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Abstract

Background: Two alternative procedures currently exist for efficient carbon dioxide (CO_2) emissions control: carbon emission trading and proportional carbon tax. This article explores which of the two is more desirable. A prominently distinct feature of these schemes is unilateral, bilateral or universal trade. Carbon emission trading is bilateral in the sense that a country purchases the emission right from its specific counterpart. If planned optimally, the proportional carbon tax is universal because each emitting country is equally levied a fixed rate depending on its CO_2 emission weight, although the tax rate should be determined consummately after referring to mutual negative externalities. This article examines how these differences affect the efficacy of the CO_2 emission control.

Results: Compared to emission trading, the equilibrium under a universal proportional carbon tax achieves Pareto superior allocation. In addition, the price of a unit of CO_2 emissions is much higher in the case of a universal proportional carbon tax than for bilateral emissions trading. This is because although the bilateral emissions trading scheme can attain the Pareto efficient allocation between the two concerned countries, the mechanism does not consider the negative externalities of these countries' emissions to the rest of the world. Indeed, such trading does not curb emissions; rather, it generates net additional emissions.

On the other hand, as the name suggests, a universal proportional carbon tax rate would be applied globally, and as far as it is obeyed by the modified Samuelson (1954) rule, the secondary negative externalities arising from the emissions trading can be entirely excluded. Thus, the first best allocation is achieved.

Conclusions: This paper reveals the superiority of a universal proportional carbon tax over the emissions trading in the efficacy of CO_2 emissions control. Although deciding and abiding by a universal tax rate is likely to require great political will, any efforts in this direction will be worthwhile. Unlike emissions trading which relies solely on private economic incentives, a universal proportional carbon tax is the urgent need of

the hour for limiting the advance of global warming.

Keywords: CO_2 emissions, Proportional carbon tax, Emissions trading, Modified Samuelson Rule, Nonlinear carbon tax

Background

Carbon markets are jerry-built in nature. They cannot be sufficiently sustained without government assistance and intervention. According to Lovins and Cohen (2011, p.227), ``In the wake of the world's failure to agree on a new trading legislation in Copenhagen and the U.S. Senate failure to pass binding legislation, the price of carbon fell drastically. In the United States it fell almost to nothing. In Europe, the price of carbon fell from a high of € 25 to € 8 a ton. By October 2010, prices had started to rebound , hitting € 12. Subsequent measures have only strengthened the price per ton."

First, this article explores the source of such fragility. We find that the decision of the economic agents emitting carbon dioxide (CO_2) is isolated from direct and indirect damages due to such emissions, owing to the genuine property of their external diseconomy of the emission, therefore, at the very least, the market cannot remain sustainable without expectations of strengthened regulation in the future.

Second, we examine the properties of bilateral offset carbon trading, which is currently the most popular trading scheme and is not subject to regulation concerning total emission amounts. Unlike Otaki (2013), although the analysis is static, the extension to dynamic analysis is not impossible, since the latter is an application of cap trading which regulates total emissions. Moreover, analysis pertaining to cap trading, which seems to be popular within the European market, requires the consideration of only one additional constraint concerning total emissions in the model.

Finally, we compare the function of bilateral carbon trading with that of a universal proportional carbon tax. The article concludes that a wide-ranged universal proportional carbon tax is superior to bilateral carbon trading, although the procedure of income redistribution between developed and developing countries becomes rather complicated for a proportional tax, because some progressive or digressive tax rate should be adopted.

Results and Discussions

The Model

Assume that two types of countries exist. One of the countries is an advanced industrialized or developed country which emits CO_2. The other country is a developing country that does not emit CO_2. There are n pairs of a developed and a developing country $\left(d_j, u_j\right)$ within which the emission right is traded, where d_j and u_j are the jth developed and developing country respectively, comprising the jth block of emission trading.

Each country has the same utility function U_j^i:

$$U_j^i \equiv c_j^i - \Psi\left(e_1^d, \cdots, e_j^d, \cdots, e_n^d, e_1^u, \cdots, e_j^u, \cdots, e_n^u\right) \equiv c_j^i - \Psi\left(\overrightarrow{e^d}, \overrightarrow{e^u}\right), \qquad (1)$$

where c_j^i denotes the consumption level of the j th country that belongs to type i $\left(i = d, u\right)$. Ψ represents the disutility from the CO_2 emission via the production process of a consumption good. We assume that Ψ is linear homogenous, quasi concave and *symmetric* in the following sense. That is,

$$\Psi\left(\cdot\cdot, e_l^k, \cdots, e_{l'}^{k'} \cdot\cdot\right) \equiv \Psi\left(\cdot\cdot, e_{l'}^{k'}, \cdots, e_l^k \cdot\cdot\right), \forall k, k', l, l'. \qquad (2)$$

This symmetric assumption implies that the disutility derived from the emission does not depend on where it is emitted. It is a plausible assumption when we consider the diffusion speed of CO_2 in the atmosphere.

