a high cash flow generation ability have strong incentives to reduce the probability of liquidation, so they tend to follow a low risk strategy. A higher rate of cash flow generation also makes it easier to fund investments, a valuable option that should not be risked by an aggressive strategy, especially when leverage is not too high.

Cash balances (inside equity) relate to risk management in a straightforward way: Lower cash balances make firms act conservatively in order to reduce the likelihood of bankruptcy, a policy that becomes relatively more important than the exercise of a distant (for lack of funds) option to invest. At higher levels of cash balances, the option to invest becomes more feasible, and taking risks can help speed up the investment. This behavior differs from the behavior of firms that are financially unconstrained. In an unconstrained firm, equityholders of a levered firm take on more risk when the value of the assets-in-place falls low enough and the value of the abandonment option increases.

Firms with investment opportunities do not necessarily have low leverage ratios. Leverage helps to fund investment. The incentive to borrow to add to cash balances needed for investing might outweigh the cost of having more debt. What is not obvious is that a firm with an opportunity to invest right away if it issues more debt will necessarily do so. At times, the amount of debt needed to invest sooner is just too much, and speeding up investment to attain greater operating cash flows, can later on put the firm in greater danger from the higher debt obligations. Clearly, achieving larger size by levering up does not necessarily translate into higher valuation.

The profile of an investment matters greatly to the risk management policy of the firm. Consider two opposite cases: on one hand, a discrete irreversible investment opportunity that requires significant up-front costs, similar to a real option; on the other hand, small investments that are made continuously. These are simplifications, but their different characteristics can help us draw interesting conclusions. When the investment is like a real option, adopting a high risk strategy can be optimal for a financially constrained firm. The lumpiness of the investment creates the incentive to take risks when these can improve the chance that the firm can get enough cash to make the investment. On the other hand, if the investment occurs continuously in small increments, adopting a high risk strategy can hurt the firm's cash balances, in which case a risk management policy adds little value. Hence, cash flow volatility hurts firm value when investment is small and occurs with some regularity. For a firm with no investment opportunities, there is only downside in following a risky strategy, unless the firm is highly leverered and is financially unconstrained, in which case equityholders will try to bet their way out of bankruptcy.

Risk management appears to be relatively insensitive to the profitability of an investment. This may seem unusual. The explanation lies in the fact that a more profitable investment creates two effects that impact the incentives to change risk in opposite ways: On the one hand, more risk increases the probability of investing when investment is lumpy. On the other, more risk increases the likelihood of bankruptcy. In most circumstances, one effect does not seem to dominate the other.

Many papers have found that firms' financing relies mainly on internal equity and debt. This choice is captured in our basic model, which is extended to allow also outside equity issuance at the time of the investment. Additional equity mitigates the firm's financial constraint and speeds up investment. Unlike debt, equity increases cash balances but not the risk of default. But equity is expensive to issue, so firms issue additional equity only when the reduction in the expected cost of default is greater than the issuing costs. Whether the firm issues debt or equity to fund investment matters to its risk policy. More equity financing is reflected in a more conservative risk strategy before the investment, similar to the behavior of a firm that finances investment with debt but has a low debt to cash flow coverage ratio.

Once the investment takes place, there is no incentive for the firm to adopt a risky strategy. In essence, relaxing the financial constraints by allowing the firm to issue equity at low issuance costs to fund investment makes the firm behave more conservatively. A high risk strategy adds little to help fund investment that low cost equity issuance does not provide.

Next, we modify the model to incorporate asymmetries in information between fully informed insiders (existing equityholders) and imperfectly informed stock market investors. Outside investors cannot observe the quality of the assets in place, measured by the cash flow growth rate, and whether the firm has growth opportunities; instead they form expectations about the firm based on publicly available information. The actions of insiders managing the firm provide important signals that help investors revise their valuation of the firm. Investors observe the cash balances, but understand that these are volatile, and therefore have trouble figuring out the firm's true cash flow rate. Finally, investors can see when the firm starts paying dividends, and whether the firm behaves more conservatively or more aggressively.

In such a setting, investors observe changes in the firm's cash balances over time. Suppose cash balances gradually increase. If at some level of cash balances investors see the firm announcing a dividend payout, they conclude that the firm must not have growth opportunities in the foreseeable future. If, instead, investors notice that the firm switches from a low risk strategy to a more aggressive one, they conclude that the higher risk following an increase in cash balances cannot be the result of some harmful desire to gamble on bankruptcy, but instead an attempt to accelerate investment. A firm that keeps accumulating cash flows beyond that level and neither switches risk nor announces dividends must be a firm that has an investment opportunity but may have the true cash flow rate different from what they had expected.

Our results show how risk management and dividend policy can reveal important information that helps investors figure out the type of the firm and its fundamental value. When cash balances are low, financially constrained firms do not pay dividends and follow low risk strategies. As cash balances increase, firms with investment opportunities switch to a high risk strategy, while still refraining from making dividend payments, in order to conserve cash for future investments. Switching to a higher risk strategy increases the likelihood of investing with internal cash balances. Firms start to pay dividends only after the investment is made, but only insofar as the cash remaining in the firm after the investment is high enough: Once the investment is implemented, the value of the assets in place increases, and therefore the opportunity cost of going bankrupt also increases. Firms with no investment opportunities will continue to use a low risk strategy and do not pay dividends unless cash balances are high enough to avoid bankruptcy. An important implication is that a risk management policy provides information to outside investors about the value of the investment opportunities a firm has, and correspondingly affects their valuation of the firm.

In a recent paper closely related to ours, Bolton *et al* (2009) also explore the interactions among corporate investment, financing and risk management. There are, however, important distinctions between our work and theirs. First, in their model firms continuously adjust investment, and thus have no real options. Given that data show that investment is lumpy, we believe that discrete and significant investment outflows are important. We show that the characteristics of the firm's investment matter to the risk strategy. In Bolton *et al.* hedging affects the value of the firm through the costs of margin requirements. When these costs are low, presumably firms could eliminate all systematic risk. In our model, when investment implies fixed adjustment costs, a risk taking strategy can be optimal even in a frictionless world.

Second, in our model firms decide on what risk to take by evaluating the impact of the choice on the value of the firm and its claims, whereas in Bolton *et al* (2009) the i.i.d nature of the shocks allows them to analyze how risk management affects the next instant's cash flows. Obviously, managing the risk of next instant liquidity and managing the risk of the firm's value are two very different objectives.

We begin with a basic model of a financially constrained firm, financed both with debt and equity. After the initial investment, equityholders are assumed to have no more funds to inject if the firm has a negative cash flow. If the firm exhausts its cash reserves, it goes bankrupt and is liquidated, and both equityholders and bondholders lose everything. This creates incentives to carry cash balances in the firm. The firm has the opportunity to make an investment that requires the payment of a lump sum amount and the investment is irreversible. Once the investment is made, the firm can issue additional debt and revise its optimal capital structure. Besides deciding on the optimal capital structure and investment, the firm must also choose its dividend policy as well as how much risk it bears. Risk management is executed by adjusting the volatility of the firm's cash flows produced by current operations. The assumptions of the model are outlined in Section 2. Section 3 computes the values of the equity and the debt, both the initial debt after the investment is made and the additional debt issued when the investment takes place. Section 4 determines the values of the firm's claims before investment and the initial cash balances. Section 5 examines a firm with no investment opportunities and contrasts it with the firm in the previous sections. Section 6 provides a numerical example to illustrate the interactions among the various policies of the firm and how they relate to risk management. Section 7 looks at investments with different characteristics. Section 8 considers asymmetry of information between insiders and outside investors, and evaluates the information content of risk management, as well as dividend decisions. Section 9 relaxes the constraint on additional equity issuance. Section 10 highlights the main conclusion.

2 Assumptions of the Model

Consider a firm managed by equityholders who maximize the value of their claim. At time $t = 0$, equityholders contribute an amount X_0 to the firm. The firm also sells debt with a perpetual coupon c to increase its initial capital from X_0 to $X_0^{'}$. After $t = 0$, equityholders are assumed to be unable to make additional capital contributions, and outside financing is possible only with debt, and occurs when the firm takes on a new investment. This is meant to replicate firms refinancing decisions that appear to be largely associated with important events, such as investments. This also captures the notion that it is usually more difficult for a firm to borrow when it has a low cash balance and is close to liquidation than when it has a high cash balance and is making investments.

The firm generates cash flows from its operations at a rate of α. The firm's cash balances, X, evolve according to a Brownian motion with a drift:

$$dX = (\alpha - c(1 - \tau) - d)dt + \sigma dZ, \tag{1}$$

where c is the coupon payment to debtholders, d is the dividend payment to equityholders, σ is the instantaneous volatility rate of the cash flows, and $Z \sim N(0, t)$. Since debt is tax deductible, net coupon payments equal $c(1 - \tau)$, where τ is the corporate tax rate.

There is a constant instantaneous riskless interest rate of ρ. For simplicity, it is assumed that the cash balances held in the firm earn no interest; otherwise the firm would receive an additional cash flow equal to $Xrdt$ from investing its cash balances, where r is the instantaneous interest rate earned on X.

It is assumed that when cash balances become zero ($X = 0$), the firm is declared insolvent and is liquidated. Although there may be some value left in the firm, equityholders are assumed not to have pockets deep enough to rescue the firm. Liquidation at zero cash balances also assumes that strategic default is not feasible. Any changes in these assumptions would change the event of bankruptcy.

Equityholders decide the firm's capital structure as well as when to invest and when to pay dividends. They also determine the risk level of the firm's operations. At all times they can costlessly change the risk of the cash flows by choosing the volatility rate σ, where σ may take any values between σ_L (low risk) and σ_H (high risk).

The firm has one irreversible project that will increase the instantaneous cash flows by a factor of $\nu > 1$ (i.e., α increases to $\nu\alpha$). To make the investment, the firm must spend a lump-sum equal to $I > 0$. At the time of the investment, equityholders have the opportunity to alter the firm's capital structure and issue additional perpetual debt to partly finance the investment. Denote $c1$ as the coupon of the original debt and $c2$ as that of the new debt, then the new debt will increase the total coupon payments from $c1$ to $c1 + c2$. Newly issued debt is assumed to have the same seniority as outstanding debt. Later we generalize the model to allow for additional equity finance and analyze a different type of investment. Instead of a one-time fixed investment opportunity, we will look at many small investments. Although corporate investments appear to be lumpy, we wish to understand how the characteristics of the investment impact risk management and payout policies.

In the setting just described, the objective of the equityholders is to maximize the value of the equity by choosing the firm's financial policy (the initial capital structure and the refinancing upon making the new investment), the dividend policy, the timing of the investment in the new project and the risk management.

3 Values of the Equity and Debt after Investment

After the investment is made, both the remaining cash balances and the firm's cash flows change, and so do the value functions describing the firm's securities. Therefore, we solve the model backward, starting first with the value functions right after the investment, and then proceed to determine the value functions before the investment. We denote E^A, $D1^A$ and $D2^A$ as the value functions after the investment for the equity, the initial debt, and the additional debt, respectively. Later we will consider that additional outside equity is also available.

3.1 Value of the Equity After Investment

After the investment is made, the company has both the initial and the new debt, so the total coupon payments are $c1+c2$. At that time, equityholders maximize E^A, the value of the equity after investment, by choosing the dividend payout and the risk level. E^A , d, and σ have to satisfy the following expression:

$$\rho E^A = \max_{\sigma,d}\{d + (\nu\alpha - (c1+c2)(1-\tau) - d)E^A_X + \frac{1}{2}\sigma^2 E^A_{XX}\} \tag{2}$$

The first-order condition with respect to the dividend d is $E^A_X = 1$. This means that the dividend policy switches between paying no dividends to paying maximum dividends. The first-order condition with respect to the level of risk σ is $E^A_{XX} = 0$, which implies that the risk policy switches between high and low risk levels.

Define the dividend threshold X_d as the point at which the firm switches between paying no dividends to paying dividends. Then the following proposition summarizes the dividend and risk policies after the firm makes the investment:

Proposition 1 *After the investment, the firm pays no dividend if $X \leq X_d$, and pays as dividends all the cash left after paying coupons on the debt if $X > X_d$. At all times the firm chooses a low risk strategy, σ_L.*

Proof. See Milne and Robertson (1996). ■

From proposition 1, in the region $0 < X < X_d$, Equation (2) can be re-written as follows:

$$\rho E^A = (\nu\alpha - (c1+c2)(1-\tau))E^A_X + \frac{1}{2}\sigma_L^2 E^A_{XX} \tag{3}$$

There are three boundary and smoothness conditions that define the equity value function and the optimal dividend policy:

Condition 1: $E^A|_{X=0} = 0$. This is the boundary condition at liquidation; it says that when cash balances are zero ($X = 0$), the firm is liquidated and the equityholders receive nothing. Whatever the value of the equity at that point, it is lost. Therefore equityholders bear a direct deadweight cost if they let X go to zero.

