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Global Warming, Proportional Carbon Taxes, and International Fund for Atmospheric Stabilization

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Abstract

In this paper, we prove in terms of a simple dynamic model of global warming that sustainable time-paths of the atmospheric accumulations of carbon dioxide are obtained as the standard market equilibrium under the system of proportional carbon taxes, where the carbon taxes are levied at the rate that is proportional to the per capita national income of each country, with the discounted present value of the impact coefficient of global warming as the coefficient of proportion.

Keywords

dynamic model of global warming

imputed price

sustainable development

proportional carbon taxes

1. Introduction

The atmospheric concentration of greenhouse gases, particularly carbon dioxide, has been increasing since the times of the Industrial Revolution, with an accelerated rate in the last several decades. According to the IPCC reports (1991a, 1991b, 1992, 1996a, 1996b, 2002, 2007), it is estimated that, if the emission of carbon dioxide and other greenhouse gases and the disruption of tropical rain forests were to continue at the present pace, global average air surface temperature toward the end of the twenty-first century would be $3-6$ ˚C higher than the level prevailing before the Industrial Revolution, resulting in drastic changes in climatic conditions and accompanying disruption of the biological and ecological environments.

The problems of global warming are genuinely dynamic. From past human activities we inherit an excess concentration of atmospheric CO_2, and the choices we make today concerning the use of fossil fuels and related activities significantly affect all future generations through the phenomenon of global warming that is brought about by the atmospheric concentrations of CO_2 due to the combustion of fossil fuels today. Thus, we explicitly have to take into account the decreases in the welfare levels of all future generations caused by the increases in the atmospheric accumulations of CO_2 today.

In this paper, the economic analysis of global warming is developed within the theoretical framework of dynamic analysis of global warming, as introduced in Uzawa (1991, 1993, 2003, 2005). We are particularly concerned with the policy arrangements of a proportional carbon tax scheme under which the tax rate is made proportional to the levels of the per capita national income of the countries where greenhouse gases are emitted. In the first part of this paper, we consider the case in which the oceans are the only reservoir of CO_2 on the earth, whereas, in the second part, we explicitly take into consideration the role of the terrestrial forests in moderating processes of global warming by absorbing the atmospheric accumulation of CO_2, on the one hand, and in affecting the level of the welfare of people in the society by providing a decent and cultural environment, on the other.

Finally, an International Fund for Atmospheric Stabilization is proposed to narrow the divergence in economic performance between developed countries and developing countries that has been steadily widening in the last several decades. Many institutional and policy measures that have been devised internationally or bilaterally have not had much impact in narrowing the gap between these two groups of countries. The International Fund for Atmospheric Stabilization is an institutional framework in which it is possible to combine an international arrangement to stabilize atmospheric equilibrium with a redistributive scheme to help developing countries to accelerate processes of economic development.

2. Atmospheric Concentrations of Carbon Dioxide

We denote by V_t the amount of CO_2 that has accumulated in the atmosphere at time t. The quantity V_t is measured in actual tons (in weights of carbon content) or in terms of the density of CO_2 in the atmosphere. We also adopt as the origin of the measurement the stable pre-Industrial Revolution level of 600 GtC (=10^9 tons in carbon content), approximately corresponding to the density of 280 ppm (= part per million). The current level of 760 GtC (approximately 380 ppm) is expressed as V_t = 160 GtC.

The premises of the dynamic model introduced in this paper are primarily based on the scientific findings on global warming and related subjects referred to in Uzawa (1991, 2003). A detailed description of the model was described in Uzawa (2003, 2005).

Changes of CO_2 Accumulations to Natural Causes

The atmospheric accumulations of CO_2 change over time as the result of natural and anthropogenic factors. A certain portion of atmospheric concentrations of CO_2 is absorbed by the oceans (roughly estimated at 50%) and to a lesser extent by living terrestrial plants. In the simple dynamic model postulated in this section, the exchange of carbon between the atmosphere and the terrestrial biosphere is not taken into consideration.

Approximately 75-90 GtC of carbon are annually exchanged between the atmosphere and the surface oceans. The studies made by several meteorologists and oceanologists — in particular, Keeling (1968, 1983) and Takahashi et al. (1980) — suggest that the rate of ocean uptake is closely related to the atmospheric accumulations of CO_2 in excess of the stable, pre-Industrial Revolution level of 280 ppm.

In the simple dynamic model introduced in this section, we assume that the amount of atmospheric CO_2 annually absorbed by the oceans is given by μV_t, where V_t is the atmospheric concentrations of CO_2 measured in actual tons of CO_2 with the pre-Industrial Revolution level of 600 GtC as the origin of measurement. The rate of absorption μ would have a magnitude 2—4%. In what follows, we assume that μ = 0.04.

In the simple dynamic model, we assume that the anthropogenic change in atmospheric CO_2 is exclusively caused by the combustion of fossil fuels in connection with industrial, agricultural, and urban activities.

The change in the atmospheric level of CO_2 is given by

$$\dot{V}_t = a_t - \mu V_t,$$

where a_t is the annual rate of increase in the atmospheric level of CO_2 due to anthropogenic activities and μV_t is the amount of atmospheric CO_2 annually absorbed by the oceans.

The rate of anthropogenic change in the atmospheric level of CO_2, a_t, is determined by the combustion of fossil fuels and is closely related to the levels of production and consumption activities conducted during the year observed. In what follows, the time variable t is often omitted for the sake of expository brevity.

3. The Simple Dynamic Model of Global Warming

We postulate that each greenhouse gas is measured so as to equate the greenhouse effect with that of CO_2. We postulate that the welfare effect of global warming is measured in relation to the stock of greenhouse gases accumulated in the atmosphere.

There are a finite number of individual countries in the world that share the earth's atmosphere as social common capital. Each country is generically denoted by $\nu = 1, \ldots, n$.

The behavioral characteristics of individual countries are expressed in the aggregate by two representative economic agents−the consumers, who are concerned with the choice of economic activities related to consumption, on the one hand, and the producers, who are in charge of the choice of technologies and levels of productive activities, on the other.

Specifications of the Utility Functions

We assume that the utility function for each country ν is expressed in the following manner:

$$u^\nu = u^\nu(c^\nu, a_c^\nu, V),$$

where $c^\nu = (c_j^\nu)$ is the vector of goods consumed in country ν on the per capita basis, a_c^ν is the amount of CO_2 emissions released during the processes of consumption, and V is the atmospheric concentration of CO_2 measured in tons of carbon in CO_2.

Remark: To simplify the exposition, our discussion is carried out as if the size of population of each country ν is one. However, when we discuss the policy and institutional implications, the standard case of numerous individuals in each country will be explicitly brought out.