The semi-reduced form production function, which represents the relationship between the consumption c and the adjoined emission e, is

$$c = \alpha F\left(e\right), \quad \alpha \text{ is some positive constant.}$$

$$F' > 0, F'' < 0, \left(e < \bar{e}\right). F' = F'' = 0, \left(e \geq \bar{e}\right). \qquad (3)$$

We assume that only developed countries possess the production technology. No developing country can access such an opportunity until the bilateral emission trading is settled or some type of proportional carbon tax is levied.

The Complete *Laissez-Faire* Situation

Assume that, in the *laissez-faire* situation, the decision of the production sectors within a country are separated from the consumer sector; they do not consider the nuisance incurred by their emission. Hence, they produce goods which amount to the maximum level $F\left(\bar{e}\right)$. In such a case, the unit price of carbon is zero.

As noted by Lovins and Cohen (2011, ch.8) and evidenced by history, this fact implies that without government intervention, the carbon market will surely collapse and the unit price of carbon will fall to become negligibly.

Unilateral Proportional Carbon Tax: Implementation of the Nash Equilibrium

Formatting a Unilateral Proportional Carbon Tax

Moving from the complete *laissez-faire* situation, each country is separately incentivized to levy a carbon tax to suppress its emissions for its own wellbeing, and thus, a unilateral proportional carbon tax is formatted by the solution of the following optimization problem:

$$\max_{e_l^k}\left[\alpha F\left(e_l^k\right)-\Psi\left(e_l^k,\overrightarrow{e_l^{k*}}\right)\right],\overrightarrow{e_l^{k*}}\equiv\left(e_l^{k'*},\vec{e}_2^{\,*},\cdots,\vec{e}_{l-1}^{\,*},\vec{e}_{l+1}^{\,*},\cdots,\vec{e}_n^{\,*}\right),$$
$$\vec{e}_j^{\,*}\equiv\left(e_j^{d*},e_j^{u*}\right).$$

Since the solution of this problem satisfies

$$\alpha F'\left(e_l^{k*}\right)=\frac{\partial\psi}{\partial c_1^d}\Big|_{\varepsilon_l^k=\frac{1}{n}},\tag{4}$$

we find that the implemented allocation by a unilateral carbon tax is identical to that implemented by the Nash equilibrium of an international game concerning CO_2 emission. In this sense, a unilateral proportional carbon tax belongs to a kind of international *laissez-faire* scheme and does not require any cooperative behavior between countries. However, it is evident that the resource allocation is improved compared with the complete *laissez-faire* situation, because the equilibrium emissions are determined by weaving the social disutility from excess emissions.

Bilateral Emissions Trading

Formatting the Bilateral Emission Trading Scheme

Bilateral emissions trading between the developed and the developing country in the j th pair is defined by the following optimization problem. That is,

$$\max_{e_j^d,e_j^u,P}\{\left[\alpha F\left(e_j^d\right)+\alpha F\left(e_j^u\right)-P-\Psi\left(e_j^d,e_j^u,e_1^{d*},e_1^{u*}\cdots e_n^{d*},e_n^{u*}\right)\right]$$
$$+\lambda\left[P-\Psi\left(e_j^u,e_j^d,e_1^{d*},e_1^{u*}\cdots e_n^{d*},e_n^{u*}\right)-\overline{U}\right]\},$$

where P is the total payment for the carbon emissions. * denotes the optimal contract emissions of the other pairs. λ is the Lagrangean multiplier of this problem. $\overline{U}$ denotes the reservation utility of the corresponding developing country. The first term of (4) is the developed-country's utility derived from this emission trading, and the term within the square brackets of the second term is the net welfare gain of the developing country.

