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# **The Role of Money: Credible Asset or Numeraire?**

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## **Abstract**

It is well known that money is neutral if (i) people hold the extraneous belief that it is only a numeraire and does not possess intrinsic value, and (ii) new money is injected into an economy as its own interest in the OLG model under perfect information (Lucas [1] Theorem 2). We find that whenever (i) is not satisfied and money is rationally held to have substance value, it becomes non-neutral even if we use the same model as Lucas [1].

## **1 Introduction**

This article shows that the multiplicity of rational beliefs concerning the value of money decides whether money is neutral under perfect information structures in the two-period OLG model. The result is kept intact, even if the new money injected into the economy is subject to the model of Lucas [1]. This result contrasts with those of Otani [2] and Otaki [3], [4], [5].

If the nominal rate of interest on money increases (i.e., money supply increases), people can consistently believe that the purchasing power of money (the inverse of the next period price level) is retained.<sup>1</sup> Then, the real interest rate becomes higher, thereby matching the supply, and the demand for money increases.

From assumptions concerning the utility function, This situation also implies the reduction of current consumption and leisure. Thus, the monetary expansion increases current total output, and hence, money becomes non-neutral.

We must note that the attained equilibrium is stationary in the sense that the values of real endogenous variables, such as current/future consumption and leisure, are entirely time-independent. This assertion holds, since once the real interest rate is raised because of a change of the nominal interest rate (increment of nominal money supply per capita), one may expect that the change in the inflation rate will equal that of the nominal interest rate; the heightened real interest rate is kept intact, and thus, the equilibrium becomes self-enforcing and stationary.

The rest of paper is organized as follows. In Section 2, I construct the same model as Lucas [1], except for the formation of rational expectations concerning the value of money, and proves the non-neutrality of money. A welfare economics implication is also analyzed. Section 3 contains brief concluding remarks.

## **2 The Model**

### **2.1 The Structure of the Model**

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<sup>1</sup>Otaki [5] defines such a rational expectation as money being credible.

We use essentially the same model as Lucas [1], excluding uncertainty. In every period a unit individual is born and lives two periods. Each individual has an identical utility function  $\tilde{U}$  :

$$\tilde{U} \equiv U(c_1, n) + V(c_2) \quad (1)$$

where  $c_1$  and  $c_2$  are the current and future consumption level, and  $n$  denotes the hours worked per individual.

Furthermore,  $U$  and  $V$  satisfy the following properties:

$$U_c > 0, \quad U_n < 0, \quad (2)$$

$$U_{cc} < 0, \quad U_{nn} < 0, \quad U_{cc} + U_{cn} < 0, \quad U_{cn} + U_{nn} < 0, \quad (3)$$

$$V''(c_2)c_2 + V'(c_2) > 0, \quad \frac{c_2 V''(c_2)}{V'(c_2)} \leq -a < 0, \quad (4)$$

$$\lim_{c_2 \rightarrow 0} V'(c_2) = +\infty, \quad \lim_{c_2 \rightarrow \infty} V'(c_2) = 0. \quad (5)$$

## 2.2 The Maximization Problem of Representative Individual

Each individual maximizes his/her lifetime utility  $\tilde{U}$ , subject to the following budget constraint:

$$n_t \leq c_{1t} + \frac{m_{t-1}}{p_t}, \quad p_{t+1}c_{2t+1} \leq m_{t-1}x, \quad (6)$$

$$\Leftrightarrow n_t \leq c_{1t} + \rho_t c_{2t+1}, \quad (7)$$

$$\rho_t \equiv \frac{p_{t+1}}{p_t x}, \quad (8)$$

where  $x$  denotes the increment of money supply per capita, and  $\rho_t$  is the inverse of the real interest rate.  $m_{t-1}$  is the money per capita carried over from the previous period.

The Kuhn-Tucker conditions imply that the optimal decision  $(c_{1t}^*, c_{2t}^*, n_t^*)$  satisfies

$$\frac{dn_t^*}{dc_{1t}^*} = -\frac{U_c(c_{1t}^*, n_t^*)}{U_n(c_{1t}^*, n_t^*)} = 1, \quad (9)$$

$$\frac{V'(c_{2t}^*)}{U_c(c_{1t}^*, n_t^*)} = \rho_t \quad (10)$$

$$n_t^* = c_{1t}^* + \rho_t c_{2t}^*. \quad (11)$$

## 2.3 Market Equilibrium

There are two markets in the above model: the money market and goods market. By Walras' law, we can neglect the equilibrium condition for the goods market. The money market equilibrium condition is

$$m_{t-1}x = p_{t+1}^* c_{2t+1}^*. \quad (12)$$

Instead of the quantity-theoretic equilibrium price function imposed by Lucas [1],

let us assume that money is *credible* in the sense of Otaki [5] -- that is, the rational expectation concerning the current purchasing power of money  $\frac{1}{P_{t+1}^*}$  is not perturbed by an increase of  $x$ :

$$\frac{dp_{t+1}}{dx} = 0. \quad (12)$$

The general equilibrium of markets is attained by five equations: (8), (9), (10), (11) and (12). Endogenous variables are  $(c_{1t}^*, c_{2t}^*, p_t^*, p_{t+1}^*, n_t^*)$ .