Condition 2: $E^A_X|_{X=X_d} = 1$. This is the continuity condition, stating that at the dividend threshold ($X = X_d$), the marginal value of cash kept in the firm or the first derivative of the equity value function with respect to X, E^A_X, equals the marginal value of cash paid out, 1. For a one time lump-sum dividend payment, this condition would be sufficient to guarantee the optimality of X_d [see Dixit and Pindyck (1994)]. Dumas (1991), however, points out that for continuous dividend payments, the order of differentiation increases by one, so Condition 2 holds for any X_d. This implies that an additional condition is required to guarantee the optimality of X_d.

Condition 3: $E^A_{XX}|_{X=X_d} = 0$. This is the optimality condition for X_d; At the dividend threshold ($X = X_d$), the second derivative of the equity value function with respect to X, E^A_{XX}, must be equal to zero. For more details, see Dumas (1991) and Dixit (1993).

The solution to the value of the equity takes the general form:

$$E^A(X) = A_1 e^{m_1^A X} + A_2 e^{m_2^A X}, \tag{4}$$

where

$$m_1^A = \frac{-(\nu\alpha - (c1 + c2)(1-\tau)) + \sqrt{(\nu\alpha - (c1 + c2)(1-\tau))^2 + 2\rho\sigma_L^2}}{\sigma_L^2}, \tag{5}$$

and

$$m_2^A = \frac{-(\nu\alpha - (c1 + c2)(1-\tau)) - \sqrt{(\nu\alpha - (c1 + c2)(1-\tau))^2 + 2\rho\sigma_L^2}}{\sigma_L^2}. \tag{6}$$

The coefficients A_1 and A_2 are obtained from Conditions 1 and 2, or explicitly:

$$E^A(X) = \frac{e^{m_1^A X} - e^{m_2^A X}}{m_1^A e^{m_1^A X_d} - m_2^A e^{m_2^A X_d}}, \tag{7}$$

and X_d is obtained from Condition 3:

$$X_d = \frac{2\log(\frac{m_1^A}{m_2^A})}{m_1^A - m_2^A}. \tag{8}$$

For $X \geq X_d$, the cash balances are more than enough to pay out dividends, and the firm pays all the excess cash above X_d to equityholders as dividends, keeping only X_d in the firm. The equity value has the form

$$E^A(X) = X - X_d + \frac{e^{m_1^A X_d} - e^{m_2^A X_d}}{m_1^A e^{m_1^A X_d} - m_2^A e^{m_2^A X_d}}. \tag{9}$$

Delaying dividend payments by choosing a high threshold for cash balances, X_d, allows the firm to accumulate more cash and reduce the risk of liquidation ($X = 0$), but it also reduces the value of the dividend payments to the equityholders. Therefore, the optimal dividend condition requires that at X_d , the marginal benefit of retaining one extra dollar of cash in the firm equals the marginal costs of paying that dollar out as a dividend.

Jensen and Meckling (1976) show that equityholders of levered firms have an incentive to increase risk after debt has been issued. The call feature of the equity in a levered firm increases in value with volatility. When equityholders bear some of the costs of financial distress, however, they tend to favor low risk strategies. For example, Purnanandam (2008) has shown that if a firm faces potential losses to competitors when its value declines, equityholders have incentives to reduce firm risk even if the firm is partly financed with debt. In our model, bankruptcy destroys the cash flows to equity from continuation, as well as the equityholders' exit option. The assumption that equityholders are cash constrained makes them choose a low risk strategy for the firm. Following Shleifer and Vishny (1995) we assume that firms find it difficult to attract additional outside equity after investors observe a history of poor performance.

3.2 Value of Initial Debt after Investment

After the investment, for cash balances $X \geq X_d$, the value of the original debt $D1^A$ satisfies:

$$\rho D1^A = c1 + (\nu\alpha - (c1 + c2)(1 - \tau))D1^A_X + \frac{1}{2}\sigma^2_L D1^A_{XX}. \tag{10}$$

The general solution to Equation (10) has the form

$$D1^A(X) = B^A_1 e^{m^A_1 X} + B^A_2 e^{m^A_2 X} + \frac{c1}{\rho}, \tag{11}$$

where the term $\frac{c1}{\rho}$ represents the present value of the coupon payments if the firm is never liquidated, and the term $B^A_1 e^{m^A_1 X} + B^A_2 e^{m^A_2 X}$ represents the expected value lost due to liquidation. The coupon $c1$ is set when the debt is issued at $t = 0$, and debtholders receive $c1$ payments continuously unless cash balances become zero. Formally, a boundary condition applies to $D1^A$ at $X = 0$.

Condition 4: $D1^A|_{X=0} = 0$. This condition specifies that when cash balances are zero, the firm is liquidated, and the debt value becomes zero. Consequently, debtholders also bear deadweight costs of bankruptcy.

Because the firm will not keep cash balances above X_d, the value of the debt becomes constant for $X > X_d$. Therefore, the next condition applies to $D1^A$ at $X = X_d$:

Condition 5: $D1^A_X|_{X=X_d} = 0$. This is the smooth-pasting condition at X_d. It means that the change in the debt is zero once X reaches X_d. Note that because X_d does not maximize $D1^A(X)$, there is no optimality condition associated with $D1^A$, suggesting there are agency costs of debt financing.

Conditions 4 and 5 give B^A_1 and B^A_2. Explicitly:

$$D1^A(X) = \frac{c1}{\rho}\left(1 - \frac{e^{m^A_1 X_d} e^{m^A_2 X} m^A_1 - e^{m^A_2 X_d} e^{m^A_1 X} m^A_2}{e^{m^A_1 X_d} m^A_1 - e^{m^A_2 X_d} m^A_2}\right) \tag{12}$$

It is easy to verify that $D1^A(X)$ is increasing in X_d, which means that debtholders prefer that equityholders keep more cash in the firm.

3.3 Value of Additional Debt after Investment

After investment, the value of additional debt issued to fund the investment, $D2^A$ satisfies

$$\rho D2^A = c2 + (\nu\alpha - (c1 + c2)(1-\tau) - d)D_X^A + \frac{1}{2}\sigma_L^2 D_{XX}^A. \tag{13}$$

Similar to $D1^A$, Equation (13) has the general solution:

$$D2^A(X) = C_1^A e^{m_1^A X} + C_2^A e^{m_2^A X} + \frac{c2}{\rho}, \tag{14}$$

with two boundary and smooth-pasting conditions:

Condition 6: $D2^A|_{X=0} = 0$. When the cash balances are zero, the value of the additional debt is also zero. Original debt and additional debt have equal priority.

Condition 7: $D2_X^A|_{X=X_d} = 0$. This is the smooth-pasting condition at X_d, meaning that the change in the value of the new debt is zero when the firm starts paying cash as dividends, at X_d.

C_1^A and C_2^A can be solved using Conditions 6 and 7. Explicitly,

$$D2^A(X) = \frac{c2}{\rho}\left(1 - \frac{e^{m_1^A X_d} e^{m_2^A X} m_1^A - e^{m_2^A X_d} e^{m_1^A X} m_2^A}{e^{m_1^A X_d} m_1^A - e^{m_2^A X_d} m_2^A}\right). \tag{15}$$

4 Values of Equity and Debt before Investment

Define E^B and D^B as the values of the equity and the debt before the investment, respectively. Cash balances, an instant before the investment, are denoted by X_i, and an instant after the investment by $X_i^{'}$. $X_i^{'}$ depends on the cost of the investment, I , and also on the amount raised from issuing new debt, $D2^A$, which in turn depends on $X_i^{'}$:

$$X_i^{'} = X_i - I + D2^A(X_i^{'}). \tag{16}$$

This equation allows us to write X_i in terms of $X_i^{'}$: $X_i = X_i^{'} + I - D2^A(X_i^{'})$, which shows that the investment threshold is determined by: 1) the cash balances after the investment $X_i^{'}$, and 2) the amount of additional debt the firm issues to finance the investment, itself a function of $X_i^{'}$ and $c2$. Thus, the investment policy is jointly determined by $X_i^{'}$ and $c2$. In other words, how much cash balances the firm optimally plans to have after investing and how much debt the firm issues to finance the investment determine when the firm invests.

The value of the equity an instant before investing and an instant after investing must be equal, leading to the value-matching condition:

Condition 8: $E^B|_{X=X_i} = E^A|_{X=X_i^{'}}$.

Because investment is lumpy, the smooth-pasting condition is sufficient to guarantee the optimality of the investment policy, and because the investment policy is set by X_i' and $c2$, there are two related optimality conditions:

Condition 9: $E^B_X|_{X=X_i}(1 - D2^A_X|_{X=X_i'}) = E^A_X|_{X=X_i'}$. This is the smooth-pasting condition that guarantees the optimality of X_i. To understand this condition, first consider the case of no new debt to fund investment. Then $E^B_X|_{X=X_i} = E^A_X|_{X=X_i'}$. This condition requires that the marginal value of cash in the firm, before and after investment, is the same. When the firm has low cash balances (X is low), the marginal value of cash in the firm is high; as cash balances increase, the marginal value of cash in the firm declines; when the firm accumulates enough cash so that the marginal value of cash before and after investment is the same, the firm invests.

Next, consider that the firm issues an amount of new debt equal to $D2^A$ to fund the investment. By increasing cash at the investment threshold, the firm in effect reduces the marginal value of cash before investment, hence the term $(1 - D2^A_X)$. This is true even if there are issuance costs. How much additional debt the firm will issue is determined by the next condition:

Condition 10: $E^B_X|_{X=X_i} D2^A_{c2}|_{X=X_i'} = -E^A_{c2}|_{X=X_i'}$. This condition sets the optimal coupon $c2$, or the optimal amount of additional debt. The left-hand side of the expression represents the marginal benefits from issuing additional debt: The increase in the coupon $c2$ increases the cash balances by $D2^A_{c2}$. The higher cash balances then affect marginal benefit of the new debt through the term E^B_X. Hence, the product of the two terms represents the total marginal benefit of the new debt.

The marginal costs of issuing new debt are seen on the right-hand side of Condition 10. Debt requires higher coupon payments in the future, which reduces the value of equity after investment by $E1^A_{c2}$.

Condition 11: $E^B|_{X=0} = 0$. This is the boundary condition at liquidation; it means that when the firm runs out of cash, the value of equity is zero.

4.1 Value of Equity Before Investment

Before investment, E^B, d, and σ must satisfy the expression:

$$\rho E^B = \max_{d,\sigma}\{d + (\alpha - c1(1-\tau) - d)E^B_X + \frac{1}{2}\sigma^2 E^B_{XX}\} \tag{17}$$

The following proposition establishes the optimal dividend policy before investing I:

Proposition 2 *It is not optimal to pay dividends before investing.*

Proof. See Appendix 1 ∎

The intuition behind Proposition 2 is straightforward: If the firm chooses to pay maximum dividend, it will not have enough cash balance to invest. If the investment project is profitable (the net present value is strictly positive), it is optimal to conserve cash and increase the probability of investing; hence, it is not optimal to pay dividends before investment.

The next proposition establishes the optimal risk strategy before investing I.

Proposition 3 *There is a unique risk switching point X_s at which the firm chooses σ_L if $X \leq X_s$, and chooses σ_H if $X > X_s$.*

Proof. See Appendix 2 ∎

From Proposition 2 and 3 when $0 < X < X_s$, the firm chooses σ_L, and Equation (17) becomes:

$$\rho E^B = (\alpha - c1(1-\tau) - d)E^B_X + \frac{1}{2}\sigma^2_L E^B_{XX}. \tag{18}$$

Define E^{BL} as the equity value function when σ_L is chosen. Then the general solution to Equation (18) has the form

$$E^{BL}(X) = A^L_1 e^{m^{BL}_1 X} + A^L_2 e^{m^{BL}_2 X}, \tag{19}$$

where

$$m^{BL}_1 = \frac{-(\alpha - c1(1-\tau)) + \sqrt{(\alpha - c1(1-\tau))^2 + 2\rho\sigma^2_L}}{\sigma^2_L}, \tag{20}$$

and

$$m^{BL}_2 = \frac{-(\alpha - c1(1-\tau)) - \sqrt{(\alpha - c1(1-\tau))^2 + 2\rho\sigma^2_L}}{\sigma^2_L}. \tag{21}$$

When $X_s < X < X_i$, the firm chooses σ_H, and Equation (17) becomes:

$$\rho E^B = (\alpha - c1(1-\tau) - d)E^B_X + \frac{1}{2}\sigma^2_H E^B_{XX}. \tag{22}$$

Define E^{BH} as the equity value function when σ_H is chosen. Then for $X_s < X \leq X_i$, the general solution to the previous expression has the form

$$E^{BH}(X) = A^H_1 e^{m^{BH}_1 X} + A^H_2 e^{m^{BH}_2 X}, \tag{23}$$

where

$$m^{BH}_1 = \frac{-(\alpha - c1(1-\tau)) + \sqrt{(\alpha - c1(1-\tau))^2 + 2\rho\sigma^2_H}}{\sigma^2_H}, \tag{24}$$

and

$$m^{BH}_2 = \frac{-(\alpha - c1(1-\tau)) - \sqrt{(\alpha - c1(1-\tau))^2 + 2\rho\sigma^2_H}}{\sigma^2_H}. \tag{25}$$

The boundary, smooth-pasting and optimality conditions are respectively:

Condition 12: $E^{BL}|_{X=X_s} = E^{BH}|_{X=X_s}$. This value-matching condition guarantees the continuity of the value function at the risk switching point.