Let us denote

$$\varepsilon_j^l\equiv\frac{e_j^l}{\sum_{l,k=1}^{n}e_k^l}\equiv\frac{e_j^l}{E}.$$

By using ε_j^l and E, Ψ can be transformed into

$$\Psi = \psi\left(\overrightarrow{\varepsilon^d}, \overrightarrow{\varepsilon^u}\right), E \equiv \psi\left(\vec{\varepsilon}\right)E.$$

Then by the symmetry of Ψ, the optimality condition for the above-mentioned contract problem under perfect information and symmetric equilibrium can be represented as

$$F'\left(e_l^{k*}\right) = \frac{2}{\alpha}\frac{\partial \psi}{\partial c_1^l}|_{\varepsilon_j^{l*}=\frac{1}{n}}, \forall j, l, P^* = \overline{U} + 2ne^*\psi|_{\varepsilon_j^{l*}=\frac{1}{n}}. \quad (5)$$

Universal Proportional Carbon Tax

Formatting a Universal Proportional Carbon Tax Scheme

A universal proportional carbon tax is formatted by the following optimization problem:

$$\max_{\vec{\varepsilon}}\left[\alpha F\left(e_1^d\right) - \Psi\left(\vec{\varepsilon}\right)E + \sum_{(l,k)\neq(1,d)} \lambda_l^k\left[\alpha F\left(e_l^k\right) - \Psi\left(\vec{\varepsilon}\right)E - \overline{U^m}\right]\right].$$

It is clear that the attained allocation under such a scheme is Pareto efficient by definition. By the symmetry of the problem, it is also clear that every Lagrangean multiplier under optimal planning takes the value unity[i]. Thus, we obtain the following formula concerning the optimal emission:

$$F'\left(e^*\right) = \frac{n}{\alpha}\frac{\partial \psi}{\partial c_1^d}|_{\varepsilon_j^{l*}=\frac{1}{n}} \quad (7)$$

This is the modified Samuelson (1954) rule concerning the optimal public good (bad) provision: The marginal benefit accrued from the country's emissions should be equalized to the sum of the marginal disutility diffused all over the world. The right-hand side of (7) is the optimal tax rate common to all constituents.

Welfare Ordering for Various Emission Suppressing Measures

Although it is clear that a proportional carbon tax, with the rate as is expressed by (7), is the first-best policy, how are the other two measures ordered in terms of Pareto efficiency? We can deal with this problem by using the symmetry and linear homogeneity of Ψ.

The utility of each country U_l^k can be written as

$$U_l^k\left(j\right) = \alpha F\left(e^*\left(j\right)\right) - \frac{\partial \psi}{\partial c_1^l}|_{\varepsilon_l^{k*}=\frac{1}{n}^*} \cdot E^*\left(j\right) = \alpha F\left(e^*\left(j\right)\right) - n\frac{\partial \psi}{\partial c_1^l}|_{\varepsilon_l^{k*}=\frac{1}{n}^*} \cdot e^*\left(j\right). \quad (8)$$

Employing the envelop theorem,

$$\frac{dU_l^k(j)}{dj} = \frac{\partial \psi}{\partial c_1^l}\Big|_{\varepsilon_l^k = \frac{1}{n}} \cdot e^*(j) > 0, (1 \le j < n)$$

$$\frac{dU_l^k(j)}{dj} = 0, (j = n)$$

$$\frac{dU_l^k(j)}{dj} = \frac{\partial \psi}{\partial c_1^l}\Big|_{\varepsilon_l^k = \frac{1}{n}} \cdot e^*(j) < 0, (n < j)$$

holds. Thus, it is clear from the above equations that the bilateral emission trading scheme Pareto-dominates a unilateral carbon tax scheme, whereby a country can set a proportional carbon tax rate at its discretion.

This fact implies that although a proportional carbon tax possibly attains the first-best allocation, emission trading is the second-best measure, unless all counties concur about the seriousness of global warming, in which case a much higher carbon tax rate than that in the unilateral case can be adopted. In addition, since n is likely far exceed two, the suppression effect of emission trading is estimated to be rather restrictive from the view point of the first-best allocation.

Welfare Analysis of Emission-Saving Technological Progress

Consider the effect of emission-saving technological progress to the world economy as a whole. This progress is expressed by an increase in α in this model. Before proceeding to the general equilibrium analysis, we must note that every trading pair increases emissions in conjunction with technological progress. Although it seems to be counterintuitive, if we note the fact that technological progress makes the imputed price of CO_2 cheaper as shown by the right-hand side of the left-half of (4), (5), and (7), it is natural that emission-saving technological progress conversely heightens the accumulation of CO_2.

