DBJ Discussion Paper Series, No.1307

Income Disparity, Uneven Economic Opportunities, and Verifiability

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January 2014

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Abstract

The function of income in educational investment is considered under asymmetric information on individuals' effort. High income strengthens deep commitment to investment, and hence facilitates access to the capital market. Low income individuals tend to be excluded for the same reason even though they have the same abilities as wealthy individuals. Thus, disparity of income produces uneven economic opportunities. Some income redistribution policy is necessary to cure this social inefficiency. The expenditure of transferred income should, however, be limited to educational investment to avoid wasteful consumption.

1. Introduction

Human capital investment, especially educational investment, heavily depends on an individual's own income and/or wealth, and thus, non-wealthy individuals are excluded from economic opportunities that not only enhance their intelligence but also increase their future incomes. This article explores why income disparity hinders equality of economic opportunities.

Verifiability of one's effort to succeed plays a key role. Wealthy people, who can invest sufficient money in education, are naturally incentivized to engage in high level efforts regardless of whether it is verifiable or not. This is because they would lose much money when their educational investment fails. Thus, high income and/or wealth signal the soundness of the investment plan to financial intermediaries. Hence, high income and/or wealth facilitate lending towards wealthy individuals' investment.

Since non-wealthy individuals cannot be incentivized by the cost incurred when their educational investment fails, and efforts towards cultivation via education are generally not verifiable, financial intermediaries quote a high interest rate to compensate for such a high risk. Therefore, financial intermediaries' lending to non-wealthy individuals cannot be accomplished easily. Thus, uneven economic opportunities coexist with disparity of income as such.

This article is organized as follows. In Section 2, we construct a model that exhibits the coexistence of uneven economic opportunities and income disparity. Section 3 explains how this problem resolved. Section 4 presents some concluding remarks.

2. The Model

2.1 Structure of the model

The model is an application of the theory of moral hazard to limited liability in financial deals. It originates from Arrow (1963) and Stiglitz and Weiss (1981).

We assume lenders (financial intermediaries) and borrowers (individuals who intend to invest in education) are both risk neutral, and their concerns are confined to expected return. The investment initially requires a unit of money. The probability of success in the investment, which is controllable by the borrower's effort, is p. The investment generates X amounts of goods when it succeeds, and nothing when it fails.

The cost function of the effort to ensure the success probability, p, in terms of money, $c(p)$,has the following properties.

$$c(0)=c'(0)=0, c'>0, c''>0, \text{ if } c>0. \qquad (1)$$

Furthermore, we denote the lending interest rate as $1+r$, and the deposit rate is equal to unity. Finally, we assume the following inequality.

$$X > 1+r. \tag{2}$$

This inequality is necessary for all educational investments to be meaningful.

Based on this setting, the borrower's payoff function π^B can be defined as

$$\pi^B \equiv p\left[X-\left[1+r\right]\left[1-m\right]\right]-\left[1-p\right]m-c\left(p\right). \tag{3}$$

The first term in the left-hand side of Equation (3) is the net expected revenue from the investment whose income is m. The second term corresponds to the expected loss when the investment fails. The third term is the disutility of the effort to ensure the success probability, p, which is measured in terms of money.

Since an individual maximizes his or her payoff, we obtain the following by differentiating Equation (2) with respect to p.

$$c'\left(p\right) = X-\left[1+r\right]\left[1-m\right]+m, \tag{4}$$

The first term in the right-hand side of Equation (4) represents the gain from lightening his or her redemption. The second term is the effect that reduces the loss from investment failure. Equation (4) is illustrated by Figure 1, where the success probability p is clearly a monotonously increasing function of m. Let us denote this relationship as follows.

$$p = \psi\left(m\right), \psi' > 0. \tag{5}$$

On the other hand, the payoff function of a lender is

$$\pi^L\left(m\right) \equiv \psi\left(m\right)\left[1+r\left(m\right)\right]-1. \tag{6}$$

The first term in the right-hand side of Equation (6) is the expected interest revenue from lending. The second term is the redemption of a deposit. We assume that the deposit market is competitive, and the equilibrium profits from a loan are zero. That is,

$$\psi\left(m\right)\left[1+r\left(m\right)\right]-1=0 \tag{7}$$

holds.

2.2 Comparative statics and welfare implications

Presuming the relationship in Equation (7), and employing the envelope theorem, we can show that

$$\frac{d\pi^B}{dm}=\frac{\partial\pi^B}{\partial m}=\psi(m)\left[1+r(m)\right]-\left[1-\psi(m)\right]=\psi(m)>0 \qquad (8)$$

To summarize, we obtain the following theorem.