For each country ν, we also assume that utility function $u^\nu(c^\nu, a_c^\nu, V)$ is strongly separable with respect to (c^ν, a_c^ν) and V in the sense originally introduced by Goldman and Uzawa (1964); that is,

$$u^\nu = \varphi^\nu(V)\, u^\nu(c^\nu, a_c^\nu).$$

As in the case of the models of social common capital introduced in Uzawa (2005), the function $\phi^\nu(V)$ expresses the extent to which people in country ν are adversely affected by global warming. It is referred to as the impact index of global warming. We assume that the impact index function $\phi^\nu(V)$ of global warming for each country ν satisfies the following conditions:

$$\phi^\nu(V) > 0,\ \phi^{\nu}{}'(V) < 0,\ \phi^{\nu}{}''(V) < 0 \text{ for all } V > 0.$$

The *impact coefficient* of global warming for country ν is the relative rate of the marginal change in the impact index due to the marginal increase in the atmospheric accumulation of CO_2; that is,

$$\tau^\nu(V) = -\frac{\phi^{\nu}{}'(V)}{\phi^\nu(V)}.$$

We assume that the impact coefficients of global warming $\tau^\nu(V)$ are identical for all countries ν:

$$\tau^\nu(V) = \tau(V) \text{ for all } \nu.$$

The impact coefficient function $\tau(V)$ satisfies the following conditions:

$$\tau(V) > 0,\ \tau'(V) > 0 \text{ for all } V > 0.$$

The impact index function $\phi(V)$ of the following form is often postulated:

$$\phi(V) = (\hat{V} - V)^\beta, \quad 0 < V < \hat{V},$$

where $\hat{V}$ is the critical level of the atmospheric accumulation of CO_2 and β is the sensitivity parameter $(0 < \beta < 1)$. The critical level $\hat{V}$ of the atmospheric accumulation of CO_2 is usually assumed to be twice the level prevailing before the Industrial Revolution; that is, $\hat{V}$ = 600 GtC. The impact coefficient $\tau(V)$ is given by

$$\tau(V) = \frac{\beta}{\hat{V} - V}.$$

Utility functions $u^\nu(c^\nu, a_c^\nu)$ satisfy the following neoclassical conditions:

(U1) Utility function $u^\nu(c^\nu, a_c^\nu)$ is defined, positive, continuous, and continuously twice-differentiable for all $(c^\nu, a_c^\nu) \geqq 0$.

(U2) Marginal utilities are positive both for the consumption of goods c^ν and CO_2 emissions a_c^ν :

$$u_{c^\nu}^\nu(c^\nu, a_c^\nu) \geqq 0,\ u_{a_c^\nu}^\nu(c^\nu, a_c^\nu) \geqq 0 \quad \text{for all } (c^\nu, a_c^\nu) \geqq 0.$$

(U3) Utility function $u^\nu(c^\nu, a_c^\nu)$ is strictly quasi-concave with respect to $(c^\nu, a_c^\nu) \geqq 0$.

(U4) Utility function $u^\nu(c^\nu, a_c^\nu)$ is homogeneous of order 1 with respect to c^ν:

$$u^\nu(tc^\nu, ta_c^\nu) = tu^\nu(c^\nu, a_c^\nu) \quad \text{for all } (c^\nu, a_c^\nu) \geqq 0.$$

The Euler identity holds:

$$u^\nu(c^\nu, a_c^\nu) = u_{c^\nu}^\nu(c^\nu, a_c^\nu)c^\nu + u_{a_c^\nu}^\nu(c^\nu, a_c^\nu)a_c^\nu \quad \text{for all } (c^\nu, a_c^\nu) \geqq 0.$$

The Consumer Optimum

The world markets for produced goods are assumed to be perfectly competitive and prices of goods are denoted by a nonzero, nonnegative vector p ($p \geq 0$). Carbon taxes at the rate θ^ν are levied upon the emission of CO_2 in each country ν. Carbon tax rate θ^ν is assumed to be nonnegative: $\theta^\nu \geqq 0$.

Suppose national income of country ν in units of world prices is given by y^ν. Then, the consumers in country ν would choose consumption vector c^ν and CO_2 emissions a_c^ν that maximize country ν' s utility function

$$u^\nu(c^\nu, V) = \phi^\nu(V)u^\nu(c^\nu, a_c^\nu)$$

subject to the budget constraints

$$pc^\nu + \theta^\nu a_c^\nu = y^\nu, \quad (c^\nu, a_c^\nu) \geqq 0.$$

The optimum combination of consumption vector c^ν and CO_2 emissions a_c^ν is characterized by the following marginality conditions:

$$\alpha^\nu \phi^\nu(V) u_{c^\nu}^\nu(c^\nu, a_c^\nu) \leqq p \qquad (\text{mod}.\, c^\nu) \tag{1}$$

$$\alpha^\nu \phi^\nu(V) u_{a_c^\nu}^\nu(c^\nu, a_c^\nu) \leqq \theta^\nu \qquad (\text{mod}.\, a_c^\nu), \tag{2}$$

where α^ν is the inverse of the marginal utility of income y^ν of country ν.

If we note the Euler identity for the utility function $u^{\nu}(c^{\nu}, a_c^{\nu})$ of each country ν, we have the following relation:

$$\alpha^{\nu}\phi^{\nu}(V)u^{\nu}(c^{\nu}, a_c^{\nu}) = y^{\nu}. \tag{3}$$

Specifications for Production Possibility Sets

The conditions concerning the production of goods in each country ν are specified by the production possibility set T^{ν} that summarizes the technological possibilities and organizational arrangements for country ν; the endowments of factors of production available in country ν are given.

We assume that there are a finite number of factors of production that are essentially needed in the production of goods. They are generically denoted by f ($f = 1, \cdots, F$). Without loss of generality, we may assume that the factors of production needed in productive activities are the same for all countries involved.

The endowments of factors of production available in each country ν are expressed by a vector $K^{\nu} = (K_f^{\nu})$, where $K^{\nu} \geq 0$.

In each country ν, the minimum quantities of factors of production required to produce goods by the vector of production x^{ν} with CO_2 emissions at the level a_p^{ν} are specified by a vector-valued function:

$$f^{\nu}(x^{\nu}, a_p^{\nu}) = (f_f^{\nu}(x^{\nu}, a_p^{\nu})).$$

We assume that marginal rates of substitution between the production of goods and the emission of CO_2 are smooth and diminishing, that there always exist trade-offs between them, and that the conditions of constant returns to scale prevail. That is, we assume

(T1) $f^{\nu}(x^{\nu}, a_p^{\nu})$ are defined, positive, continuous, and continuously twice-differentiable for all $(x^{\nu}, a_p^{\nu}) \geqq 0$.

(T2) $f_{x^{\nu}}^{\nu}(x^{\nu}, a_p^{\nu}) > 0$, $f_{a_p^{\nu}}^{\nu}(x^{\nu}, a_p^{\nu}) \leqq 0$ for all $(x^{\nu}, a_p^{\nu}) \geqq 0$.

(T3) $f^\nu(x^\nu, a_p^\nu)$ are strictly quasi-convex with respect to (x^ν, a_p^ν) for all $(x^\nu, a_p^\nu) \geqq 0$.