The partial equilibrium of labor  $n_t^*$  and the younger-generation's consumption  $c_{1t}^*$  is illustrated by Figure 1. The downward sloping curve  $AA$  is the locus of Equation (8), which is easily derived from Assumption (3).

The upward sloping curve  $BB$  is the locus of Equation (10), which is combined with Equation (9). The procedure is as follows: Substituting Equation (9) into (10), we obtain

$$n_t^* = c_{1t}^* + c_{2t}^* \frac{V'}{U_c}.$$

In differentiating both sides of the above equation,

$$\begin{aligned} \frac{dn_t^*}{n_t^*} &= \frac{dc_{1t}^*}{c_{1t}^*} + \frac{d[V'c_{2t}^*]}{V'c_{2t}^*} - \frac{U_{cc}dc_{1t}^* + U_{cn}dn_t^*}{U_c} \\ &= \frac{dc_{1t}^*}{c_{1t}^*} + \frac{d[V'c_{2t}^*]}{V'c_{2t}^*} - \frac{c_{1t}^*[U_{cc} + U_{cn}\frac{dn_t^*}{dc_{1t}^*}]}{U_c} \frac{dc_{1t}^*}{c_{1t}^*} \\ \Leftrightarrow \frac{dn_t^*}{n_t^*} &= [1 - \frac{c_{1t}^*[U_{cc} + U_{cn}]}{U_c}] \frac{dc_{1t}^*}{c_{1t}^*} + \frac{d[V'c_{2t}^*]}{V'c_{2t}^*} \end{aligned} \quad (13)$$

holds. Hence, Curve  $BB$  is upward sloping for any fixed  $c_{2t}^*$ . When the money market equilibrates, the equilibrium consumption of the younger generation and output is determined at the intersection of Curves  $AA$  and  $BB$  (Point  $E_0$ ).

Whenever money is *credible*, it is facile to depict the property of money market equilibrium. From Equations (11) and (12), we obtain

$$\frac{dc_{2t}^*}{c_{2t}^*} = \frac{dx}{x}. \quad (14)$$

Since  $V'c_{2t}^*$  is an increasing function of  $c_{2t}^*$  from Assumption (4), Equations (13) and (14) imply that Curve  $BB$  shifts toward the south-east like, Curve  $B'B'$  by an increase of  $x$ . Thus, the economy moves from Point  $E_0$  to  $E_1$ .

Accordingly, as long as money is *credible*, a monetary expansion increases the output  $n_t^*$  and future consumption  $c_{2t}^*$ , and decreases the younger-generation's consumption  $c_{1t}^*$ . To sum up:

**Theorem 1.** *If money is a credible asset, it becomes non-neutral to the real economy. An acceleration of monetary growth heightens the real interest of money, and hence stimulates future consumption and output/labor supply, and economizes current consumption.*

Next, we shall show that the equilibrium depicted above is a stationary rational expectation equilibrium. Suppose that the economy is located at Point  $E_1$  by an increase of  $x$ , and individuals believe that the higher equilibrium real interest rate  $1/\rho^*$  prevails thereafter.

Then by the definition of  $\rho$  (Equation (7)),

$$\frac{d\rho^{1*}}{\rho^{1*}} = \frac{dx}{x} - \left[ \frac{dp_{t+j+1}^*}{p_{t+j+1}^*} - \frac{dp_{t+j}^*}{p_{t+j}^*} \right] = 0, \quad j \geq 1 \quad (15)$$

holds. Since  $m_{t+j-1}x = p_{t+j+1}^*c_{2t+j+1}^*$  and  $m_{t+j} = m_{t+j-1}x$ ,

$$\frac{dx}{x} = \left[ \frac{dp_{t+j+1}^*}{p_{t+j+1}^*} - \frac{dp_{t+j}^*}{p_{t+j}^*} \right] + \left[ \frac{dc_{2t+j+1}^*}{c_{2t+j+1}^*} - \frac{dc_{2t+j}^*}{c_{2t+j}^*} \right] \quad (16)$$

also holds. Combining Equation (16) with (15), we finally obtain

$$\frac{dc_{2t+j+1}^*}{c_{2t+j+1}^*} = \frac{dc_{2t+j}^*}{c_{2t+j}^*}, \quad j \geq 1. \quad (17)$$

Thus, the equilibrium consumption of an old individual  $c_2^*$  is time-independent.