Condition 13: $E_X^{BL}|_{X=X_s} = E_X^{BH}|_{X=X_s}$. This smooth-pasting condition ensures the continuity of the first derivative. It means that the marginal value of following a high risk strategy is equal to the marginal value of following a low risk one. Because the control is instantaneous, this condition is not sufficient to guarantee the optimality of X_s. We need a higher derivative for such optimality [see Dumas (1991) for details].

Condition 14: $E_{XX}^{BL}|_{X=X_s} = E_{XX}^{BH}|_{X=X_s}$. This condition guarantees the optimality of X_s. It means that the change in the marginal value of the high risk strategy should equal the change in the marginal value of the low risk strategy.

Also, note that when X is approaching zero, equityholders choose low risk, σ_L, so the value function at liquidation is E^{BL}. Therefore, Condition 11 becomes $E^{BL}|_{X=0} = 0$.

The coefficients A_1^L, A_2^L, A_1^H and A_2^H are obtained from Conditions 8 and 11-13. They can be solved analytically and are given in Appendix 3.

The optimal investment threshold X_i (as determined by $X_i^{'}$ and $c2$) is obtained from Conditions 9 and 10, but it is not possible to solve the problem analytically, and we must resort to a numerical solution.

Finally, the optimal risk policy X_s is obtained by Condition 14, and is given by

$$X_s = \frac{2\log(\frac{m_1^{BL}}{m_2^{BL}})}{m_1^{BL} - m_2^{BL}}. \tag{26}$$

It is interesting that, from Equation (26), for a given coupon $c1$, the risk switching threshold X_s is independent of the multiplier ν of the cash flows after the investment I is made. At first, this result may seem counter-intuitive: The higher ν is, the more profitable the investment, so the firm should be more conservative and increase X_s. To understand this, consider the marginal benefits and costs of changing risk (through changing X_s). An increase in ν heightens the incentives not only to reduce the probability of liquidation (by increasing X_s) but also to increase the probability of investing (by lowering X_s). The two effects seem to offset each other, and leave X_s unchanged.[1]

This is a bit more complicated, however, because although v does not directly affect X_s, it does so indirectly through $c1$, as the next proposition states:

[1] Another way to see this result is to recognize that $E^{BL}(X)$ can be written as $E^{BL}(X) = E^A(X_i^{'})F(X)$, and $E^{BH}(X) = E^A(X_i^{'})G(X)$, where $E^A(X_i')$ is a function of ν, and $F(X)$ and $G(X)$ are independent of ν. Condition 14 can then be written as $E^A(X_i^{'})\,[F_{XX}(X_s) - G_{XX}(X_s)] = 0$, which implies that a change in ν affects equally $F_{XX}(X_s)$, and $G_{XX}(X_s)$.

Proposition 4 *Everything else constant, for a low α, X_s is a monotonically decreasing function in $c1$. For a high α, there is a unique $\overline{c1} \geq 0$, such that X_s is monotonically increasing with $c1$ for $c1 < \overline{c1}$, and monotonically decreasing with $c1$ for $c1 \geq \overline{c1}$.*

Proof. See Appendix 4. ■

Proposition 5 states that the effect of leverage on risk management differs, depending on whether firms have a high or a low cash flow generating ability relative to their debt obligations (interest rate coverage ratio). First, for firms with a low coverage ratio - high leverage (high $c1$) or low current cash flows (low α) - there is a small chance that there will be enough cash balance to make the investment, so as leverage increases, these firms tend to seek higher risks in order to increase the likelihood of investing. An increase in leverage only makes these firms more aggressive. (Higher $c1$ leads to lower X_s.)

Next, consider firms with high cash flow generating ability (high α). These firms have strong incentives to reduce the probability of being liquidated, so when the level of debt is not too high (low $c1$), an increase in leverage will make equityholders more conservative (Higher $c1$ leads to higher X_s).

4.2 Value of Initial Debt Before Investment

Since σ changes at X_s, the value function of the initial debt also changes. Define D^{BL}as the debt value function for $0 < X < X_s$. $D^{BL}(X)$ satisfies the equation:

$$\rho D1^{BL} = c1 + (\nu\alpha - (c1 + c2)(1 - \tau))D1^{BL}_X + \frac{1}{2}\sigma_L^2 D1^{BL}_{XX}, \tag{27}$$

with the general solution:

$$D1^A(X) = B_1^{BL} e^{m_1^{BL} X} + B_2^{BL} e^{m_2^{BL} X} + \frac{c1}{\rho}, \tag{28}$$

Define $D1^{BH}(X)$ as the debt value function for $X_s < X < X_i$. $D1^{BH}$ satisfies the equation:

$$\rho D^{BH} = c1 + (\nu\alpha - (c1 + c2)(1 - \tau))D1^{BH}_X + \frac{1}{2}\sigma_H^2 D1^{BH}_{XX}, \tag{29}$$

with the general solution:

$$D1^A(X) = B_1^{BH} e^{m_1^{BH} X} + B_2^{BH} e^{m_2^{BH} X} + \frac{c1}{\rho}, \tag{30}$$

and the boundary and smoothness conditions:

Condition 15: $D1^{BL}|_{X=0} = 0$. This condition means that at liquidation the value of the debt is zero.

Condition 16: $D1^{BL}|_{X=X_s} = D1^{BH}|_{X=X_s}$. This condition guarantees the continuity of the value function at X_s.

Condition 17: $D1_X^{BL}|_{X=X_s} = D1_X^{BH}|_{X=X_s}$. This condition at X_s ensures the continuity of the first derivative. Notice that since X_s is chosen to maximize the value of the equity ex post, there is no optimality condition for the initial debt.

Condition 18: $D1^{BH}|_{X=X_i} = D1^{A}|_{X=X_i^{'}}$. This is the value-matching condition at X_i. Again, X_i is chosen to maximize the value of equity ex post, so there is no optimality condition.

These four conditions determine the coefficients B_1^{BL}, B_2^{BL}, B_1^{BH}, and B_2^{BH} analytically. (see Appendix 5).

4.3 Initial Cash Balances

At $t = 0$, the firm decides how much cash balances it must have. Equityholders are cash constrained and only contribute with X_0. The rest comes from issuing debt. Define X_0 as the cash balances just before the initial debt is issued, and $X_0^{'}$ as the cash balances after the initial debt is issued. Because the amount of cash raised from the issuance of the initial debt $D1^B$ depends on the level of cash balances after debt is issued $X_0^{'}$, we write

$$X_0^{'} = X_0 + D1^B(X_0^{'}). \tag{31}$$

The level of cash balances before debt is issued, X_0, is given exogenously, so how much debt the firm initially issues determines the cash balances it will have, $X_0^{'}$. In other words, there is only one degree of freedom in Equation (31), and the choice of the coupon payments $c1$ determines $D1^B(X_0^{'})$ and $X_0^{'}$, for a given cash flow generating process defined in Equation (1). Formally, at $t = 0$, the equityholders choose $c1$ to maximize equity value, subject to the constraint that $X_0^{'}$ satisfies Equation (31), and that $X_i^{'}$, X_s and $c2$ maximize equity value, given that the initial debt and cash balances $X_0^{'}$ are already decided. That is, the equityholders

$$\max_{c1} E^B(X_0^{'}), \tag{32}$$

subject to:

1) Condition 3: The optimality condition that determines the dividend threshold, X_d.

2) Condition 9: The smooth-pasting condition that determines the investment threshold, $X_i^{'}$.

3) Condition 10: The smooth-pasting condition that determines the optimal amount of new debt at the moment of investing, $c2$.

4) Condition 14: The optimality condition that determines the risk switching point, $X_s \leq X_i^{'}$

5) Equation (31): The constraint that determines the initial cash balance $X_0^{'}$.

6) The bounds of the choice variable: $0 \leq X_i^{'}, X_s, X_d, c1, c2 < \infty$. This restriction implies that the choice variables must be finite and non-negative.

Because the value function is continuous, the optimal $c1$ always exists, but we cannot rule out cases with corner-solutions. The lowest coupon the firm can choose is zero, and the maximum must be finite. The solution to the equityholders' maximization problem can only be obtained numerically.

5 Value of Equity with No Investment

Equityholders may forgo the investment opportunity altogether if the value created by investing is not high enough to justify the cost of investing. In this case, the instantaneous cash flow rate is α, as long as the firm keeps operating. Define E^N and D^N to be the value functions for the equity and the debt of a firm with no investment opportunities, respectively. Denote c_N as the coupon payment to creditors, and X_d^N as the dividend threshold for such a firm. E^N can be viewed as a special case of E^A in which $\nu = 1$, $I = 0$, $c1 = c_N$ and $c2 = 0$ i.e., for $X < X_d^N$:

$$E^N(X) = \frac{e^{m_1^N X} - e^{m_2^N X}}{m_1^N e^{m_1^N X_d^N} - m_2^N e^{m_2^N X_d^N}}, \tag{33}$$

and

$$D^N(X) = \frac{c_N}{\rho}\left(1 - \frac{e^{m_1^N X_d^N} e^{m_2^N X} m_1^N - e^{m_2^N X_d^N} e^{m_1^N X} m_2^N}{e^{m_1^N X_d^N} m_1^N - e^{m_2^N X_d^N} m_2^N}\right), \tag{34}$$

where $m_1^N = m_1^{BL}$, and $m_2^N = m_2^{BL}$, and

$$\begin{aligned} X_d^N &= \frac{2\log(\frac{m_1^N}{m_2^N})}{m_1^N - m_2^N} \\ &= X_s. \end{aligned} \tag{35}$$

If $X \geq X_d^N$,

$$E^N(X) = X - X_d^N + E^N(X_d^N), \tag{36}$$

and

$$D^N(X) = \frac{c_N}{\rho}\left(1 - \frac{e^{m_1^N X_d^N} e^{m_2^N X_d^N} m_1^N - e^{m_2^N X_d^N} e^{m_1^N X_d^N} m_2^N}{e^{m_1^N X_d^N} m_1^N - e^{m_2^N X_d^N} m_2^N}\right). \tag{37}$$

At $t = 0$ the firm chooses X_0^N, the level of initial cash balances it plans to have after the debt D^N is issued. The change in cash balances is:

$$X_0^N = X_0 + D^N(X_0^N). \tag{38}$$

So at $t = 0$, the equityholders solve the problem:

$$\max_{c_N} E^N(X_0^N), \tag{39}$$

subject to:

1) Equation (35): This is the equation that sets the dividend threshold, X_d^N.

2) Equation (36): This is the constraint that determines the initial cash balance, X_0^N.

3) The bounds of the choice variables: $0 \leq X_d^N, c_N < \infty$, which must have finite and non-negative values.

It is worth noting that $X_d^N = X_s$ is no coincidence. That is, the dividend threshold of a firm with no investment opportunities is the same as the risk switching point of an otherwise identical firm but with investment opportunities. A proof of this is provided in Appendix 2.

Firms keep cash balances up to the point where the probability of default is sufficiently low. Recall that cash balances held have an instantaneous opportunity cost of ρ, for they do not earn any interest. In this regard, firms with and without investment opportunities have the same threshold. Yet, once firms accumulate enough cash balances, the actions of the two types of firms start to diverge: Firms with investment opportunities continue to accumulate cash, but since they already have enough cash to avoid bankruptcy, they can adopt a riskier strategy to speed up investment. Firms without investment opportunities, on the other hand, have no reason to accumulate more cash, and therefore decide to pay out all excess cash as dividend.

It is often argued that firms with investment opportunities should have lower leverage than firms without investment opportunities, or that $c_N > c1$. There are two reasons for this: First, servicing the debt reduces the cash flows and gives rise to underinvestment. Second, debt increases the probability of bankruptcy and can destroy the value of investment opportunities. These reasons are clearly seen in our model. Notice that from Equation (19), $c1$ reduces the net cash flows and hence slows down the accumulation of cash in the firm. Also, from Equations (20) and (21), $c1$ affects the terms m_1 and m_2, and increases the probability of default.

In our setting, however, it is possible that the firm with investment opportunities operates with higher leverage than the firm with no investment opportunities, $c_N < c_1$, because cash balances partly funded with debt increase the probability of investing. To verify this, differentiate Equations (19) and (23) with respect to X: $E_X^{BH}, E_X^{BL} > 0$, which implies that an increase in X increases the probability of investing and thus increases the value of the equity both in high and low-risk strategies. Although initial debt reduces future cash flows and increases the probability of default, it does help to increase initial cash balances.