(T4) $f^\nu(x^\nu, a_p^\nu)$ are homogeneous of order 1 with respect to (x^ν, a_p^ν) for all $(x^\nu, a_p^\nu) \geqq 0$.

The following Euler identity holds:

$$f^\nu(x^\nu, a_p^\nu) = f_{x^\nu}^\nu(x^\nu, a_p^\nu)x^\nu + f_{a_p^\nu}^\nu(x^\nu, a_p^\nu)a_p^\nu \quad \text{for all } (x^\nu, a_p^\nu) \geqq 0.$$

The production possibility set of each country ν, T^ν, is composed of all combinations (x^ν, a_p^ν) of vectors of production x^ν and CO_2 emissions a_p^ν that are possibly produced with the organizational arrangements and technological conditions in country ν and the given endowments of factors of production K^ν of country ν. Hence, it may be expressed as

$$T^\nu = \left\{(x^\nu, a_p^\nu):\ (x^\nu, a_p^\nu) \geqq 0,\ \ f^\nu(x^\nu, a_p^\nu) \leqq K^\nu\right\}.$$

Postulates (T1—3) imply that the production possibility set T^ν is a closed, convex set of $J+1$-dimensional vectors (x^ν, a_p^ν).

The Producer Optimum

The producers in country ν would choose those combinations (x^ν, a_p^ν) of vectors of production x^ν and CO_2 emissions a^ν that maximize net profits

$$px^\nu - \theta^\nu a_p^\nu$$

over $(x^\nu, a_p^\nu) \in T^\nu$.

Conditions (T1−3) postulated above ensure that, for any combination of price vector p and carbon tax rate θ^ν, the optimum combination (x^ν, a_p^ν) of vector of production x^ν and CO_2 emissions a_p^ν always exists and is uniquely determined.

To see how the optimum levels of production and CO_2 emissions are determined, let us denote by r^ν $(r^\nu \geqq 0)$ the vector of imputed rental prices of factors of production. Then the optimum conditions are

$$p \leqq r^\nu f_{x^\nu}^\nu(x^\nu, a_p^\nu) \qquad (\text{mod}.\, x^\nu) \tag{4}$$

$$\theta^\nu \geqq r^\nu[-f_{a_p^\nu}^\nu(x^\nu, a_p^\nu)] \qquad (\text{mod}.\, a_p^\nu) \tag{5}$$

$$f^{\nu}(x^{\nu}, a_p^{\nu}) \leqq K^{\nu} \qquad (\mathrm{mod.}\, r^{\nu}). \tag{6}$$

The first condition (4) means the familiar principle that the choice of production technologies and the levels of production are adjusted so as to equate marginal factor costs with output prices.

The second condition (5) similarly means that CO_2 emissions are controlled so that the marginal loss due to the marginal increase in CO_2 emissions is equal to carbon tax rate θ^{ν} when $a_p^{\nu} > 0$ and is not larger than θ^{ν} when $a_p^{\nu} = 0$.

The third condition (5) means that the utilization of factors of production does not exceed the endowments, and the conditions of full employment are satisfied whenever rental price r_f^{ν} is positive.

We have assumed that the technologies are subject to constant returns to scale (T4), and thus, in view of the Euler identity, conditions (2) − (4) imply that

$$px^{\nu} - \theta^{\nu} a_p^{\nu} = r^{\nu} [\, f_{x^{\nu}}^{\nu}(x^{\nu}, a_p^{\nu})\, x^{\nu} + f_{a_p^{\nu}}^{\nu}(x^{\nu}, a_p^{\nu})\, a_p^{\nu}]$$

$$= r^{\nu} f^{\nu}(x^{\nu}, a_p^{\nu}) = r^{\nu} K^{\nu}.$$

That is, the net evaluation of output is equal to the sum of the rental payments to all factors of production.

Suppose all factors of production are owned by individual members of the country ν. Then, national income y^{ν} of country ν is equal to the sum of the rental payments $r^{\nu}K^{\nu}$ and the tax payments $\theta^{\nu}a^{\nu}$, $a^{\nu} = a_c^{\nu} + a_p^{\nu}$, made for the emission of CO_2 in country ν, all on the per capita basis; that is,

$$y^{\nu} = r^{\nu}K^{\nu} + \theta^{\nu}a^{\nu} = (px^{\nu} - \theta^{\nu}a_p^{\nu}) + \theta^{\nu}a^{\nu}$$

$$= pc^{\nu} + \theta^{\nu}a_c^{\nu}.$$

Hence, national income y^{ν} of country ν is equal to the sum of the expenditures, thus conforming with the standard practice in national income accounting.

Market Equilibrium and Global Warming

We consider the situation in which carbon taxes at the rate of θ^ν are levied on the emission of CO_2 in each country ν.

Market equilibrium for the world economy is obtained if we find the prices of goods p at which total demand is equal to total supply:

$$\sum_\nu c^\nu = \sum_\nu x^\nu .$$

Total CO_2 emissions a are given by

$$a = \sum_\nu a^\nu , \quad a^\nu = a_c^\nu + a_p^\nu .$$

(i) Demand conditions in each country ν are obtained by maximizing utility function

$$u^\nu = \phi^\nu (V)\, u^\nu (c^\nu , a_c^\nu)$$

subject to budget constraints

$$pc^\nu + \theta^\nu a_c^\nu = y^\nu ,$$

where y^ν is the national income of country ν.

(ii) Supply conditions in each country ν are obtained by maximizing net profits

$$px^\nu - \theta^\nu a_p^\nu$$

over $(x^\nu , a_p^\nu) \in T^\nu$.

(iii) Total CO_2 emissions in the world, a, are given as the sum of CO_2 emissions in all countries; that is,

$$a = \sum_\nu a^\nu , \; a^\nu = a_c^\nu + a_p^\nu .$$

4. Imputed Price and Sustainable Development

In the dynamic analysis of global warming, a crucial role is played by the concept of the imputed price of the atmospheric accumulations of CO_2. The imputed price ψ_t^ν of the atmospheric accumulations of CO_2 in units of utility for country ν at time t measures the extent to which the marginal increase in the atmospheric accumulations of CO_2 at time t induces the marginal decrease in the welfare level of country ν, including those of all future generations.

Suppose $\psi_t{}^\nu$ is the price in units of utility charged to the emission of CO_2 in country ν at time t. The resulting pattern of consumption, production, and the level of CO_2 emissions in country ν are obtained as the optimum solution for the following maximum problem:

Find the combination $(c^\nu, x^\nu, a_c^\nu, a_p^\nu, a^\nu)$ of consumption victor c^ν, production vector x^ν, and CO_2 emissions, a_c^ν, a_p^ν, a^ν, that maximizes the utility of country ν

$$u_*^\nu = \phi^\nu(V)\, u^\nu(c^\nu, a_c^\nu) - \psi^\nu a^\nu$$

subject to the constraints that

$$pc^\nu = px^\nu \tag{7}$$

$$f^\nu(x^\nu, a_p^\nu) \leqq K^\nu \tag{8}$$

$$a^\nu = a_c^\nu + a_p^\nu, \tag{9}$$

where V and ψ^ν are given.