With this precaution in mind, we shall proceed with the general equilibrium analysis, into which the mutual negative externalities between trading pairs are woven. Then, from (8) and the envelop theorem, we obtain

$$\frac{dU_j^d}{d\alpha} = F(e^*) - [n-j]\frac{\partial \psi}{\partial e_1^d}\Big|_{\varepsilon_l^k = \frac{1}{n}} \frac{de^*}{d\alpha} > \frac{1}{\alpha}\left[\alpha F(e^*) - \eta \Psi\right], \eta \equiv \frac{de^* \big/ e^*}{d\alpha \big/ \alpha}, \forall j,$$
$$\frac{dU_n^d}{d\alpha} > 0, \tag{9}$$

where η is the elasticity of the emission volume to the unit of the technological progress.

Since $\alpha F\left(e^{*}\right)-\Psi>0$, if η is small enough and the increase in the emission generated by the technological progress is not so serious[ii], the advance in the emission-saving technology improves worldwide utility, although this advance also increases the total amount of CO_2 emissions. This fact is underscored in that when we extend the scope of analysis to dynamic and intergenerational emission allocation (e.g., Otaki 2013), we may have to modify the obtained result, because the acceleration in emissions diffuses the negative externality to future generations.

In other words, although the emission-saving technological progress lowers the imputed price of CO_2 and stimulates the current generation's consumption, such current prosperity may conversely worsen the descendants' utility via the resulting massive emissions. However, such a dynamic prospect is beyond of the scope of this article, and it requires itself solving the simultaneous optimization concerning intertemporal and international emission problems.

On the Income Distribution between Countries: The Possibility of Nonlinear Pricing

Thus far, this article has assumed that a developed country directly invests in the corresponding developing country and that it receives revenues after deducting the carbon tax. Thus,

$$\alpha F\left(e^{*}(j)\right)-j\cdot\frac{\partial\psi}{\partial e_1^{l}}\Big|_{\varepsilon_l^k=\frac{1}{n}}\cdot e^{*}(j)\equiv\alpha F\left(e^{*}\right)-\tau(j)\cdot e^{*}(j),$$

where τ_j is the carbon tax rate, which is identical to the unit carbon price in emission trading. Hence, the developing country obtains tax revenues $R(j)$, which amounts to

$$R(j)\equiv\tau(j)\cdot e^{*}(j),$$

from the investing developed country.

Since every constraint concerning the joint utility from such a trading scheme binds whenever planning is optimal, the net surplus from the trading in terms of consumption becomes $\overline{U^{m}}$ [iii]. Although we have not yet analyzed the possibility of additional lump-sum transfer from the investing developed country to its counterpart developing country (or the transfer inverted direction, which is possible if the tax payment is too heavy for the developed country), one cannot envisage a universal proportional carbon tax without some fair division of the surplus earned by direct investment through this lump-sum transfer, specifically because the standard of living of the remitting /recipient country decisively depends on its share of this surplus (Uzawa 2003).

Hereafter this article analyzes both directions of the transfer and clarifies how the direction affects the *effective* tax rate. First, consider the transfer from the developed country to the developing country. This is an application of nonlinear pricing, which appears in basic microeconomics (e.g., see Tirole 1988). Let the sum of the transfer be s. Then, the total payment of a developed country to her counterpart T becomes

$$T \equiv \tau(j)e(j) + s = \left[\tau(j) + \frac{s}{e(j)}\right] \cdot e(j).$$

The term within the square brackets is the *effective* tax rate, which is illustrated by Figure 1. Thus, the effective tax rate is digressive although such a transfer enriches the developing country. This is owing to the economy of scale from the de-facto massive purchase of the right of emission.

Figure 2 illustrates the locus of the *effective* tax rate for the inverted transfer from the developing country to the developed country $(s < 0)$. It is apparent that the *effective* tax rate becomes progressive despite impoverishing the developing country. The progressive *effective* tax rate owes its existence entirely to the diseconomy of scale concerning emissions. However, we must still note that whether the *effective* tax rate is progressive or otherwise does not affect the efficacy of emission allocation. Moreover, this discussion exemplifies that the progressive tax rate is not necessarily advantageous to the developing country.