In equilibrium, the firm trades off this cost against the benefit of having initial debt. If the value of the investment opportunity is sufficiently high, it may be optimal for firms with investment opportunities to have higher leverage ratios than those without such opportunities.

6 Numerical Example

Both the extent of the problem and the path dependency of cash balances make it impossible to derive analytical solutions. We thus resort to numerical methods. There are eight input parameters to the model: $\alpha, \sigma_L, \sigma_H, \nu, I, \rho, \tau$ and X_0. The parameter values used in the numerical example are based on ten-year historical annual data of active, non-financial U.S. firms from the Compustat dataset, for the period $1997-2006$. Years $2007-09$ are not used to avoid the effects of the 2008 financial crisis. Any firms with fewer than ten-years of observations, or with total asset values lower than \$100 million are excluded. The final dataset consists of $1,489$ firms.

First, we find an estimate of α. We compute the average cash flow rate for each firm in the sample. Cash flow rate is defined as the change in earnings before interest, tax, depreciation and amortization (EBITDA) minus the change in net working capital, and minus the change in net fixed assets, divided by the average total assets of the firm. The reason why we include the change in net working capital and the change in fixed assets in the calculation is because even without a large new investment, firms have to reinvest cash in existing operations to replace depreciated assets and to finance any necessary changes to these assets. The average cash flow rate for all firms is 0.075 per year, with a standard deviation of 0.078. So α is set as 0.075.

Next, we find estimates for σ_L and σ_H. We compute the standard deviation of the cash flow rate for each firm, and then find the average standard deviation for all the firms in the sample. The average standard deviation of the cash flow rate is 0.15, with a standard deviation of 0.13. We assume that the observed standard deviation is the low volatility of the cash flow, i.e., $\sigma_L = 0.15$, and that the firm has the ability to increase the standard deviation of the cash flow to $\sigma_H = 0.20$. This 33 percent change in the standard deviation of a cash flow rate is not unreasonable. For example, this can be achieved if a firm changes its hedging by 33 percent of the output.

We then estimate the scaling factor ν by calculating the five-year average of the cash flow rate between 1996 and 2001 and between 2002 and 2006. During $1996-2001$, the estimated average of cash flow rates is 0.0635, and during $2002-2006$ it increases to 0.087. The increase is approximately 37 percent, so we set the scaling factor as $\nu = 1.37$.

To find an estimate for I, we calculate the five-year average of the fixed assets and working capital, divided by the total assets for the period $1997-2001$, and repeat the calculation for the period $2002-2006$. During $1997-2001$ it is 0.65, and during $2002-2006$ it increases to 0.89. We then assume that $I = 0.89 - 0.65 = 0.24$.

Next, the discount rate chosen for the firm is equal to 10 percent. The average three-month Treasury-bill rate during $1997-2006$ is about 3 percent. If we assume a market risk premium of 5 percent, then the representative firm has a beta of 1.4.

The initial cash that equityholders invest in the firm X_0 is not observable. We choose 0.08, since this number gives a leverage ratio of 0.35, the average leverage ratio of all firms in the sample.

Finally, the corporate tax rate is assumed to be 35 percent, the statutory tax rate.

Table 1 summarizes the parameters values chosen for the example. The output of the model is shown in Table 2.

At $t = 0$, the equityholders put $X_0 = 0.08$ in the firm, and the firm optimally borrows $D1^{BL} = 0.2404$ by issuing perpetual debt with $c1 = 0.0390$. As a result, the firm has an initial cash balance equal to $X_0' = X_0 + D1^{BL} = 0.08 + 0.2404 = 0.3204$. This is lower than the optimal risk switching point, $X_s = 0.3696$, so the firm chooses a low risk strategy, i.e., $\sigma_L = 0.15$. The firm will switch back and forth between the high and low risk strategies whenever the cash balance crosses X_s. At $t = 0$, the equity value of the firm E^{BL} is 0.4531, and the debt value $D1^{BL}$ is 0.2404, making the total market value of the firm equal to 0.6935. The leverage ratio is 35 percent.

When cash balances reach $X_i = 0.4221$, the firm invests $I = 0.24$. At this point the value of the equity E^A is 0.5564, and the value of original debt $D1^A$ is 0.2470. The investment costs the firm 0.24, but will increase the cash flow rate by 37 percent. The firm optimally finances the investment with additional debt of $D2^A = 0.1961$, and coupon $c2 = 0.0309$. Cash balances after investment are $X_i' = X_i - I + D2^A = 0.4221 - 0.24 + 0.1961 = 0.3782$. Because cash balances after investment are lower than the dividend threshold level $X_d = 0.3951$, the firm waits and does not pay any dividend until cash balances reach X_d.

Next, compare this firm with a second firm with the same parameters but no investment (growth) opportunity. Table 3 shows that this second firm has equity value equal to $E^N = 0.4190$, which is less than the value of the equity of the first firm which has an investment opportunity. It issues initial debt with coupon $c^N = 0.0370$, and market value $D^N = 0.2048$, making the initial cash balance $X_0' = X_0 + D^N = 0.08 + 0.2048 = 0.2848$. The leverage ratio is 0.3284. The firm always follows a low risk strategy, i.e., $\sigma_L = 0.15$. It accumulates cash balances up to the dividend threshold of $X_d = 0.3744$, after which it starts to pay dividends.

The example shows that a firm with investment opportunities does not necessarily have less leverage.

Such a firm wants to have high cash balances to be as close as possible to the investment threshold. In the example, the incentive to borrow to maintain high cash balances and invest outweighs the costs of having more debt.

Also, note that even though the firm with the investment opportunity has a choice to invest right away by issuing enough debt to finance the investment, it finds that this is not optimal, because the amount it would have to borrow would be just too much and create a debt overhang problem. As Table 4 shows, if the firm invests at $t = 0$, it has to issue debt $D1^A = 0.4188$ with a coupon $c1 = 0.0711$. The cash balance after paying the cost of the investment $X_0^{'}$ is $0.08 + 0.4188 - 0.24 = 0.2588$, and the dividend threshold $X_d = 0.3928$. The equity value $E^A = 0.4271$, less than in the base case previously illustrated. This shows that the flexibility to time the investment as well as the ability to manage risk add value to the firm (in the example just shown 5.74 percent).

In sum, financial policy, investment and risk management interact differently for firms with different characteristics.

6.1 Effect of Changes in X_0 on $X_0^{'}$, X_s, X_i, $c1$ and $c2$

How does the initial cash invested by the equityholders, X_0, change the financial, risk and investment policies of the firm? If equityholders are severely cash constrained, they can only put a relatively small amount X_0 in the firm. Usually, the firm will operate with a high leverage ratio, but the debt reduces future cash flows and potentially creates an overhang problem. Figure 1 shows how the base case results change as the cash invested in the firm by the equityholders at $t = 0$ varies.

As X_0 becomes larger, the initial cash $X_0^{'}$ becomes larger, as seen in Panel 1, and the firm does not need to borrow as much, as seen in Panel 4; $c1$ declines as X_0 increases. Also, lower initial debt levels induce the firm to take on less risk, as indicated by the higher switching point, X_s in Panel 2. This happens because for a given level of cash flow the relation between $c1$ and X_s is negative. For firms with higher operating cash flow rates, α, this might not be the case.

Also, lower initial debt allows the firm to issue more debt later on to finance the investment, as seen by the rising $c2$ in Panel 4. Both lower initial debt and higher additional debt allow the firm to invest more quickly and reduce the underinvestment problem as indicated by the lower X_i in Panel 3.

6.2 Effect of Changes in v on $X_0^{'}$, X_s, X_i, $c1$ and $c2$

How do the results change as the investment cash flow multiplier, ν, changes? Panel 1 of Figure 2 shows that as ν increases, the firm wants to have larger cash balances in order to invest sooner. So, initial cash balances $X_0^{'}$ increase with ν. As a result of the higher $X_0^{'}$, the firm is able to lower the investment threshold X_i, as seen by the declining X_i in Panel 3.

In Panel 4, an increase in ν leads to increases in $c1$ and $c2$. The higher ν is, the more able the firm is to finance it through the issuance of additional debt, and the higher is the incentive to borrow more at the initial date and start with higher cash balances. This result assumes that I does not change, so it should be interpreted as a result relating to the profitability of making the investment.

The relation between ν and X_s is shown in Panel 2. The more valuable the investment opportunity, the lower the risk switching point, X_s, which means that the firm engages in a high risk strategy sooner and for a longer period of time. This is because as ν increases, $c1$ increases (see Panel 4), and an increase in $c1$ leads to a reduction in X_s. Recall from Proposition 4 that a negative relation between $c1$ and X_s holds if α is not high.

6.3 Effect of Changes in $c1$ on X_s

The solid line in Figure 3 shows the relation between initial leverage $c1$ and risk policy, X_s, for a firm with low cash flow rate, $\alpha = 0.075$. An increase in the coupon $c1$ reduces the cash flows to equityholders. As a result, equityholders have less incentive to keep the firm operating under a low risk strategy, hence a lower X_s.

Figure 3 (dashed line) also shows that the relation between leverage, $c1$, and risk management, X_s, is not monotonic in the case of firms with high cash flow generation. In this case, equityholders have strong incentives to keep firms solvent. Therefore, if leverage increases, equityholders try to make the firms safer by pursuing a low risk strategy (increase X_s), provided that $c1$ is not too high. If leverage is very high, however, cash flows after coupon payments are low and a debt overhang problem may result. Keeping a low risk strategy is not optimal and equityholders will follow a high risk strategy more often (reduce X_s).

7 Profile of the Investment and Risk Management

So far we have dealt with investment that is lumpy. It costs the firm a non-trivial amount, I, to invest, and the firm must have the financial capacity to pay for the cost of the investment all at once. Although many corporate investments have the characteristics of real options, this is an obvious simplification.

When investment is irreversible with a contingent, lump-sum up-front cost, we have shown that it can be optimal for a financially constrained firm to adopt a high risk strategy: When the option to invest is out of the money - the cash balances X are lower than the cash balance threshold to invest, X_i - and the firm is sufficiently far from the bankruptcy threshold (X is far from 0), volatility works in favor of the firm. After the option is exercised, if the firm does not have any other investments left, it gains nothing from following a high risk strategy, and reverts to a low risk strategy. Our model predicts that financially constrained firms with investment opportunities that require large fixed adjustment costs may intentionally choose higher risk strategies.

This result contrasts with the results on risk management in other studies. Leland (1998) and Bolton *et al.* (2009) find that, unless there are large costs associated with risk reducing, firms with investment opportunities tend to follow low risk strategies. Hence the question: Do different investment profiles affect corporate risk management differently?

To analyze this question, we consider a firm that is similar in every way to the firm in our base model, but makes continuous investments, instead of a large investment. For this firm, investment costs i are paid each instant and increase the instantaneous operating cash flow by a factor of λ, so the new cash flow rate becomes $\lambda\alpha$. If $\lambda\alpha - i \geq \alpha$, the firm should invest continuously because investment increases cash flows and the value of equity.

Define E^i as the equity value function of such firm. Then E^i, σ, and d satisfy the equation:

$$\rho E^i = \max_{\sigma,d}\{d + (\lambda\alpha - c(1-\tau) - d - i)E^i_X + \frac{1}{2}\sigma^2 E^i_{XX}\} \tag{40}$$

Similar to the results in Section 3, the first-order condition with respect to d is $E^i_X = 1$, which implies that the firm maintains all cash in the firm as long as the marginal benefit of holding cash is greater than one, and pays out all cash when the marginal benefit of holding cash equals one, and the first-order condition with respect to σ is $E^i_{XX} = 0$, implying that the firm chooses σ_L if $E^i_{XX} < 0$, and chooses σ_L otherwise.

The equity value function before the firm pays dividends is $E^i = \frac{e^{m^i_1 X} - e^{m^i_2 X}}{m^i_1 e^{m^i_1 X^i_d} - m^i_2 e^{m^i_2 X^i_d}}$, where $X^i_d = \frac{2\log(\frac{m^i_1}{m^i_2})}{m^i_1 - m^i_2}$, and $m^i_1 > 0$, and $m^i_2 < 0$. It, then, can be verified that $E^i_{XX} < 0$, and $E^i_X > 1$ for $X \in [0, X^i_d)$. The firm pays dividends for cash balances above X^i_d; therefore, for $X \in [X^i_d, \infty)$, $E^i_{XX} = 0$. So E^i_{XX} is always non-positive, suggesting that the firm chooses σ_L at all times.

In other words, when the cash balance is low and $E^i_{XX} < 0$, the firm chooses a low risk strategy, and since the firm pays out all residual cash flows as dividends when $E^i_X = 1$, cash balances will never get to the level at which $E^i_{XX} \geq 0$, hence the firm will never switch to a high risk strategy.