The maximum problem is solved in terms of the Lagrangian form

$$L^\nu(c^\nu, x^\nu, a_c^\nu, a_p^\nu, a^\nu; \lambda^\nu, \lambda^\nu r^\nu, \lambda^\nu\theta^\nu) = \phi^\nu(V)\, u^\nu(c^\nu, a_c^\nu) - \psi^\nu a^\nu$$
$$+ \lambda^\nu(px^\nu - pc^\nu) + \lambda^\nu r^\nu[K^\nu - f^\nu(x^\nu, a_p^\nu)] + \lambda^\nu\theta^\nu(a^\nu - a_c^\nu - a_p^\nu),$$

where the variables λ^ν, $\lambda^\nu r^\nu$, $\lambda^\nu\theta^\nu$ are the Lagrangian unknowns associated with constraints (7), (8), and (9), respectively.

The optimum conditions are

$$\alpha^\nu\phi^\nu(V)\, u_{c^\nu}^\nu(c^\nu, a_c^\nu) \leqq p \qquad (\mathrm{mod}.\, c^\nu) \tag{10}$$

$$\alpha^\nu\phi^\nu(V)\, u_{a_c^\nu}^\nu(c^\nu, a_c^\nu) \leqq \theta^\nu \qquad (\mathrm{mod}.\, a_c^\nu) \tag{11}$$

$$p \leqq r^\nu f^\nu_{x^\nu}(x^\nu, a^\nu_p) \qquad (\mathrm{mod.}\, x^\nu) \tag{12}$$

$$\theta^\nu \geqq r^\nu[-f^\nu_{a^\nu_p}(x^\nu, a^\nu_p)] \qquad (\mathrm{mod.}\, a^\nu_p) \tag{13}$$

$$f^\nu(x^\nu, a^\nu_p) \leqq K^\nu \qquad (\mathrm{mod.}\, r^\nu), \tag{14}$$

where α^ν is the inverse of marginal utility of income, i. e., $\alpha^\nu = \dfrac{1}{\lambda^\nu}$.

Marginality conditions (13) and (14) imply that net profits

$$px^\nu - \theta^\nu a^\nu_p$$

are maximized over the technological possibility set $(x^\nu, a^\nu_p) \in T^\nu$.

The marginal decrease m^ν_t in the welfare level of country ν at time t induced by the marginal increase in the atmospheric accumulations V_t of CO_2 at time t is given by

$$m^\nu_t = -\frac{\partial}{\partial V_t}[\varphi^\nu(V_t)\, u^\nu(c^\nu_t)] = \tau(V_t)\varphi^\nu(V_t)\, u^\nu(c^\nu_t), \tag{15}$$

which, in units of market prices, may be expressed as

$$\alpha^\nu_t m^\nu_t = \tau(V_t)\alpha^\nu_t \varphi^\nu(V_t)\, u^\nu(c^\nu_t) = \tau(V_t) y^\nu_t, \tag{16}$$

where y^ν is per capita national income of country ν at time t.

Let us denote by π^ν_t the imputed price of the atmospheric accumulations of CO_2 in units of market prices for country ν at time t; that is,

$$\pi^\nu_t = \alpha^\nu_t \psi^\nu_t.$$

Then, by noting (16), we have from the definition of the imputed price that

$$\pi^\nu_t = \int_t^\infty \alpha^\nu_\tau m^\nu_\tau e^{-(\delta+\mu)(\tau-t)} d\tau = \int_t^\infty \tau(V_\tau) y^\nu_\tau e^{-(\delta+\mu)(\tau-t)} d\tau. \tag{17}$$

By differentiating both sides of (17) with respect to time t, we obtain the following differential equation:

$$\dot{\pi}^\nu_t = (\delta + \mu)\pi^\nu_t - \tau(V_t) y^\nu_t. \tag{18}$$

Differential equation (18) is nothing but the Euler-Lagrange differential equation in the calculus of variations or the Ramsey-Keynes equation in the theory of optimum economic growth.

To clarify the intrinsic meaning of the imputed price of the atmospheric accumulations of CO_2, we introduce a virtual capital market at time t that is perfectly competitive and the atmospheric accumulations of CO_2 are transacted as an asset. The imputed price π_t^v at time t is identified with the market price. Consider the situation in which the unit of such an asset is held for a short time period $[t,\ t+\Delta t]$, ($\Delta t > 0$). The gains obtained by holding such an "asset" are composed of "capital gains" $\Delta\pi_t^v = \pi_{t+\Delta t}^v - \pi_t^v$ and "earnings" $\tau(V_t)y_t^v\Delta t$, minus "depreciation charges" $\mu\pi_t^v\Delta t$; that is,

$$\Delta\pi_t^v + \tau(V_t)y_t^v\Delta t - \mu\pi_t^v\Delta t.$$

On other hand, the cost of holding such an "asset" for the time period $[t,\ t+\Delta t]$ is "interest" payment $\delta\pi_t^v\Delta t$, where the social rate of discount is identified with the "market rate of interest". Hence, on the virtual capital market, these two amounts become equal; that is,

$$\Delta\pi_t^v + \tau(V_t)y_t^v\Delta t - \mu\pi_t^v\Delta t = \delta\pi_t^v\Delta t,$$

which, by dividing both sides by Δt and taking the limit as $\Delta t \to 0$, yields differential equation (18).

It may be noted that the time derivative in the differential equation (18) refers to the fictitious time of the virtual capital market at time t and remains valid only at time t. For any future time τ ($\tau > t$), equation (18) has to be modified by taking into account the changing circumstances due to the accumulation of capital.

We define that the imputed price π_t^v of the atmospheric accumulations of CO_2 for country v is at the *sustainable* level at time t, if $\dot{\pi}_t^v = 0$ at time t. That is,

$$\pi_t^v = \theta_t^v, \quad \theta_t^v = \frac{\tau(V_t)}{\delta + \mu} y_t^v \quad \text{at time } t.$$

A time-path (V_t) of the atmospheric accumulations of CO_2 is defined sustainable for country v, if the imputed price π_t^v for country v is at the sustainable level at all times t. It is defined *sustainable*, if it is sustainable for all countries in the world.

The discussion above may be summarized as the following proposition:

Proposition 1. *Sustainable time-paths* (V_t) *of the atmospheric accumulations of* CO_2 *are obtained as the competitive market equilibrium under the system of proportional carbon taxes,* *where, in each country* ν*, the carbon taxes are levied at the rate* θ^ν *that is proportional to the per capita national income* y^ν *of each country* ν*, with the discounted present value* $\frac{\tau(V)}{\delta+\mu}$ *of the impact coefficient of global warming* $\tau(V)$ *as the coefficient of proportion; that is,*

$$\theta^\nu = \frac{\tau(V)}{\delta+\mu} y^\nu,$$

where $\tau(V)$ *is the impact coefficient of global warming,* δ *is the social rate of discount, and* μ *is the rate at which atmospheric* CO_2 *is annually absorbed by the oceans.*

5. Global Warming and Forests

The economic analysis of global warming, as developed in the previous sections, may be extended to examine the role of terrestrial forests in moderating processes of global warming, on the one hand, and in affecting the level of the welfare of people in the society by providing a decent and cultural environment, on the other.