Theorem 1

The success probability of an education investment is an increasing function of the individual's income, m *. Furthermore, the expected net revenue from the investment is also an increasing function of* m *.*

Combining Equations (2) and (7), we obtain

$$\psi(m)X>\psi(m)\left[1+r\right]>1,\forall m\Rightarrow\psi(0)X>1. \qquad (9)$$

From Theorem 1 and Inequality (9), we can ascertain that all potential educational investments are socially desirable, because they bring about positive surpluses to potential borrowers.

However, some not wealthy strata cannot access the capital market because surpluses for the investment are too small relatively to the cost incurred by the effort. Henceforth, we assume that the following relationship holds. That is,

Assumption 1

$$\psi(0)\left[X-\left[1+r\right]\right]-c\left(\psi(0)\right)<0 \qquad (10)$$

holds.[1]

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[1] The following is an example. Let $c(p)=\frac{1}{2\alpha}p^2$. Equations (4) and (7) imply that

$$\psi(0)=\alpha\left[X-\left[1+r(0)\right]\right]=\alpha\left[X-\left[1+\frac{1}{\psi(0)}\right]\right].$$

Thus, $\psi(0)$ satisfies the following quadratic equation.

$$\psi^2(0)-\alpha\left[X-1\right]\psi(0)-\alpha=0.$$

The solution is

$$\psi(0)=\frac{\alpha\left[X-1\right]+\sqrt{\alpha^2\left[X-1\right]^2-4\alpha}}{2}.$$

Consequently,

$$\pi^B(0)=\alpha\psi(0)X-1-\frac{\psi^2(0)}{2\alpha}=\frac{\left[X-1\right]\psi(0)-3}{2}.$$

Since, $\psi(0)$ is a monotonous decreasing function of α, as long as α is sufficiently

This assumption, in conjunction with Theorem 1, implies that the net expected return for borrowers whose income is located within the interval $\left[0, m^*\right)$ become negative, where m^* satisfies $\pi^B\left(m^*\right)=0$ (see Figure 2). Consequently, they have to give up the investment even though all individuals have the same innate abilities. Thus, income disparity hinders equalizing economic opportunities, and we have the following theorem.

Theorem 2

The uneven educational opportunity owing to the disparity of income is socially inefficient.

2.3 Verifiability and the law of large numbers

The above theory, similar to other economic theories under uncertainty, is based on the law of large numbers. Lenders can know borrowers' effort only statistically. That is, while the average revenue of lenders can be calculated accurately from large samples, lenders cannot anticipate the success or failure of an each individual borrower's educational investment.

This property, which is intrinsic to stochastic phenomena, causes a problem concerning the verifiability of borrowers' efforts. Lenders are unable to identify the cause of investment failure; because of misfortune or their laziness. That is, the cause of failure is not verifiable by its stochastic nature. Hence lenders depend on observable information such as incomes to infer borrowers' efforts.

This implies that even though a talented but non-wealthy individual, who has a low cost function $c\left(p\right)$, applies for a loan, his or her request is rejected because of the low income. Theorem 2 proves that such an uneven opportunity reflects the inefficiency of the society, and suggesting the acute need of that some measures to provide educational

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small, $\pi^B\left(0\right)<0$ holds.

On the other hand, $\psi\left(1\right)=\alpha\left[X+1\right]$, and thus,

$$\pi^B\left(1\right)=\alpha X\left[X+1\right]-\frac{\alpha^2}{2\alpha}\left[X+1\right]^2=\frac{\alpha}{2}\left[X+1\right]\left[X-1\right]>0.$$

Therefore, there is a value m^* such that $\pi^B\left(m^*\right)=0$ is satisfied.

opportunities to non-wealthy individuals for educational opportunity are acutely desirable.

3. Supplementary Policies

The society faces a problem of shortage of funds for non- wealthy individuals. Hence, it is sufficient that the government transfers incomes to them up to the critical value m^*. However, the government should restrict the usage of money. It should limit the usage to educational investment.

Without this restriction, those who receive the subsidy consume wasteful items. This can be easily proved as follows. From Equation (8), the marginal utility from additional investment is $\psi\left(m^*\right)<1$. On the other hand, the marginal utility of wasteful consumption is unity from the definition of the utility function (3). Accordingly, any additional money transfers to non-wealthy individuals with no condition attached are always used for wasteful expenditure. Thus, we obtain the following theorem.

Theorem 3

An optimal income redistribution policy involves income transfers from those whose income exceeds unity to non-wealthy individuals up to m^*. *However, expenditure must be restricted to educational investment.*

4. Concluding Remarks

We have analyzed the role of income in education investment. The results obtained are as follows. First, high income eases investment because it interests lenders. Excessively low income individuals are deprived of educational opportunities even though they have the same abilities as high income individuals. Such deprivation indicates social inefficiency.