The continuous investment i can be thought of as small incremental investments that the firm makes on a regular basis, such as R&D and small upgrading expenditures. Under this type of investment policy, cash flow volatility hurts firm value, since it increases the probability that the firm will not have the cash to pay for the cost of investing i. Minton and Schrand (1999) empirically find that for this type of investment, volatility reduces firm value. The former CFO of Merck, Judy Lewent, in an interview [Nichols(1994)] also indicates that cash flow stability helped Merck sustain a steady flow of investments needed for its pipeline of drugs.

8 Asymmetrical Information: What Changes in Risk Management and Dividend Reveal to Investors

Cash flows from existing assets and potential growth opportunities are the most important variables in valuing a company. Firms' choices regarding risk and dividend policies reveal something to outside investors about the firm's cash flow generating ability and its growth options.

We assume that investors observe the firm's current cash balances and dividends, as well as whether a firm changes its risk strategy. With this information, outside investors form expectations about the firm's cash flow rate from existing assets, α, and the investment opportunities.

For simplicity, the firm may have one of two possible cash flow rates, α_1 or α_2, where $\alpha_1 < \alpha_2$. The firm may or may not have investment opportunities. Thus, there are four possible types of firms, depending on the cash flow rates and the investment opportunities:

Type-1 firm has a low cash flow rate (α_1) and no investment opportunity. Type-2 firm has a low cash flow rate (α_1) and a profitable investment opportunity. Type-3 firm has a high cash flow rate (α_2) and no investment opportunity. Type-4 firm has a high cash flow rate (α_2) and a profitable investment opportunity. The equity values of the four types are denoted by E^1, E^2, E^3 and E^4, respectively.

From our analysis so far we are able to conclude that:

If the firm is of type-1, it will use a low risk strategy at all times, and will pay dividends at $X_d^{\alpha_1}$. If the firm is of type-2, it will use a low risk strategy if $X < X_s^{\alpha_1}$, otherwise, it will use high a risk strategy before it invests. If the firm is of type-3, it will use a low risk strategy at all times and will pay dividends at $X_d^{\alpha_2}$. Finally, if the firm is of type-4, it will use a low risk strategy if $X < X_s^{\alpha_2}$, otherwise it will use a high risk strategy before it invests.

It is obvious that a type-1 firm has the lowest equity value of all, and that a type-4 firm has the highest equity value of all, or $E^1 = \min[E^1, E^2, E^3, E^4]$ and $E^4 = \max[E^1, E^2, E^3, E^4]$. However, the value of a type-2 firm may be higher or lower than that of a type-3 firm, depending on the relative values of α and the investment opportunity.

Investors' expectations of the value of the equity are a function of the unobserved cash flow rate and the investment opportunities, given the observed actions of the firm and the current cash balances. The expectation can be expressed as $\overline{E}|_{X,Y}$, where X are the observed cash balances, and Y is a variable that takes on three possible values: 1 if the firm pays dividends, 2 if the firm changes its risk strategy, and 0 if the firm neither pays dividends nor switches risk.

From Equation (35), we know that the risk switching point of the firm with investment opportunities is the same as the dividend threshold of the firm without investments. As there are two possible cash flow rates, there are two levels of cash balances at which investors expect the firm either to pay dividends or to change risk. Define the first expected level as $X^{\alpha_1} = X_d^{\alpha_1} = X_s^{\alpha_1}$, and the second expected level as $X^{\alpha_2} = X_d^{\alpha_2} = X_s^{\alpha_2}$. Figure 4 shows that X^{α_1} can be higher or lower than X^{α_2}, depending on the values of α_1 and α_2: When α_1 and α_2 are low, $X^{\alpha_1} < X^{\alpha_2}$, and when they are high $X^{\alpha_1} > X^{\alpha_2}$. So, we consider two cases:

8.1 Case 1: $X^{\alpha_1} < X^{\alpha_2}$

Before X reaches X^{α_1}, outside investors observe no action ($Y = 0$) and every type of firm uses σ_L, so they learn nothing about the firm's type. The expected equity value is an equally weighted average of the values of the various possible firm types:

$$\overline{E}|_{X<X^{\alpha_1},Y=0} = \frac{E^1 + E^2 + E^3 + E^4}{4}. \tag{41}$$

Once the cash balances reach X^{α_1}, if investors observe that the firm pays dividends ($Y = 1$), they conclude that the firm has no investment opportunity in sight and must be a type-1 firm. Therefore,

$$\overline{E}|_{X=X^{\alpha_1},Y=1} = E^1. \tag{42}$$

Note that in this event, there would be a drop in the market value of the firm's equity, since $E^1 < \overline{E}|_{X<X^{\alpha_1},Y=0}$.

If at X^{α_1}, outside investors observe a change in the firm's risk strategy from σ_L to σ_H, ($Y = 2$), they might speculate that the firm is taking greater risks presumably to boost its future investment opportunities. Hence they conclude that the firm is of type-2. The equity value is:

$$\overline{E}|_{X=X^{\alpha_1},Y=2} = E^2. \tag{43}$$

This can result in a rise or fall in the equity value, depending on the relative values of E^2 and E^3.

However, if the firm does nothing ($Y = 0$), it signals to outside investors that it is generating high cash flow rates (α_2), and must be either type-3 or type-4, resulting in an expected equity value of:

$$\overline{E}|_{X=X^{\alpha_1},Y=0} = \frac{1}{2}E^3 + \frac{1}{2}E^4, \tag{44}$$

leading to an upward revision in the value of the equity.

At the next level of cash balances, X^{α_2}, a type-3 firm will pay dividends and a type-4 firm will take greater risks. Outside investors will revise their expectations of the value of the equity to E^3 for the type-3 firm and to E^4 for the type-4 firm:

$$\overline{E}|_{X=X^{\alpha_2},Y=1} = E^3, \tag{45}$$

and

$$\overline{E}|_{X=X^{\alpha_2},Y=2} = E^4. \tag{46}$$

The value of the equity either drops to E^3 or increases to E^4. We can use a numerical example to illustrate the value revisions by outside investors.

First assume that the firm has no initial debt and finances investment solely from internal cash balances. The set of parameter values are: $\alpha_1 = 0.07$, $\alpha_2 = 0.08$, $\nu = 1.5$, $I = 0.30$, $\tau = .35$, $\rho = 0.1$, $\sigma_L = 0.15$ and $\sigma_L = 0.20$. Suppose that the true type of the firm is 4 ($\alpha_2 = 0.08$), but investors do not know that. Panels 1 and 2 in Figure 5 plot the cash balances and outside investors' expectations of the equity values over time, measured in months. If the firm is seen as having $\alpha_1 = 0.07$, then $X^{\alpha_1} = X_d^{\alpha_1} = X_s^{\alpha_1} = 0.4320$, which is reached in month 26, as seen in Panel 1. If, on the other hand, the firm is seen as having $\alpha_2 = 0.08$, then $X^{\alpha_2} = X_d^{\alpha_2} = X_s^{\alpha_2} = 0.4359$, which is reached in month 31.

A type-4 firm will take on high risk only after its cash balance reaches 0.4359. So, in month 26 when the cash balance is 0.4232, it does nothing. The fact that the firm does not switch from σ_L to σ_H or pay dividends reveals to outside investors that the firm has a high cash flow rate, α_2, and therefore must be either type-3 or type-4. The market value of the equity will therefore increase on this date (month 26), as seen in Panel 2.

When cash balances increase to 0.4359 in month 31, the firm switches to a high risk strategy, and investors then conclude that it is a type-4 firm. The market value of the equity will increase again at this point. In this example, one can expect to see two upward jumps in firm value, as shown in Panel 2.

8.2 Case 2: $X^{\alpha_1} > X^{\alpha_2}$

When the cash balances reach $X^{\alpha_2} = X_s^{\alpha_2} = X_d^{\alpha_2}$, a firm with α_2 (type-3 or type-4) either pays dividends or changes risk policies (see Panels 3 and 4 in Figure 5). A firm with α_1 (type-1 or type-2) will wait until its cash balances reach $X^{\alpha_1} = X_s^{\alpha_1} = X_d^{\alpha_1}$.

When the firm's cash balance reaches X^{α_2}, if investors observe a dividend ($Y = 1$), they will conclude that the firm has α_2 and no investment opportunity; therefore, it is a type-3 firm. At this point, the equity value becomes

$$\overline{E}|_{X=X^{\alpha_2},Y=1} = E^3. \tag{47}$$

Depending on the relative values of E^2 and E^3, the equity value may go up or down at this point.

If the firm switches risk ($Y = 2$), it reveals that it has α_2 and an investment opportunity, therefore it is a type-4 firm. The equity value increases to

$$\overline{E}|_{X=X^{\alpha_2},Y=2} = E^4. \tag{48}$$

If, however, investors observe no actions ($Y = 0$), they'll conclude that the firm has α_1 and must be either type-1 or type-2. The expected equity value declines to

$$\overline{E}|_{X=X^{\alpha_2},Y=0} = \frac{1}{2}E^1 + \frac{1}{2}E^2. \tag{49}$$

At X^{α_1} the firm's true type is revealed. A dividend payment ($Y = 1$) signals that the firm has no investment opportunity and is of type-1. Risk switching ($Y = 2$) signals that the firm has an investment opportunity and is of type-2. The corresponding equity values for type-1 and type-2 are

$$\overline{E}|_{X=X^{\alpha_1},Y=1} = E^1, \tag{50}$$

and

$$\overline{E}|_{X=X^{\alpha_1},Y=2} = E^2. \tag{51}$$

At X^{α_1}, the equity value will decline if it is type-1 and increase if it is type-2.

A numerical example illustrates the equity value revisions by outside investors. Suppose that the unobservable true type of the firm is type-1. Here $\alpha_1 = 0.13$ and $\alpha_2 = 0.14$, and the rest of the parameters are as in Case 1. Panels 3 and 4 of Figure 5 plot the cash balances and the equity values against time measured in months. Investors know that a firm with $\alpha_2 = 0.14$ will pay dividends or will switch risk at $X^{a_2} = X_d^{a_2} = X_s^{a_2} = 0.4296$, which is reached in month 12 (in Panel 3). A firm with $\alpha_1 = 0.13$ has $X^{a_2} = X_d^{a_2} = X_s^{a_2} = 0.4354$, a value that is reached in month 18. A type-1 firm pays dividends only when cash balances reach 0.4354, and the firm does nothing in month 12 when cash balances are 0.4296. At this point, the market value of the equity falls because investors realize that the firm must have α_1. The equity value drops again later, in month 18, when cash balances are 0.4354, and the firm decides to pay dividends. At this point, investors know that the firm is indeed a type-1 firm. Panel 4 shows that the expected equity value drops twice in months 12 and 18.

9 When Firms Issue Equity to Finance Investment

In this section, we extend the base model and allow the firm to finance investment by issuing additional equity. This allows the firm to invest sooner. Although, equity financing reduces the share of the profits of the existing equityholders, if additional equity is sold at market price, the reduction in profits is offset by the increase in the cash received from new equity. If equity issuance were costless, the firm should finance investment with equity. In reality, equity has significant fixed issuance costs,[2] so the firm compares whether the reduction in the default probability is worth the fixed costs.

For firms that have high cash balances and are far away from the default threshold, the benefits of equity financing will not be very high. Therefore, it does not take a high equity issuance costs for these firms to forego outside equity financing altogether.

To analyze the effect of equity financing formally, we assume that the firm already has cash balances X_0, and existing debt $D1^B$ with coupon $c1$ in place. At the time of the investment, the firm can sell more equity and debt to fund the investment. The fraction of the equity sold to outside equityholders is denoted by f. The costs of equity issuance are fixed and denoted by $k > 0$.

Next, define $E1^A$ and $E2^A$ as the values, after the investment, of the existing equity and the new equity, respectively. Also, define $E1^B$ as the value before the investment of the existing equity. After investment, the value of the existing equity is reduced by the fraction sold to the new equityholders. Thus, the value of the existing equity after investment is

$$E1^A = (1 - f)E^A, \tag{52}$$

and the value of the new equity after investment is

$$E2^A = fE^A, \tag{53}$$

where E^A is the total equity value and is equal to $E1^A + E2^A$.

The cash balances after investment, $X_i^{'}$, now depend also on the costs k, and on the amount of new equity raised $E2^A$, which is a function of $X_i^{'}$, so the change in cash balances at the time of investment is given by:

$$X_i^{'} = X_i - I + D2^A(X_i^{'}) + E2^A(X_i^{'}) - k. \tag{54}$$

Because the dividend received by the existing equityholders is proportional to the fraction of the equity

[2] Equity also has dilution costs from the underpricing of new shares. Although we do not consider such costs in the model, they can be incorporated by assuming that the share of the future profits generated by the investment is allocated disproportionally to the new equityholders, who pay less than the value of these profits.

they own, the boundary, smooth-pasting and optimality conditions after investment remain the same. At the investment threshold, however, the following additional condition must hold:

Condition 19: $E1^B_X|_{X=X_i} E2^A_f|_{X=X'_i} = -E1^A_f|_{X=X'_i}$. This is a smooth-pasting condition that must apply at the time of investing in order to make f optimal. Again, this condition equates the marginal benefits and the marginal costs of changing f. The left-hand side of the condition is the marginal benefit of issuing additional equity. On the left-hand side, the term $E2^A_f$ is the change in cash from selling additional equity that results from changing f, and the term $E1^B_X$ is the change in the value of the existing equity due to a change in the cash balance. The product of the two terms represents the total marginal benefit from changing f.