In the simple, dynamic analysis of global warming introduced in the previous sections, we have assumed that the combustion of fossil fuels is the only cause for atmospheric instability and that the surface ocean is the only reservoir of carbon on the earth's surface that exchanges carbon with the atmosphere. In this section, we consider the role of terrestrial forests, particularly tropical rain forests, in stabilizing the processes of atmospheric equilibrium.

Terrestrial forests are regarded as social common capital and managed by social institutions with an organizational structure similar to that of private enterprise except for the manner in which prices of the forests themselves and products from the forests are determined. We assume that the amount of atmospheric CO_2 absorbed by the terrestrial forest per hectare in each country ν is a certain constant on the average to be denoted by γ^ν ($\gamma^\nu \geqq 0$). Then the basic dynamic equation concerning the change in the atmospheric concentrations of CO_2 may be modified to take into account the amount of atmospheric CO_2 absorbed by terrestrial forests. We have

$$\dot{V}_t = a_t - \sum_\nu \gamma_t^\nu R_t^\nu - \mu V_t, \tag{19}$$

where a_t is total CO_2 emissions in the world;

$$a_t = \sum_\nu a_t^\nu,$$

R_t^ν is the acreages of terrestrial forests of country ν, μ is the rate at which atmospheric CO_2 is absorbed by the oceans. We assume that the carbon sequester rate for temperate forests is around 7.5 tC/ha/yr and for tropical rain forests, it is assumd at 9.6-l0.0 tC/ha/yr.

The change in the acreages of the terrestrial forests R^ν in each country ν is determined first by the levels of reforestation activities and secondly by various economic activities carried out in country ν during the year in question−particularly by agricultural and lumber industries and by processes of urbanization. We denote by z_t^ν the acreages of terrestrial forests annually reforested and by b_t^ν the acreages of terrestrial forests in country ν annually lost due to economic activities. Then the acreages of terrestrial forests R_t^ν in each country ν are subject to the following differential equations:

$$\dot{R}_t^\nu = z_t^\nu - b_t^\nu.$$

Specifications for Utility Functions

We assume that the utility level u^ν of each country ν is influenced by the acreages of terrestrial forests R^ν in country ν, in addition to the atmospheric concentrations of CO_2, V. That is, the utility function for each country ν is expressed in the following manner:

$$u^\nu = u^\nu(c^\nu, a_c^\nu, R^\nu, V),$$

where c^ν is the vector of goods consumed in country ν, a_c^ν is the amount of CO_2 emitted by the consumers in country ν, R^ν is the acreages of terrestrial forests in country ν, and V is the atmospheric concentrations of CO_2 accumulated in the atmosphere, all at time t.

For each country ν, we assume that utility function $u^\nu(c^\nu, a_c^\nu, R^\nu, V)$ is strongly separable with respect to (c^ν, a_c^ν), R^ν, and V:

$$u^\nu(c^\nu, a_c^\nu, R^\nu, V) = \phi^\nu(V)\varphi^\nu(R^\nu)\, u^\nu(c^\nu, a_c^\nu).$$

As with the case discussed in the previous sections, the function $\phi^\nu(V)$ expresses the extent to which people in country ν are adversely affected by global warming, which is referred to as the impact index of global warming. Similarly, the function $\phi^\nu(R^\nu)$ expresses the extent to which people in country ν are positively affected by the presence of the terrestrial forests in country ν, which is referred to as the impact index of forests. We assume that the impact indices, $\phi^\nu(V)$ and $\varphi^\nu(R^\nu)$, satisfy the following conditions:

$$\phi^\nu(V) > 0,\ \phi^{\nu}{}'(V) < 0,\ \phi^{\nu}{}''(V) < 0;\ \varphi^\nu(R^\nu) > 0,\ \varphi^{\nu}{}'(R^\nu) > 0,\ \varphi^{\nu}{}''(R^\nu) < 0.$$

The impact coefficients of global warming and forests are, respectively, defined by

$$\tau^\nu(V) = -\frac{\phi^{\nu}{}'(V)}{\phi^\nu(V)},\quad \tau^\nu(R^\nu) = \frac{\varphi^{\nu}{}'(R^\nu)}{\varphi^\nu(R^\nu)}.$$

We assume that the impact coefficients of global warming $\tau^\nu(V)$ are identical for all countries ν:

$$\tau^\nu(V) = \tau(V) \quad \text{for all } \nu.$$

The impact coefficient functions, $\tau(V)$ and $\tau^\nu(R^\nu)$, satisfy the following conditions:

$$\tau(V) > 0,\quad \tau'(V) > 0;\ \tau^\nu(R^\nu) > 0,\quad \tau^{\nu}{}'(R^\nu) > 0.$$

We assume that, for each country ν, the utility function $u^\nu(c^\nu, a_c^\nu)$ satisfies the conditions (U1)−(U4), as introduced in Section 3.

The Consumer Optimum

The consumers in country ν would choose the vector of consumption c^ν and CO_2 emissions a_c^ν that maximizes country ν' s utility function

$$u^\nu(c^\nu, a_c^\nu, R^\nu, V) = \phi^\nu(V)\varphi^\nu(R^\nu)\, u^\nu(c^\nu, a_c^\nu)$$

subject to the budget constraints

$$pc^\nu + \theta^\nu a_c^\nu = y^\nu,$$

where y^ν is national income of country ν in units of world prices.

The optimum combination of consumption c^ν and CO_2 emissions a_c^ν is characterized by the following marginality conditions:

$$\alpha^\nu \phi^\nu(V)\varphi^\nu(R^\nu)\, u_{c^\nu}^\nu(c^\nu, a_c^\nu) \leqq p \qquad (\text{mod.}\, c^\nu) \qquad (20)$$

$$\alpha^\nu \phi^\nu(V)\varphi^\nu(R^\nu)\, u_{a_c^\nu}^\nu(c^\nu, a_c^\nu) \leqq \theta^\nu \qquad (\text{mod.}\, a_c^\nu), \qquad (21)$$

where α^ν is the inverse of the marginal utility of income y^ν of country ν.

The linear homogeneity hypothesis for the utility function $u^\nu(c^\nu, a_c^\nu)$ implies that

$$\alpha^\nu \phi^\nu(V)\varphi^\nu(R^\nu) u^\nu(c^\nu, a_c^\nu) = pc^\nu + \theta^\nu a_c^\nu = y^\nu.$$

Specifications for Production Possibility Sets

The conditions concerning the production of goods in each country ν are specified by the production possibility set T^ν in exactly the same manner as in the previous sections.