It is clear from Equations (8) and (13) that the rest of the two endogenous variables  $(c_{1t+j}^*, n_{t+j}^*)$  are also time-independent. Consequently, the equilibrium illustrated by Point  $E_1$  is stationary in the sense that every equilibrium value of endogenous variables is time-independent. One can thus affirm

**Theorem 2.** *The rational expectation equilibrium defined by Equations (8), (9), (10), (12), and (15) is stationary (i.e., time-independent). Hence the heightened real interest rate caused by an increase of the nominal interest on money  $x$  permanently affects the real variables.*

## 2.4 A Welfare Implication of the Model

By Theorem 1, a monetary expansion (an increase in  $x$ ) stimulates the equilibrium real GDP  $n^*$  through the rise of the real rate of interest. Here, we consider its welfare economics implication. Let the Lagrangean of individual decision  $L$  that is evaluated at the equilibrium value.

Then, applying the envelop theorem, we obtain

$$\frac{d\tilde{U}}{d\rho^*} = \frac{dL}{d\rho^*} = \frac{\partial L}{\partial \rho^*} = -\lambda^*(x)c_2^*(x) < 0,$$

where  $\lambda^*$  is the Lagrangean multiplier. Accordingly, a monetary expansion improves economic welfare, since it makes future goods cheaper.

### 3 Concluding Remarks

This paper shows that money is non-neutral as long as it is *credible* even if we obey the money-supply rule proposed by Lucas [1]. A monetary expansion (an acceleration of the money growth rate) surely hightens the real rate of interest of money whenever people believe that money is *credible*. The effect of intertemporal substitution leads them to work more to prepare for more future consumption, and thus, the aggregate products increases. It also implies that economic welfare is improved by a monetary expansion.

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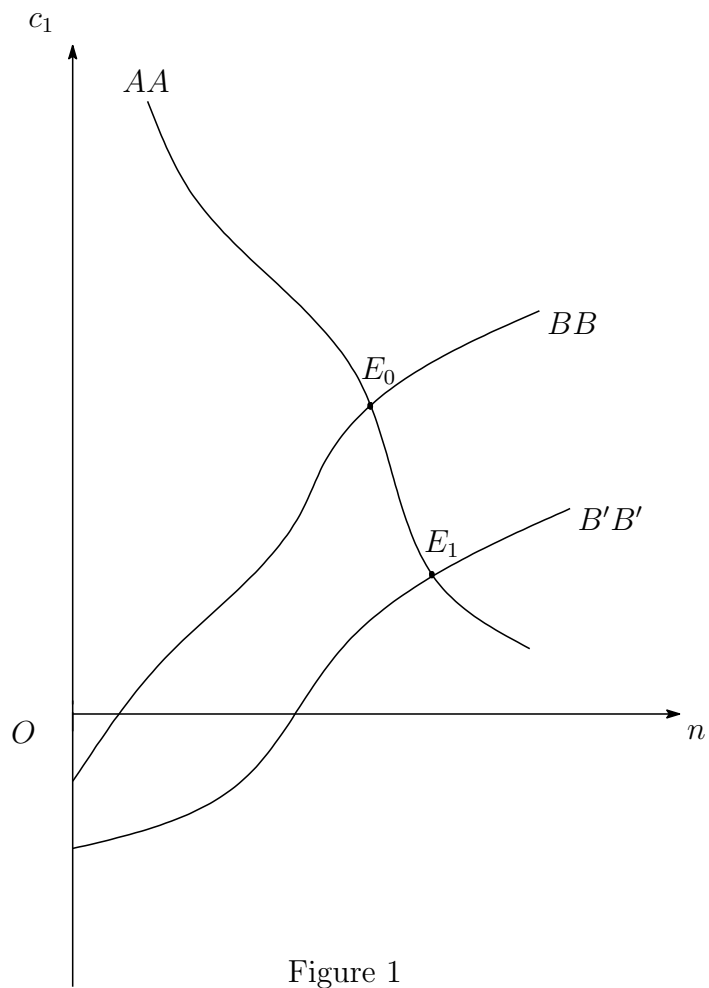


Figure 1