The term on the right-hand side $-E1^A_f$ represents the marginal cost of equity issuance, i.e., the reduction in the value of the existing equity due to an increase in f.

From Equations (52) and (53), $E1^A_f = -E^A$, $E2^A_f = E^A_f$, respectively, implying that the reduction in profits is completely offset by the increase in additional cash from new equity. So Condition 19 simplifies to

$$E1^B_X|_{X=X_i} = 1. \tag{55}$$

This equation states that the firm should issue additional equity to fund the investment up to a point where the benefit of one more dollar net of issuance costs raised from additional equity equals one. Before investment, however, the marginal benefit of one dollar in the firm is always greater than one, i.e., $E1^B_X > 1$. This implies that if k is not too high the firm should issue additional equity so that it can invest in the project immediately, i.e., the firm chooses $X_i = X_0$ at $t = 0$, and $E1^B_X|_{X=X_0} = E1^A_X|_{X=X'_i} = 1$.

The additional debt $D2^A$ with the coupon payment $c2$ is pinned down by:

Condition 20: $E1^A_X|_{X=X'_i}(D2^A_{c2}|_{X=X'_i} + E2^A_{c2}|_{X=X'_i}) = -E1^A_{c2}|_{X=X'_i}$. The left-hand side represents the marginal benefits of increasing $c2$ at the investment threshold. By increasing $c2$ the firm increases the cash holding by $D2^A_{c2}$. However, $c2$ reduces the net cash flow after debt service, and hence reduces the cash received from the new equity issued by $E2^A_{c2}$, so the total increase in the cash balance is $D^A_{c2} + E2^A_{c2}$. One dollar increase in the cash balance affects the value of the equity by E^A_X. Hence the product of the two terms represents the marginal benefits of increasing $c2$.

The marginal costs of increasing $c2$ can be seen on the right-hand side of Condition 20. The increase in the coupon payment reduces cash flows and reduces the value of the existing equity after investment by $E1^A_{c2}$.

Because at $X = X'_i$, $E1^A_X = 1$, and from Equation (52) and (53), $E1^A_{c2} = (1-f)E^A_{c2}$, and $E2^A_{c2} = fE^A_{c2}$, then Condition 20 simplifies to

$$D2^A_{c2}|_{X=X_i'} = -E^A_{c2}|_{X=X_i'}. \tag{56}$$

This means that the increase the marginal value of additional debt must be equal to the decrease in the marginal value of equity. This condition determines the optimal $c2$. Because the firm invests immediately, it follows the risk strategy described in Proposition 1, i.e., it chooses σ_L at all times. Proposition 5 formally summarizes this result.

Proposition 5 *If k is sufficiently low, the firm invests in the project immediately. To finance the investment, the firm issues additional debt $D2^A$ and additional equity $E2^A$. The optimal c is determined by the condition $D2^A_{c2}|_{X=X_i'} = -E^A_{c2}|_{X=X_i'}$, the point where the increase in the marginal value of additional debt equals the decrease in the marginal value of new equity. The optimal fraction of equity sold to outside investors, f, is determined by the condition $E^A_X|_{X=X_i'} = 1$, the point where the increase in the marginal value of additional cash raised by issuing equity is equal to 1. Also, the firm chooses a low risk strategy, σ_L, at all times.*

If k is sufficiently high, the firm does not issue additional equity to finance the investment at all. It will invest when cash balances reach an investment threshold, and then follow the strategies as described in Sections 4 and 5.

Proof. See Appendix 6. ■

If the level of debt is fixed, then the firm chooses to use either only inside equity (cash balances accumulated in the firm) or outside equity (cash raised from selling additional equity to new investors), depending on which one is cheaper. Inside equity allows existing equityholders to keep all the benefits of the investment to themselves and avoid paying fixed issuance costs, although it takes time to accumulate enough cash in the firm. Outside equity, on the other hand, allows the firm to invest sooner, but it requires paying issuance costs. In equilibrium, the firm chooses the alternative with the lowest cost to existing equityholders.

Next, consider the optimal amount of extra debt. Whether the firm uses inside or outside equity, there are benefits of using some debt to finance the investment, so the firm will issue debt so that the marginal benefits of additional debt financing equal its marginal costs.

A numerical example illustrates this point. We use the same parameters as in the base case. Also, assume that the firm already has outstanding debt with coupon $c1 = 0.0390$, and cash balances equal to $X_0' = 0.3204$.

If the cost of issuing equity, k, is zero, as Proposition 5 suggests, the firm will invest in the project right away. The output of the model is shown in Table 5. At $t = 0$, the firm issues additional equity with

$f = 0.2168$ and additional debt with $c2 = 0.0298$. The market values of additional equity and debt are $E2^A = 0.1259$ and $D2^A = 0.1909$, respectively. After paying investment costs, $I = 0.24$, the remaining cash balance after investment is $X_i' = 0.3204 + 0.1259 + 0.1909 - 0.24 = 0.3972$. This is the level of cash balances at which the firm starts to pay dividends X_d. Since the firm has no other investment opportunity in sight, it decides to follow a low risk strategy. With this strategy, the equity value is $E1^A = 0.4549$. Note that this value is only slightly higher than that of the base case, where it was assumed that the firm could not issue outside equity to fund the investment and used internal cash and debt financing, ($E^{BL} = 0.4531$).

The value of the existing equity is reduced as k increases as shown in Figure 6, and when $k = 0.0018$, the value of the existing equity of a firm that finances investment with additional equity is equal to the value of a firm that does not have this option. Note that it takes only $k = 0.0018$, or just 0.40 percent of the value of the existing equity for the firm to reject issuing outside equity to fund the investment. The reason it takes only a small issuance cost k to reject issuing outside equity lies in the fact that the firm does not face an immediate threat of liquidation, so the benefits of equity financing are not high. With costs of issuance higher than $k = 0.0018$, it is preferable to use internal cash and debt to fund investment.

10 Conclusion

In this paper, we have explored how risk management interacts with the capital structure, investment and cash management policies of a financially constrained firm. We show that risk management can increase firm value by reducing the prospects of bankruptcy and relaxing the financial constraints that delay investments. Also, we show that risk management is closely related to the dividend policy. When cash balances are low, financially constrained firms can decide not to pay out dividends; they also follow low risk strategies, but will switch to high risk strategies when cash balances become high enough.

The relation between leverage and risk management is not obvious. It depends on the level of cash balances, the rate of cash flow from existing assets and the value of the investment. Firms with high cash flow generating ability act more conservatively, but relatively low leverage and the proximity of investment can make them more aggressive.

We have shown that the profile of the investment matters to the risk strategy: Large, lumpy investments induce more aggressive behavior than small, incremental investments. Risk management appears to be insensitive to the profitability of the investment after the investment is made, but not before the investment occurs: the higher the profitability of the investment, the greater the incentive to follow a more aggressive strategy.

When outside investors cannot observe the firm's cash flow rate from existing assets and also do not know whether the firm has growth opportunities, risk management and dividend policy provide important information about the firm.

Finally, we find that debt or equity financing mitigate financial constraints and accelerate investment, but firms do not freely choose to add debt or equity. Too much debt that speed investments can lead to debt overhang after investment, and equity issuance is costly. When the firm decides to fund investment with additional equity, it always follows a conservative risk strategy as a riskier strategy to accelerate investment becomes unnecessary.

Appendix

Appendix 1

Without loss of generality, fix $c1$ and σ. Before investment, for the firm with investment opportunities, the first-order condition of Equation (17) with respect to d is $E_X^B = 1$, implying that the firm should keep all cash inside the firm when $E_X^B > 1$ and pay maximum dividends when $E_X^B \leq 1$. It is shown below that for the firm with valuable investment opportunities, $E_X^B > 1$. Therefore, it should not pay dividends before investing.

The equity value of the firm with no investment opportunities, $E^N(X)$, is given by Equation (33), and the dividend threshold, X_d^N, by Equation (35).

At $X = X_d^N$,

$$\begin{aligned} E_X^N &= \frac{\rho}{(\alpha - c1(1-\tau))} E^N \\ &= 1. \end{aligned} \tag{A-1}$$

For derivations and proofs, see Milne and Robertson (1996).

Next, consider the equity value of the firm with investment opportunities, E^B. Now, for a fixed σ and $d = 0$, Equation (17) becomes Equation (18). Before the dividend payments, the solution to the ODE in Equation (18) has the form: $E^B(X) = Z_1 e^{m_1^N X} + Z_2 e^{m_2^N X}$, and because when the firm defaults at $X = 0$, $E^B(0) = 0$, then $Z_1 = -Z_2$, and $E^B = Z_1(e^{m_1^N X} - e^{m_2^N X})$, where Z_1 is a constant determined by Condition 8.

With this form, it can be verified that at $X = X_d^N$, $E_{XX}^B = 0$, and for all X, $E_{XXX}^B > 0$. This implies that E^B attains a unique minimum at X_d^N, and at this point, Equation (18) can be rearranged to yield

$$E_X^B = \frac{\rho}{(\alpha - c1(1-\tau))} E^B. \tag{A-2}$$

Next, if the net present values of the investment opportunities are strictly positive, the equity value of the firm with investment opportunities should be greater than that of the firm without such opportunities, i.e., $E^B > E^N$, then at $X = X_d^N$,

$$\begin{aligned} E_X^B &> \frac{\rho}{(\alpha - c1(1-\tau))} E^N \\ &= E_X^N \\ &= 1. \end{aligned} \tag{A-3}$$

Since at $X = X_d^N$, E_X^B attains a unique minimum value which is greater than 1, $E_X^B > 1$ for all X. Therefore, before investment, it is not optimal for the firm to pay dividends.

Appendix 2

Before investment, for the firm with investment opportunities, the first-order condition of Equation (17) with respect to σ is $E^B_{XX} = 0$, implying that the firm should decrease σ to the minimum when $E^B_{XX} < 0$ and increase σ to the maximum when $E^B_{XX} \geq 0$.

From, Appendix 1, it is argued that $E^B_{XX} = 0$ at $X = X^N_d$. Since for all X, $E^B_{XXX} > 0$, X^N_d is the unique minimum for E^B_X. From the properties of a minimum, $E^B_{XX} < 0$, for $X < X^N_d$, and $E^B_{XX} \geq 0$, for $X \geq X^N_d$. From Equation (35), $X_s = X^N_d$; therefore, the firm chooses σ_L when $X < X_s$ and σ_H when $X \geq X_s$.

Appendix 3

Define

$$L = \begin{bmatrix} e^{m_1^{BH} X_i} & e^{m_2^{BH} X_i} & 0 & 0 \\ e^{m_1^{BH} X_s} & e^{m_2^{BH} X_s} & -e^{m_1^{BL} X_s} & -e^{m_2^{BH} X_s} \\ m_1^{BH} e^{m_1^{BH} X_s} & m_2^{BH} e^{m_2^{BH} X_s} & -m_2^{BL} e^{m_1^{BL} X_s} & -m_2^{BL} e^{m_2^{BL} X_s} \\ 0 & 0 & 1 & 1 \end{bmatrix}, \tag{A-4}$$

and

$$A = \begin{bmatrix} A_1^H \\ A_2^H \\ A_1^L \\ A_2^L \end{bmatrix}, \tag{A-5}$$

and

$$M = \begin{bmatrix} E^A(X_i^{'}) \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{A-6}$$

The coefficients A_1^H, A_2^H, A_1^L and A_2^L can be written in terms of matrix A as

$$A = L^{^{-1}} \times M. \tag{A-7}$$

Appendix 4

First, differentiating X_s with respect to $c1$ yields

$$\frac{\partial X_s}{\partial c1} = \frac{(1-\tau)}{2\rho\sigma_L^2 + (\alpha - c1(1-\tau))} H, \tag{A-8}$$

where $H = (\alpha - c1(1-\tau))\, X_s - 2\sigma_L^2$. Because $\frac{(1-\tau)}{2\rho\sigma_L^2 + (\alpha - c1(1-\tau))} > 0$, $\frac{\partial X_s}{\partial c1} = 0$, iff $H = 0$, and the sign of $\frac{\partial X_s}{\partial c1}$ is the same of that of H.

If α is sufficiently small, $H < 0$, and $\frac{\partial X_s}{\partial c1} < 0$, and X_s is monotonically decreasing in $c1$.