In each country ν, the minimum quantities of factors of production needed to produce goods by the vector of production x^ν with the use of the natural resources of the forests by the amount b^ν and the CO_2 emission at the level a_p^ν are specified by a vector-valued function

$$f^\nu(x^\nu, b^\nu, a_p^\nu) = (f_f^\nu(x^\nu, b^\nu, a_p^\nu)).$$

Similarly, the minimum quantities of factors of production needed to engage in reforestation activities at the level z^ν are specified by a vector-valued function $g^\nu(z^\nu)$.

We assume that marginal rates of substitution between the production of goods, the use of the natural resources of forests, reforestation activities, and the emission of CO_2 are smooth and diminishing, trade-offs always exist between them, and the conditions of constant returns to scale prevail. That is, we assume that

(T'1) $f^\nu(x^\nu, b^\nu, a_p^\nu)$ and $g^\nu(z^\nu)$ are defined, positive-valued, continuous, and continuously twice-differentiable for all $(x^\nu, b^\nu, a_p^\nu) \geqq 0$ and $z^\nu \geqq 0$, respectively.

(T'2) $f^{\nu}_{x^{\nu}}(x^{\nu}, b^{\nu}, a^{\nu}_p) \geqq 0$, $f^{\nu}_{b^{\nu}}(x^{\nu}, b^{\nu}, a^{\nu}_p) < 0$, $f^{\nu}_{a^{\nu}_p}(x^{\nu}, b^{\nu}, a^{\nu}_p) < 0$ for all $(x^{\nu}, b^{\nu}, a^{\nu}_p) \geqq 0$

$$g^{\nu}_{z^{\nu}}(z^{\nu}) \geqq 0 \text{ for all } z^{\nu} \geqq 0.$$

(T'3) $f^{\nu}(x^{\nu}, b^{\nu}, a^{\nu}_p)$ and $g^{\nu}(z^{\nu})$ are strictly quasi-convex with respect to $(x^{\nu}, b^{\nu}, a^{\nu}_p)$ and z^{ν}, respectively.

(T'4) $f^{\nu}(x^{\nu}, b^{\nu}, a^{\nu}_p)$ and $g^{\nu}(z^{\nu})$ are homogeneous of order 1 with respect to $(x^{\nu}, b^{\nu}, a^{\nu}_p)$ and z^{ν}, respectively.

The production possibility set T^{ν} is given by

$$T^{\nu} = \left\{(x^{\nu}, z^{\nu}, b^{\nu}, a^{\nu}_p) : (x^{\nu}, z^{\nu}, b^{\nu}, a^{\nu}_p) \geqq 0, f^{\nu}(x^{\nu}, b^{\nu}, a^{\nu}_p) + g^{\nu}(z^{\nu}) \leqq K^{\nu}\right\},$$

where K^{ν} is the vector of endowments of fixed factors of production in country ν.

Postulates (T'1-T'3), as specified above, imply that the production possibility set T^{ν} is a closed convex set of $(x^{\nu}, z^{\nu}, b^{\nu}, a^{\nu}_p)$.

The Producer Optimum

Suppose that prices of goods are given by p and the imputed price of forests in each country ν by π^{ν}, whereas carbon taxes at the rate θ^{ν} are levied on the emission of CO_2 in each country ν.

Forests are regarded as social common capital, and there are no markets on which either the ownership of forests or the entitlements for the products from forests are transacted. Hence, prices of forests are generally not market prices, but rather imputed prices. The imputed price of the ownership of a particular forest is the discounted present value of the stream of the marginal utilities of the forest and the expected value of the entitlements for the natural resources in forest in the future.

The producers in country ν would choose those combinations $(x^{\nu}, z^{\nu}, b^{\nu}, a^{\nu}_p)$ of vectors of production x^{ν}, levels of reforestation z^{ν}, use of resources of forests b^{ν}, and CO_2 emissions a^{ν}_p that maximize net profits

$$px^{\nu} + \pi^{\nu}(z^{\nu} - b^{\nu}) - \theta^{\nu} a^{\nu}$$

over $(x^{\nu}, z^{\nu}, b^{\nu}, a_p^{\nu}) \in T^{\nu}$.

Marginality conditions for the producer optimum are

$$p \leqq r^{\nu} f_{x^{\nu}}^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu}) \qquad (\text{mod.}\, x^{\nu}) \tag{22}$$

$$\pi^{\nu} \geqq r^{\nu}[-f_{b^{\nu}}^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu})] \qquad (\text{mod.}\, b^{\nu}) \tag{23}$$

$$\theta^{\nu} \geqq r^{\nu}[-f_{a_p^{\nu}}^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu})] \qquad (\text{mod.}\, a_p^{\nu}) \tag{24}$$

$$\pi^{\nu} \leqq r^{\nu} g_{z^{\nu}}^{\nu}(z^{\nu}) \qquad (\text{mod.}\, z^{\nu}) \tag{25}$$

$$f^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu}) + g^{\nu}(z^{\nu}) \leqq K^{\nu} \qquad (\text{mod.}\, r^{\nu}), \tag{26}$$

where r^{ν} is the vector of imputed rental prices of factors of production.

The meaning of these conditions is simple. Condition (22) means that the choice of production technologies and the levels of production are adjusted so as to equate marginal factor costs with output prices.

Condition (23) means that the use of resources of forests is determined so that the marginal gain due to the marginal increase in the use of resources of forests is equal to imputed price of forests π^{ν} when $b^{\nu} > 0$ and is not larger than π^{ν} when $b^{\nu} = 0$.

Condition (24) means that CO_2 emissions are determined so that the marginal gain due to the marginal increase in CO_2 emissions is equal to carbon tax rate θ^{ν} when $a_p^{\nu} > 0$ and is not larger than θ^{ν} when $a_p^{\nu} = 0$.

Condition (25) means that the level of reforestation activities is determined so that the marginal gain due to the marginal increase in the use of resources of forests is equal to the imputed price of forests π^{ν} when $z^{\nu} > 0$ and is not larger than π^{ν} when $z^{\nu} = 0$.

Condition (26) means that the employment of factors of production does not exceed the endowment, and conditions of full employment are satisfied whenever rental price is positive.

We have assumed that the technologies are subject to constant returns to scale, and thus, in view of the Euler identity, conditions $(22)-(26)$ imply that

$$px^{\nu} + \pi^{\nu}(z^{\nu} - b^{\nu}) - \theta^{\nu} a_p^{\nu}$$

$$= r^{\nu}[\, f_{x^{\nu}}^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu}) x^{\nu} + f_{b^{\nu}}^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu}) b^{\nu} + f_{a_p^{\nu}}^{\nu}(x^{\nu}, b^{\nu}, a_p^{\nu}) a_p^{\nu} + g_{z^{\nu}}^{\nu}(z^{\nu}) z^{\nu}\,]$$

$$= r^v[\, f^v_{x^v}(x^v, b^v, a^v_p)x^v + g^v_{z^v}(z^v)z^v - f^v_{b^v}(x^v, b^v, a^v_p)b^v - f^v_{a^v_p}(x^v, b^v, a^v_p)a^v_p]$$

$$= r^v[\, f^v(x^v, b^v, a^v_p) + g^v(z^v)] = r^v K^v.$$

That is, the net evaluation of output is equal to the sum of the rental payments to all factors of production.