Conclusions

This article compared the static efficiency of consumption/emission allocation of three alternative emission control measures (proportional carbon tax (unilateral or universal) and emission trading)[iv]. A unilateral carbon tax, akin to symmetric Nash equilibrium of the worldwide consumption/emission game, is less efficient than bilateral carbon emission trading. Although a universal proportional carbon tax, into which the entire negative externalities of emissions of a country are woven, achieves the Pareto efficient allocation, realizing such an ideal tax system at this time appears very difficult. That is because the tax rate would become extremely high, much higher than the incumbent tax rate or price of emission right. As such, gradualism seems inevitable. The transition from bilateral to multilateral emission trading is desirable. Thus, a universal proportional carbon tax should be considered as the ultimate solution and political arena are needed to gradually establish the system.

Finally, it is inevitable that an emission-saving technological progress stimulates not only consumption but also emissions. Such technological progress cheapens the imputed price of CO_2. Although this possibly bodes well the current generation, it accelerates CO_2 accumulation and translates into a negative inheritance for our descendents.

Methods

Three measures for suppressing CO_2 emission are compared from the viewpoint of Pareto efficiency: a unilateral proportional carbon tax, a bilateral emission trading, and a universal proportional carbon tax. Heightened consumption within a country increases its utility at the cost of scattering more voluminous CO_2 all over the world and accelerating global warming. In this sense, there is a serious trade-off between consumption and CO_2 emissions, and thus this induces excess consumption/emission owing to the negative externalities, which are inherent to emissions.

The above three measures are modeled using a basic social planning theory under certainty, and are developed for suppressing excessive economic activity. The characteristics of these measures are classified by the extent of the negative externalities which a developed country cares to address when it emits CO_2. A unilateral proportional carbon tax scheme only concerns with the country's own disutility from its emissions. A bilateral emission trading scheme limits the country's concern to itself and its counterpart developing country, while a universal proportional carbon tax requests each affiliate to calculate the worldwide disutility caused by the country's emissions.

Competing interests

The author declares that he has no competing interests.

Author's contribution

MO carried out the modeling and analyzed the model. He also drafted the manuscript. MO read and approved the manuscript.

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Endnotes

[i] The first-order condition requires that

$$\alpha\lambda_l^k F'\left(e_l^{k*}\right)=\left[1+\sum_{j\neq l,k}\lambda_j^k\right]\frac{\partial\Psi}{\partial\varepsilon_1^d}\Big|_{\varepsilon_j^k=\frac{1}{n}}.$$

Accordingly, $\lambda_j^k=1,\forall j,k$ satisfies this condition.

If $\lambda_{j'}^{k'}\neq 1$ for some p pairs of $(j",k")$, $\frac{1}{\lambda_{s"}^{k"}}\left[1+\sum_{j,k}\lambda_j^k\right]=1+\frac{n-p+\sum_{(j',k')\neq(j",k")}\lambda_{j'}^{k'}}{\lambda_{s"}^{k"}}$ holds for $(j",k")$.

Otherwise $\frac{1}{\lambda_l^k}\left[1+\sum_{j\neq l,k}\lambda_j^k\right]=\left[n-p\right]+\sum_{(l',k')}\lambda_{s'}^{k'}$ holds.

Let us define $S_p\equiv\sum_{(l",k")}\lambda_{s"}^{k"}$. Then, the symmetric assumption requires

$$1+\frac{n-p+S_p-\lambda_{s"}^{k"}}{\lambda_{s"}^{k"}}=\left[n-p\right]+S_p\Rightarrow\lambda_{s"}^{k"}=1.$$

This is a contradiction. Therefore, $\lambda_l^k=1,\forall l,k$.

[ii] Otherwise, the welfare is conversely aggravated by the emission-saving technological progress under the second-best emission control systems. This is a *fallacy of composition*.

[iii] Since our utility function is quasi linear, the surplus is equivalent to that in terms of the utility. In addition, since we presume that the equilibrium is symmetric, the reservation utility $\overline{U^m}$ is endogenously determined. This result comes from the required property that all optimal Lagrangean multipliers should take the value unity.

[iv] This article studies how far emissions of CO_2 are permissible with the remaining given volume of CO_2 in the atmosphere. As such, the dynamic and cumulative effects of CO_2 are beyond the scope of this article. While Otaki (2013) considered this problem, he neglected the problem of the international efficient allocation of emissions.