If α is sufficiently large, there exists $c1 = \overline{c1}$ such that at $\overline{c1}$, $H = 0$. To see this, first, let $c1 = \frac{\alpha}{1-\tau}$, $H = -2\sigma_L^2 < 0$. Next, let $c1 = 0$, $H = \alpha X_s - 2\sigma_L^2$. It can be verified that as $\alpha \to +\infty$, $\lim\ H \to +\infty$. By continuity of H, if α is sufficiently large, at $c1 = 0$, $H > 0$.

Because H is a continuous function, there exists $\overline{c1} \in (0, \frac{\alpha}{1-\tau})$, such that $H = 0$.

Next, we show that H is strictly decreasing in $c1$, so $\overline{c1}$ is unique. Differentiating H with respect to $c1$ yields

$$\frac{\partial H}{\partial c1} = -\frac{2\sigma_L^2(1-\tau)}{2\rho\sigma_L^2 + (\alpha - c1(1-\tau))^2}\left(\rho X_s + (\alpha - c1(1-\tau))\right) < 0. \tag{A-9}$$

Therefore, if α is sufficiently high, X_s is monotonically decreasing in $c1$ for $c1 \in [0, \overline{c1})$, and monotonically increasing in $c1$ for $c1 \in (\overline{c1}, \infty)$.

Appendix 5

Define

$$B = \begin{bmatrix} B_1^H \\ B_2^H \\ B_1^L \\ B_2^L \end{bmatrix}, \tag{A-10}$$

and

$$N = \begin{bmatrix} D1^A(X_i^{'}) - \frac{c1}{\rho} \\ 0 \\ 0 \\ -\frac{c1}{\rho} \end{bmatrix}. \tag{A-11}$$

The coefficients B_1^H, B_2^H, B_1^L, and B_2^L can be written in terms of matrix B as

$$B = L^{^{-1}} \times N, \tag{A-12}$$

where L is define in Equation (A-4).

Appendix 6

First, note that k is a fixed cost that does not create any benefits, so clearly $E1^B$ is strictly decreasing in k.

Now, assume that $k = 0$. If the firm issues equity, Condition 19 must hold: $E1^B_X|_{X=X_i} E2^A_f|_{X=X_i'} = -E1^A_f|_{X=X_i'}$. From Equations (52) and (53), $E1^B_f = -E2^A_f = E^A_f$. Plugging these results into Condition 20 yields $E1^B_X|_{X=X_i} = 1$. This implies that the firm should issue equity as long as $E1^B_X > 1$, and stop issuing equity when $E1^B_X \leq 1$. From Proposition 2 it is always the case that before investment, $E1^B_X > 1$. This means that the firm should keep issuing equity until $E1^B_X = 1$, and make the investment immediately, i.e., $X_i = X_0^{'}$, and $E1^B_X|_{X=X_0} = E1^A_X|_{X=X_i'}$. The optimal f is pinned down by the condition

$$E1^A_X|_{X=X_i'} = 1. \tag{A-13}$$

The optimal coupon $c2$ of the additional debt is pinned down by Condition 20: $E1^A_X|_{X=X_i'}(D2^A_{c2}|_{X=X_i'} + E2^A_{c2}|_{X=X_i'}) = -E1^A_{c2}|_{X=X_i'}$. Because at $X = X_i^{'}$, $E1^A_X = 1$, and from Equation (52) and (53), $E1^A_{c2} = (1-f)E^A_{c2}$, and $E2^A_{c2} = fE^A_{c2}$, then Condition 20 simplifies to

$$D2^A_{c2}|_{X=X_i'} = -E^A_{c2}|_{X=X_i'}. \tag{A-14}$$

Because the firm invests immediately, it chooses σ_L at all times.

Next, consider the case in which $k > 0$. If the firm does not issue additional equity to finance investment, its equity value is E^B. The firm will issue additional equity only if so doing increases the value of the existing equity $E1^A$ above E^B. Because $E1^A$ is strictly decreasing in k, if $k \to \infty$, $E1^A \to 0$. So there is a cut-off point, $\overline{k}$, such that at $\overline{k}$, $E1^A = E^B$, and if k is lower than $\overline{k}$, the firm will issue additional equity to finance investment; otherwise, it will not issue additional equity at all, i.e., it chooses $f = 0$, and follows the strategies described in Sections 3 and 4.

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Table1 : Input Parameters

Parameter	Description	Value
X_0	Cash investment by the equity holder	0.080
α	Cash flow rate	0.075
ν	Scaling factor to cash flows after investment	1.370
τ	Tax rate	0.350
ρ	Discount rate	0.100
I	Lump-sum fixed investment costs	0.240
σ_L	Low cash flow volatility	0.150
σ_H	High cash flow volatility	0.200

Table 2: Outputs: Base Case

Variable	Description	Value
E^{BL}	Equity value	0.4531
$D1^{BL}$	Initial debt value	0.2404
E^A	Equity value at investment threshold	0.5564
$D1^A$	Initial debt value at investment threshold	0.2470
$D2^A$	Additional debt value at investment threshold	0.1961
c1	Coupon payments of the initial debt	0.0390
c2	Coupon payments of the additional debt	0.0309
X'_0	Initial cash balance	0.3204
X_s	Risk switching point	0.3696
X_i	Investment threshold	0.4221
X'_i	Cash balances after investment	0.3782
X_d	Dividend threshold	0.3951

Table 3: Outputs: No Investment Opportunity

Variable	Description	Value
E^N	Equity value	0.4190
D^N	Initial debt value	0.2048
cN	Coupon payments of the initial debt	0.0370
X'_0	Initial cash balance	0.2848
X_d	Dividend threshold	0.3744

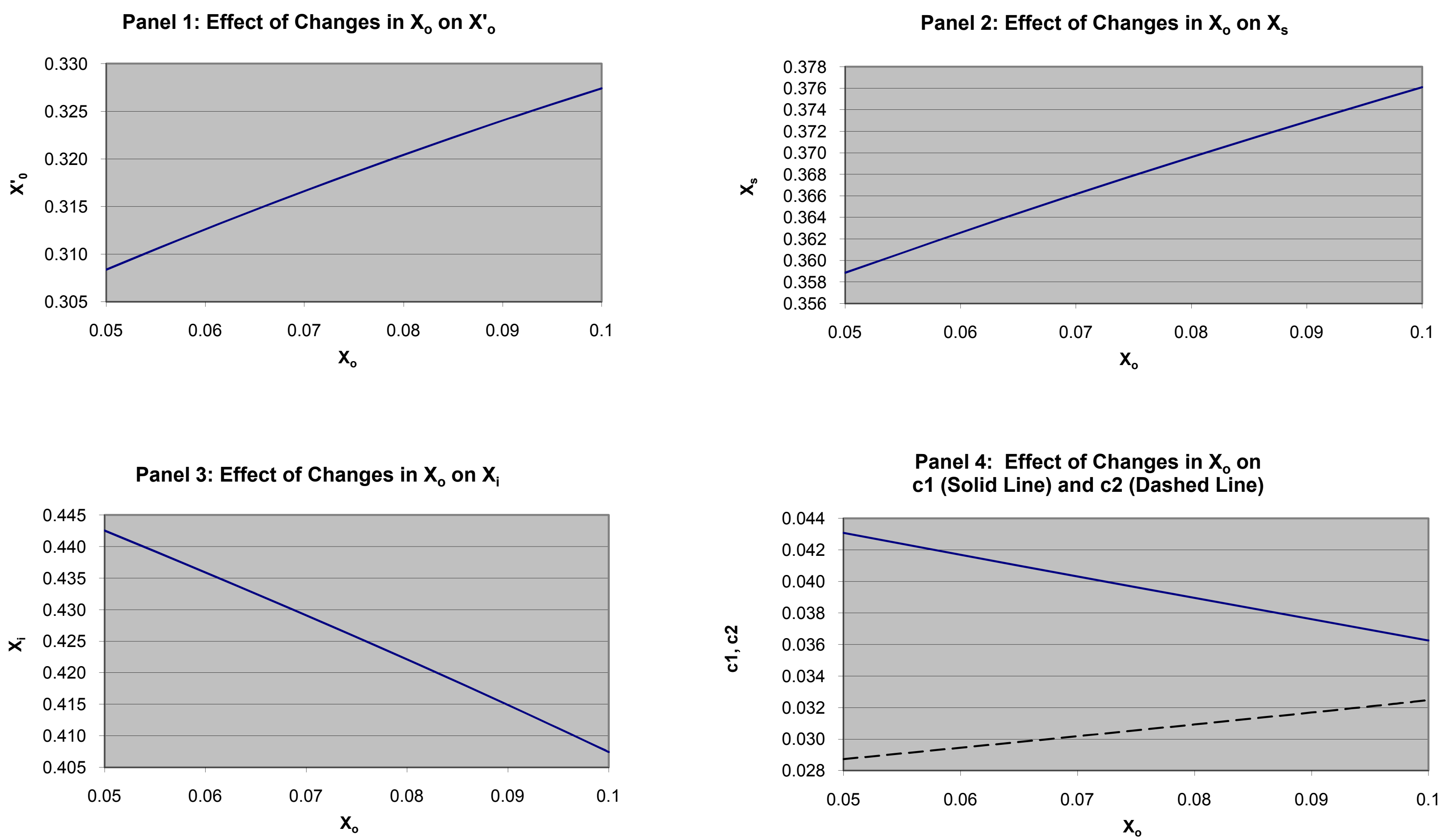
Table 4: Outputs: Immediate Investment

Variable	Description	Value
E^A	Equity value	0.4271
$D1^A$	Initial debt value	0.4188
c1	Coupon payments of the initial debt	0.0711
X'_0	Initial cash balance	0.2588
X_d	Dividend threshold	0.3928

Table 5: Outputs: Equity Financing

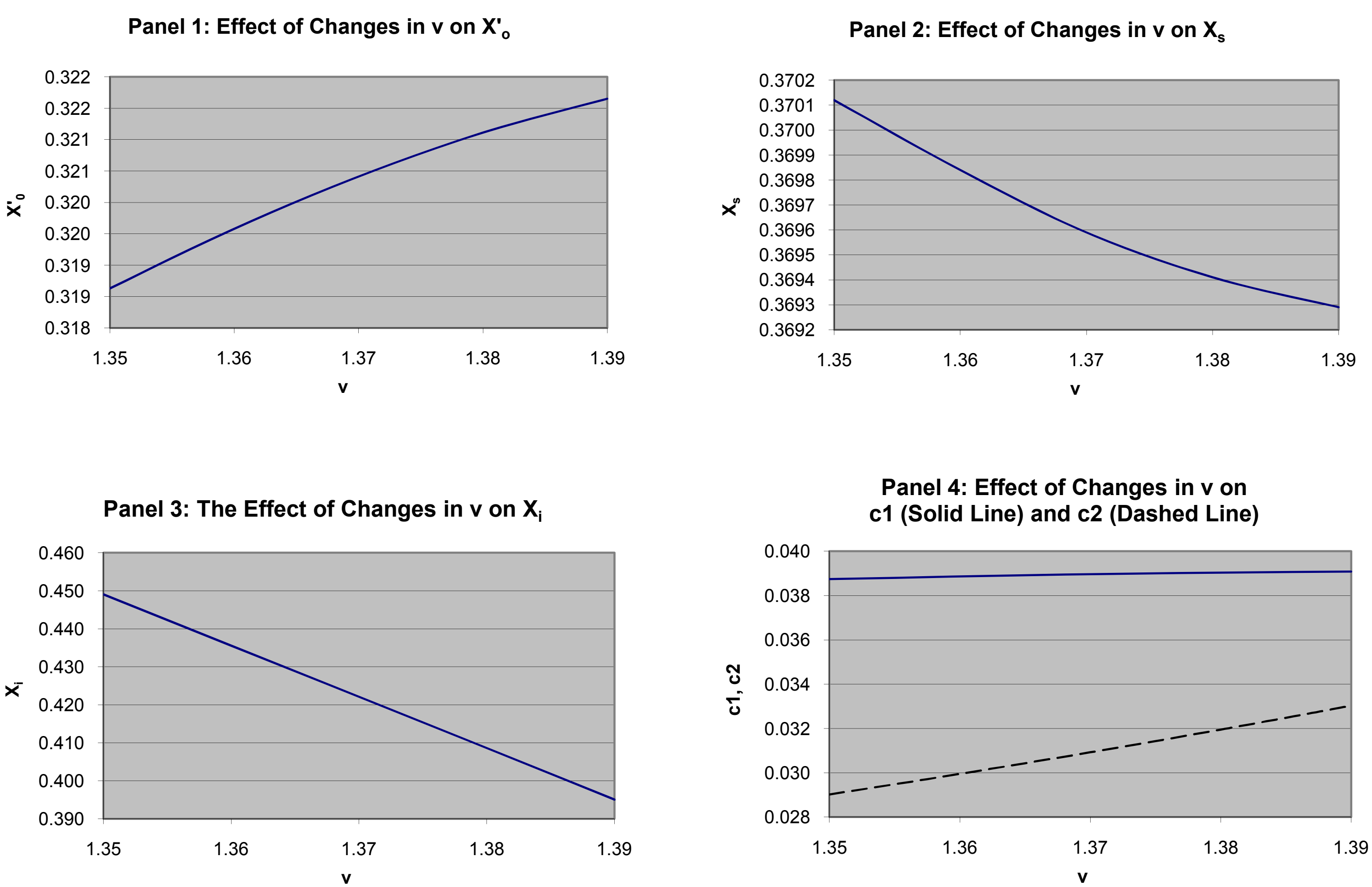
Variable	Description	Value
$E1^A$	Equity value	0.4549
$D1^A$	Initial debt value	0.2499
$E2^A$	Additional equity value	0.1259
$D2^A$	Additional debt value	0.1909
c1	Coupon payments of the initial debt	0.0390
c2	Coupon payments of the additional debt	0.0298
f	Fraction of equity sold to the public	0.2168
X'_0	Initial cash balance	0.3204
X'_i	Cash balances after investment	0.3972
X_d	Dividend threshold	0.3972

Figure 1: Effect of Changes in X_0 on X'_0, X_s, X_i, c1, and c2



X_0 represents the initial cash invested by the equityholders. X'_0 represents the cash in the firm from both the equityholders and original debtholders. X_s is the optimal risk switching threshold point. X_i is the optimal investment threshold point. c1 and c2 are the coupon amounts of the original debt and the new debt issued to fund the investment, respectively. Parameter values are: X_0=0.080, α=0.075, I=0.24, ν=1.37, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20%.