6. Imputed Prices of Capital in General

The imputed price, in units of the utility, of each kind of capital at time t, ψ_t, is the discounted present value of the marginal increases in total utility in the future due to the marginal increase in the stock of that kind of capital at time t. When we denote by r_τ the marginal increase in the total utility at future time τ, the imputed price at time t, ψ_t, is given by

$$\psi_t = \int_t^\infty r_\tau e^{-(\delta+\mu)(\tau-t)} d\tau. \tag{27}$$

By differentiating both sides of (27) with respect to time t, we obtain the following differential equation:

$$\dot{\psi}_t = (\delta + \mu)\psi_t - r_t. \tag{28}$$

We suppose that capital is transacted as an asset on a virtual capital market that is perfectly competitive and the imputed price ψ_t is identified with the market price at time t. Consider the situation in which the unit of such an asset is held for the short time period $[t,\ t+\Delta t]$ $(\Delta t > 0)$. The gains obtained by holding such an asset are composed of "capital gains" $\Delta\psi_t = \psi_{t+\Delta t} - \psi_t$ and "earnings" $r_t \Delta t$; that is,

$$\Delta\psi_t + r_t \Delta t.$$

On other hand, the costs of holding such an asset for the time period $[t,\ t+\Delta t]$ consist of "interest payments" $\delta\psi_t \Delta t$ and "depreciation charges" $\mu\psi_t \Delta t$, where the social rate of discount δ is identified with the market rate of interest; that is,

$$\delta\psi_t \Delta t + \mu\psi_t \Delta t.$$

On the virtual capital market, these two amounts become equal; that is,

$$\Delta\psi_t + r_t \Delta t = \delta\psi_t \Delta t + \mu\psi_t \Delta t.$$

By dividing both sides of this equation by Δt and taking the limit as $\Delta t \to 0$, w obtain relation (28).

We define that the imputed price ψ_t is at the sustainable level at time t, if it remains stationary at time t; i. e.,

$$\dot{\psi}_t = 0 \quad \text{at time } t,$$

where it may be reminded that $\dot{\psi}_t$ refers to the time derivative with respect to the time of the virtual capital market at time t.

From the basic differential equation (28), the imputed price ψ_t is at the sustainable level at time t, if, and only if,

$$\psi_t = \frac{r_t}{\delta + \mu} \quad \text{at time } t,$$

where r_t is the marginal increase in total utility due to the marginal increase in the stock of capital of that kind at time t.

7. Imputed Prices of Atmospheric Concentrations of CO_2 and Forests

Consider the situation in which a combination $(c^\nu, a_c^\nu, x^\nu, z^\nu, b^\nu, a_p^\nu)$ of vectors of consumption and production, c^ν, x^ν, level of reforestation activities z^ν, CO_2 emissions a_c^ν, and a_p^ν, $a^\nu = a_c^\nu + a_p^\nu$, is chosen in country ν. Imputed prices of atmospheric concentrations of CO_2 and forests are defined as follows.

Suppose CO_2 emissions in country ν, a^ν, are increased by a marginal amount. This would induce a marginal increase in the aggregate amount of CO_2 emissions in the world, causing a marginal increase in the atmospheric level of CO_2. The resulting marginal increase in the degree of future global warming would cause a marginal decrease in country v's utility.

The marginal decrease in country v's utility due to the marginal increase in CO_2 emissions today in country v is given by the partial derivative, with minus sign, of utility of country v

$$u^v = \phi^v(V)\varphi^v(R^v)\,u^v(c^v, a_c^v)$$

with respect to atmospheric accumulations of CO_2, V; that is,

$$-\frac{\partial u^v}{\partial V} = \tau(V)\phi^v(V)\varphi^v(R^v)\,u^v(c^v, a_c^v),$$

where $\tau(V)$ is the impact coefficient of global warming.

We assume that future utilities of country v are discounted at the social rate of discount δ that is assumed to be positive and identical for all countries in the world. We have assumed that the rate at which atmospheric carbon dioxide is annually absorbed by the oceans is a certain constant μ. Hence, for each country v, the sustainable imputed price ψ^v of the atmospheric accumulations of CO_2, in units of utility of country v, is given by the discounted present value of the marginal decrease in utility of country v due to the marginal increase in CO_2 emissions in country v today ; that is,

$$\psi^v = \frac{\tau(V)}{\delta + \mu}\phi^v(V)\varphi^v(R^v)\,u^v(c^v, a_c^v).$$

Hence, the sustainable imputed price θ^v of the atmospheric accumulations of CO_2 for country v, in units of world prices, is given by

$$\theta^v = \alpha^v\psi^v = \frac{\tau(V)}{\delta + \mu}y^v, \qquad (29)$$

where α^v is the inverse of the marginal utility of country v.

Similarly, the imputed prices of forests, in units of world prices, are defined as follows. Suppose the acreages of forests of country v, R^v, are increased by a marginal amount. This would induce a marginal increase in the level of the utility of country v, on the one hand, and a marginal increase in the utility of country v in the future due to the marginal decrease in the atmospheric level of CO_2 induced by the absorbing capacity of forests in country v, on the other.

The first component is the marginal utility with respect to the acreages of forests of country ν, R^ν, in units of world prices. It is given by

$$\frac{\partial(\alpha^\nu u^\nu)}{\partial R^\nu} = \tau^\nu(R^\nu)\alpha^\nu \phi^\nu(V)\varphi^\nu(R^\nu)u^\nu(c^\nu, a_c^\nu) = \tau^\nu(R^\nu)y^\nu.$$

The second component is the marginal increase in country ν's utility in the future due to the marginal decrease in the atmospheric level of CO_2 induced by the absorbing capacity of forests in country ν. It is given by

$$\gamma^\nu \theta^\nu = \gamma^\nu \frac{\tau(V)}{\delta + \mu} y^\nu.$$

Hence, the sustainable imputed price π^ν of forests of country ν, in units of world prices, is given by the discounted present value of the sum of these two components:

$$\pi^\nu = \frac{1}{\delta}[\tau^\nu(R^\nu) + \gamma^\nu \frac{\tau(V)}{\delta + \mu}]y^\nu.$$

The discussion above may be summarized in the following proposition.