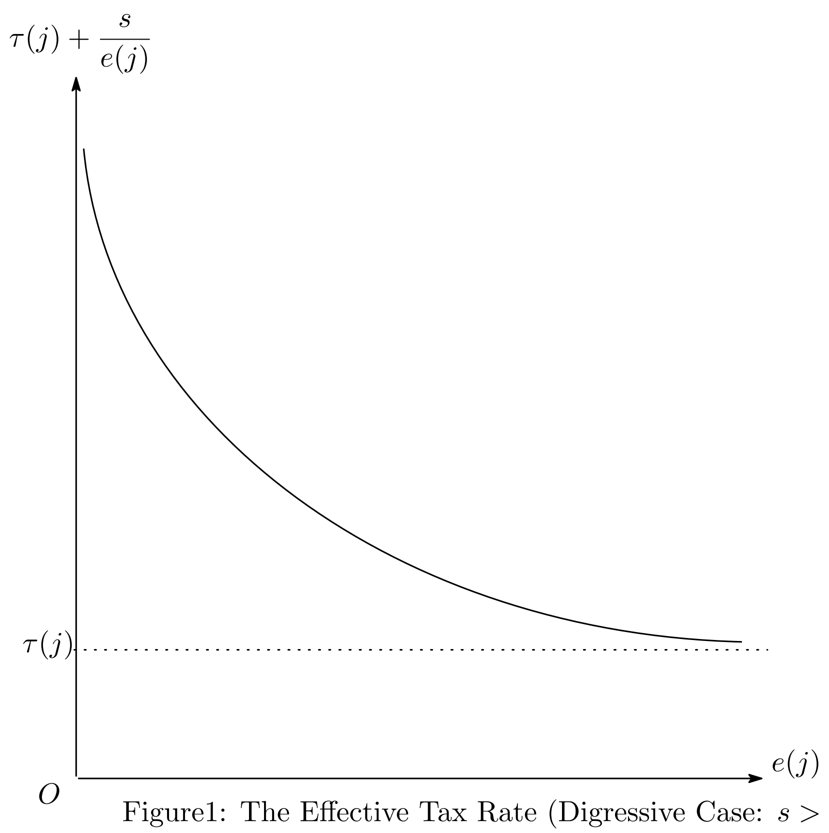

Figure1: The Effective Tax Rate (Digressive Case: $s > 0$)

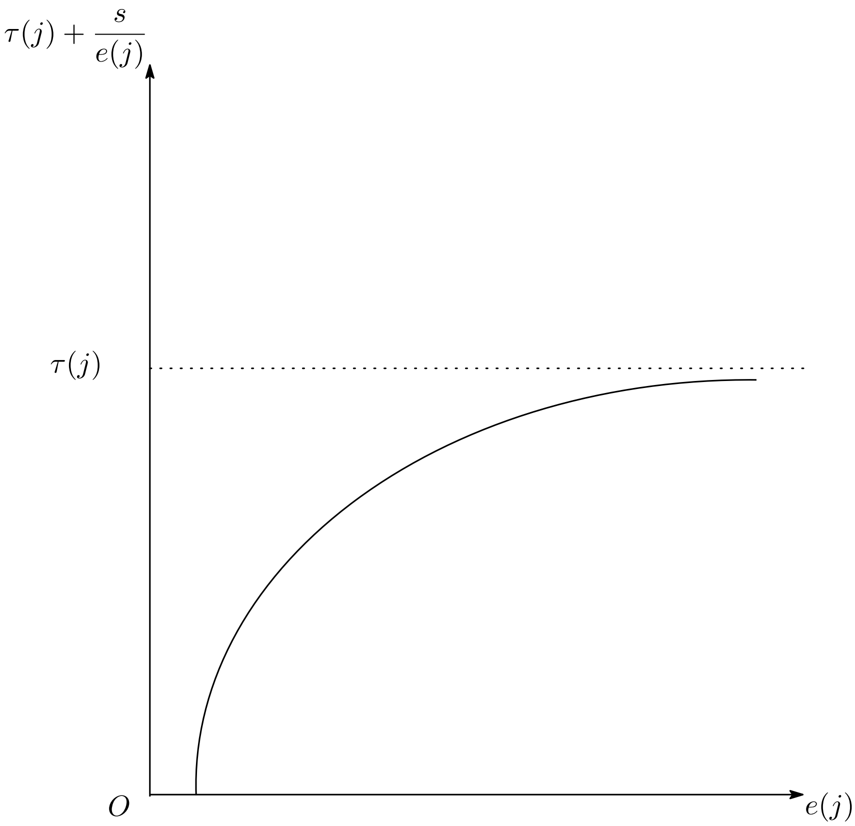

Figure 2: The Effective Tax Rate (Progressive Case: $s < 0$)