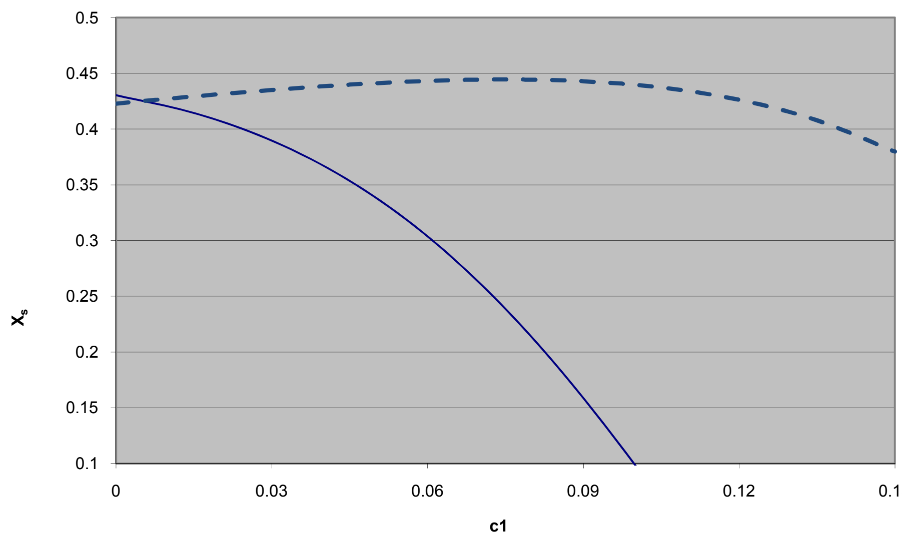
Figure 2: Effect of Changes in v on X'_o, X_s, X_i, c1, and c2



v is the factor by which the firm's instantaneous cash flows increase upon making the investment. X_0 represents the initial cash invested by the equityholders. X'_0 represents the cash in the firm from both the equityholders and original debtholders. Xs is the optimal risk switching threshold point. X_i is the optimal investment threshold point. c1 and c2 are the coupon amounts of the original debt and the new debt issued to fund the investment, respectively. Parameter values are: X_0=0.080, α=0.075, I=0.24, v=1.37, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20%.

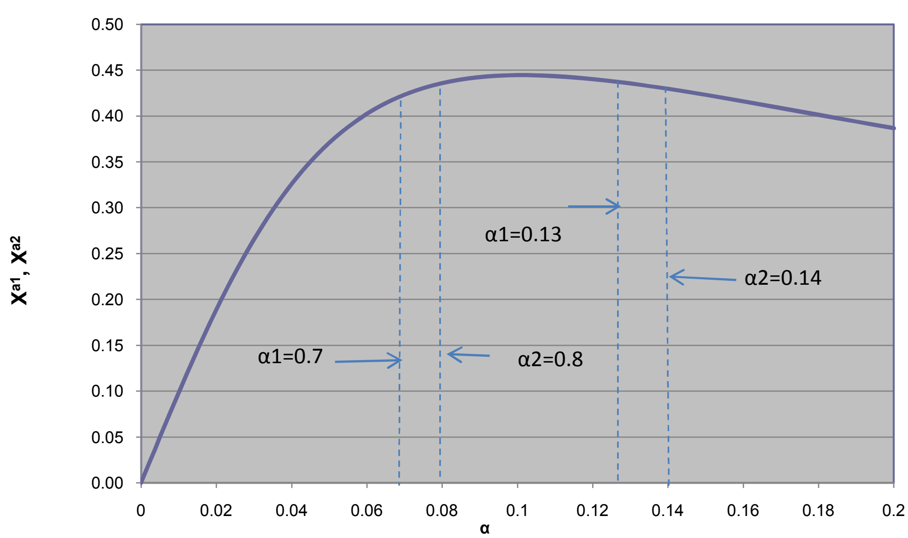
Figure 3: Effect of Changes in c1 on X_s

α=0.075 (Solid Line) and α=0.15 (Dashed Line)



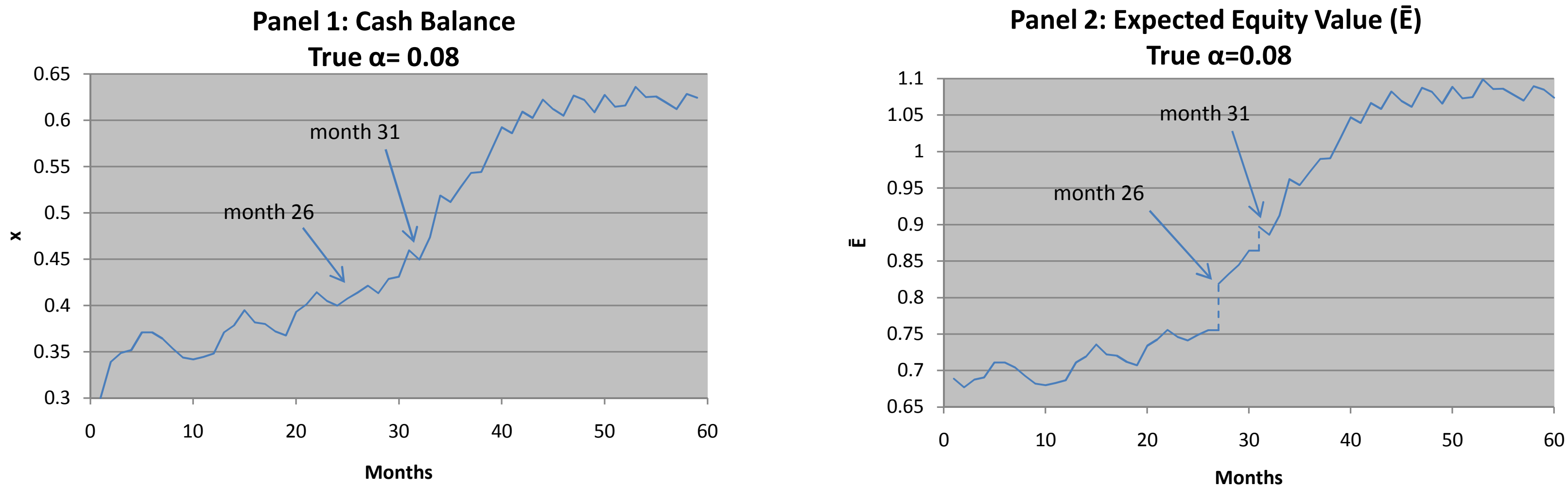
c1 is the coupon amount on the initial debt. Xs is the optimal risk switching threshold point. Parameter values are: X_0=0.080, α=0.075, I=0.24, v=1.37, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20%.

Figure 4: $X_s^{\alpha 1}$ and $X_s^{\alpha 2}$ versus α1 and α2

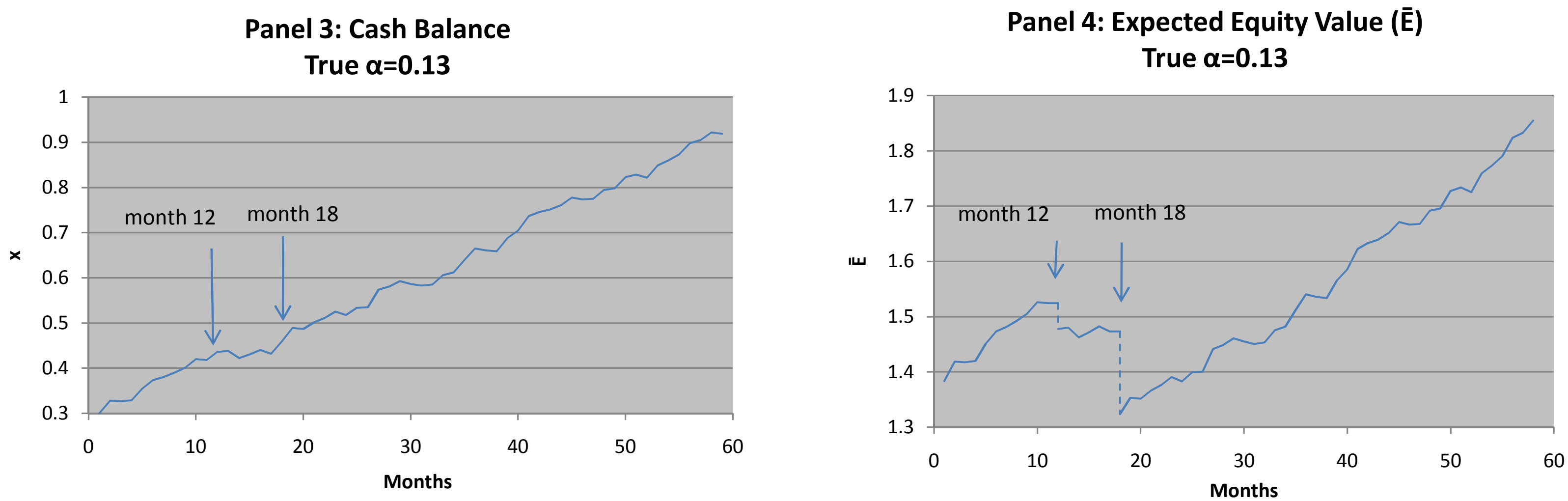


$X^{\alpha 1}$ and $X^{\alpha 2}$ are expected levels of risk and dividend switching threshold points, which depend on the values of the cash flow rates, α1 and α2. Parameter values are: X_0=0.080, I=0.24, v=1.37, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20%.

Figure 5: Cash Balance (X) and Expected Equity Value (Ē)When Investors Cannot Observe α and Investment Opportunity

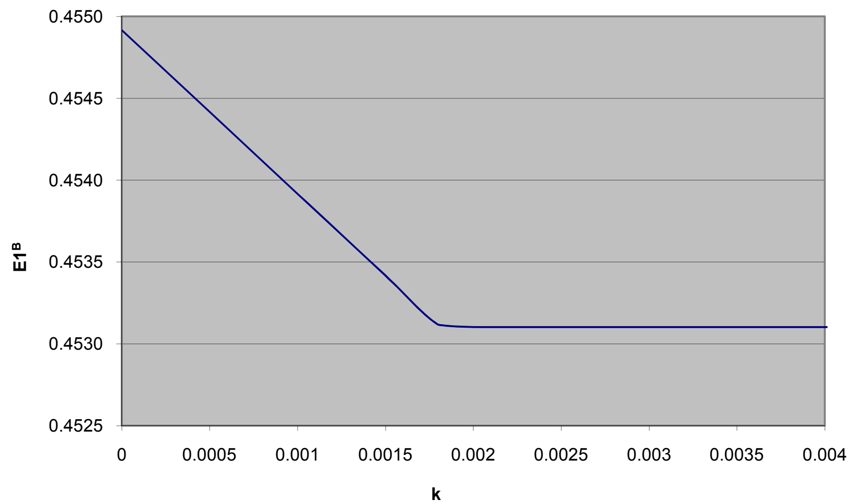


Parameter values in panels 1 and 2 are α_1 =0.07, α_2=0.08, I=0.30, v=1.5, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20% .



Parameter values in panels 3 and 4 are α_1 =0.13, α_2=0.14, I=0.30, v=1.5, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20% .

Figure 6: Equity Issuing Costs (k) versus Equity Value ($E1^B$)



k are the new equity issuing costs. E_1^B is the value of the existing equity before the investment. Parameter values are: X_0=0.080, α=0.075, I=0.24, ν=1.37, τ=35%, ρ=0.1, σ_L= 15%, σ_H=20%.

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