Proposition 2. *The optimum conditions for the sustainable time-path of atmospheric* concentrations of CO_2 *corresponds precisely to the optimum conditions for the competitive market equilibrium under the following system of proportional carbon taxes for the emission of* CO_2 *and tax-subsidy measures for the reforestation and depletion of resources of forests:*

(i) *In each country* ν, *the carbon taxes are levied with the rate* θ^ν *that is proportional to the per capita national income* y^ν*:*

$$\theta^\nu = \frac{\tau(V)}{\delta + \mu} y^\nu,$$

where $\tau(V)$ *is the impact coefficient of global warming,* δ *is the social rate of discount, and* μ *is the rate at which atmospheric* CO_2 *is annually absorbed by the oceans.*

(ii) *In each country* ν, *tax-subsidy arrangements are made for the depletion of resources and reforestation of forests with the rate* π^ν *that is proportional to the national income* y^ν, *to be given by*

$$\pi^\nu = \frac{1}{\delta}[\tau^\nu(R^\nu) + \gamma^\nu \frac{\tau(V)}{\delta + \mu}]y^\nu,$$

where $\tau^\nu(R^\nu)$, γ^ν *are, respectively, the impact coefficient and carbon sequester rate for forests* *in country* ν.

8. International Fund for Atmospheric Stabilization

The divergence in economic performance between developed countries and developing countries has steadily widened in the last several decades, and various institutional and policy measures that have been devised internationally or bilaterally have not had much impact in narrowing the gap between these two groups of countries. The introduction of the proportional carbon tax system as envisioned here, in spite of the implicit recognition of the equity aspect in its design, may tend to worsen the relative position of developing countries, at least in the short-run. It would be desirable, therefore, to supplement the carbon tax system with the international redistributive scheme that would have significant impact in narrowing the gap between the stages of economic development of various countries involved.

The International Fund for Atmospheric Stabilization is an institutional framework in which it is possible to combine an international arrangement to stabilize atmospheric equilibrium with a redistributive scheme to help developing countries to accelerate processes of economic development.

The International Fund for Atmospheric Stabilization presupposes that each country adopts the proportional carbon tax system under which emissions of carbon dioxide and other greenhouse gases are charged a levy evaluated at the imputed prices proportional to the per capita level of national income and a charge (or a subsidiary payment) is made for the depletion (or the afforestation) of terrestrial forests, again based upon the evaluation at the imputed prices of terrestrial forests that are proportional to the per capita level of national income, as in detail discussed in the present paper.

The tax revenues from the proportional carbon tax system are principally put into the general revenue account of each government, preferably to be partly earmarked for the purposes of restoring the natural and ecological environments, and for encouraging private economic agents to develop those technological and institutional knowledge that are crucial in restoring equilibrium conditions in the global environment.

Each country then transfers a fixed portion, say 5%, of the net revenue from the carbon tax system to the International Fund for Atmospheric Stabilization . The total amount transferred to the International Fund for Atmospheric Stabilization from individual countries then would be allocated to developing countries according to a certain predetermined schedule, properly taking into account the per capita levels of national income and the size of population. Developing countries may use the amounts transferred from the International Fund for Atmospheric Stabilization for the purposes which they think appropriate, preferably for compensating those who would suffer from the phenomena of global environmental disequilibrium and incur the hardships by the implementation of the carbon tax system, for restructuring industrial organizations and social infrastructure, and for introducing substitutional energy sources and energy-saving technologies.

It is difficult to imagine that the International Fund for Atmospheric Stabilization or similar international arrangements on the global scale may be instituted in any immediate future. Whether such international arrangements may be effectively implemented or not depends to a significant extent upon the degree of awareness on the part of the general public concerning the enormous burden and costs future generations will have to suffer from the phenomena of global warming and other global environmental disequilibrium.

The strenuous effort by a large number of geo-scientists, ecologists, and other scientists to clarify the mechanism of global warming and to identify the specific implications of global warming and other environmental issues for ecological, biological, social, and cultural life on Earth has had a significant impact to the awareness and consciousness of the general public and the national governments. The numerous conferences and symposia organized by various international organizations, such as the 1991 Rio Conference and the Intergovernmental Panel on Climate Change, particularly the Kyoto Protocol of 1997, have substantially altered the perception of the international community as regards the plausibility and danger of global warming and other atmospheric disequilibria.

All these help the national governments involved to search for those policy and institutional arrangements that will make the practical implementation of the International Fund for Atmospheric Stabilization or similar international agreements feasible from economic, social, and political points of view. It would not be too optimistic to expect to have the International Fund for Atmospheric Stabilization or a similar framework to be instituted within a foreseeable period, though not in the immediate future.

9. A Hypothetical Case

A hypothetical case of the incidences of the proportional carbon taxes under the system of proportional carbon taxes for the emission of CO_2 and tax-subsidy measures for the reforestation and depletion of resources of forests are presented, all in terms of the statistical data of 2005 (in US$).

Table 1

Incidences of proportional carbon taxes with the coefficient of proportion 0.01

including all radiative forcing agents (RFA)

Country	National Income per capita	RFA Increase per capita(Ct)	Imputed Price per Ct	Carbon Taxes per capita
United States	42,000	5.90	420	2,500
Canada	34,000	6.20	340	2,100
United Kingdom	32,000	3.00	320	950
France	31,000	2.20	310	680
Germany	31,000	3.20	310	980
Italy	28,000	2.20	280	600
Netherlands	35,000	3.60	350	1,200
Sweden	32,000	1.90	320	610
Norway	48,000	1.60	480	760
Finland	31,000	2.00	310	610
Denmark	34,000	3.20	340	1,100
Indonesia	3,100	1.70	30	50
Japan	31,000	2.70	310	840
Korea	21,000	2.60	210	560
Malaysia	11,000	1.90	110	210
Philippines	3,200	0.30	30	8
Singapore	40,000	3.20	400	1,300
Thailand	6,900	1.20	70	80
India	2,200	0.30	20	7
China	4,100	1.10	40	40
Australia	33,000	7.10	330	2,300
New Zealand	23,000	3.50	230	790

Sources) UNFCCC, World Development Indicators, etc.

Table 2

Incidences of Tax-Subsidy Measures for the Reforestation

and Depletion of Resources of Forests

Country	Forest and Woodlands [Million ha]	Net Annual Reforestation [1000 ha]	Imputed Price [Per ha]	Assessment Total [Million Dollars]	Assessment Per Capita [Dollars]
United States	303	159	42,000	6,627	22
Canada	310	0	34,000	0	0
United Kingdom	3	10	32,000	321	5
France	16	41	31,000	1,264	21
Germany	11	0	31,000	0	0
Italy	10	106	28,000	2,929	50
Netherlands	0	1	35,000	35	2
Sweden	28	11	32,000	351	39
Norway	9	17	48,000	808	175
Finland	23	5	31,000	153	29
Denmark	1	3	34,000	102	19
Indonesia	88	-1,871	9,300	-17,120	-77
Japan	25	-2	31,000	-62	0
Korea	6	-7	21,000	-149	-3
Malaysia	21	-140	33,000	-4,679	-179
Philippines	7	-157	9,600	-1,507	-18
Singapore	0	0	40,000	0	0
Thailand	15	-59	20,700	-1,220	-19
India	68	29	2,200	64	0
China	197	4,058	4,100	16,678	13
Australia	164	-193	33,000	-6,319	-310
New Zealand	8	17	23,000	388	94

Source) World Resources Institute, etc.

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