Second, some income redistribution policy is unavoidable to cure the inefficiency of the society. Nevertheless, the usage of the transferred money should be confined to educational purposes.

References

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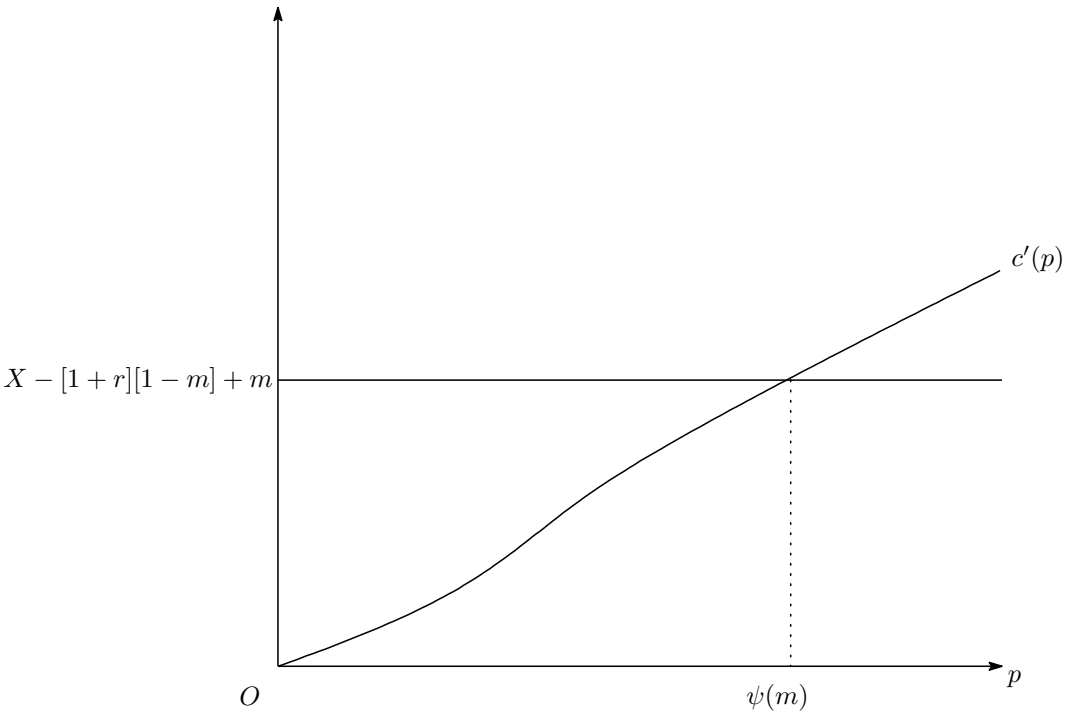

Figure 1: Optimal Effort

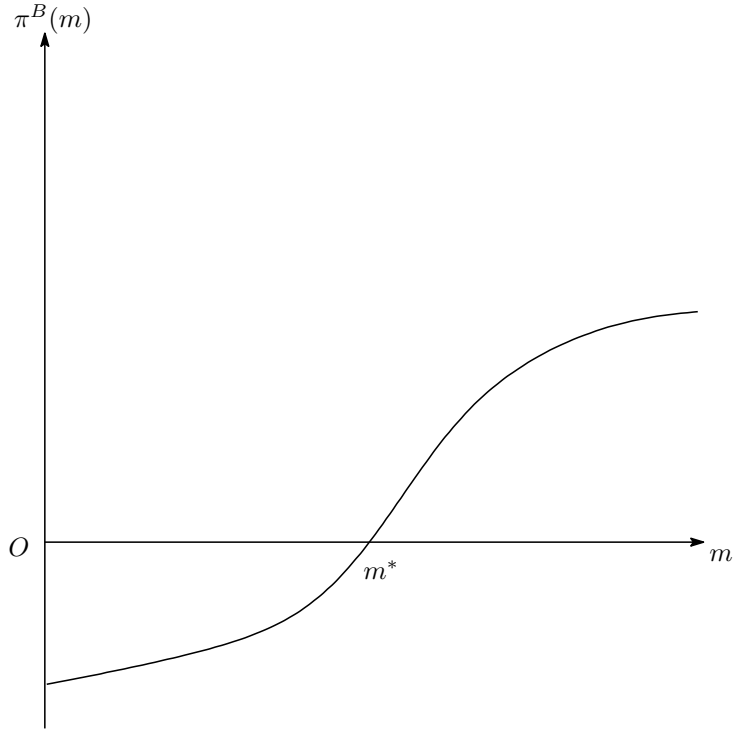

Figure 2: Borrower's Payoff Function