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# Trade, Resource Use and Pollution: A Synthesis

Akihiko Yanase
(Nagoya University)
Gang Li
(Toyo University)

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# Trade, Resource Use and Pollution: A Synthesis*

Akihiko Yanase[†] Gang Li[‡]

Nagoya University Toyo University

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**Abstract**

We develop a two-sector dynamic model of trade and the environment, in which both sectors are environmentally harmful and one sector is environmentally sensitive in the sense that its productivity depends on the environmental stock. The model yields a rich set of insights regarding specialization pattern, environmental impacts, and welfare effects of trade between countries subject to industrial pollution as well as resource extraction. In the short run, labor allocations and world specialization pattern are dependent on country size, technology, preference, and environmental stocks. The environmental stocks evolve over time and, depending on the type of each trading country (revealed by whether the environmentally sensitive sector is more environmentally harmful), the steady-state environmental stocks and world specialization pattern are determined, deriving environmental and welfare consequences of trade. At least one country gains from trade in the long run if two countries are of the same type. If two countries are of different types, however, both may lose from trade by exporting their respective "dirtier" goods to one another.

*Keywords:* Renewable resource; pollution; environmental stock; long-run supply curve; long-run PPF; world specialization pattern; long-run gains from trade

JEL classification: F18; Q27

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[†]Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8601, Japan. E-mail: yanase@soec.nagoya-u.ac.jp

[‡]Department of Economics, Toyo University, 5-28-20 Hakusan, Bunkyo-ku, Tokyo 112-8606, Japan. E-mail: ligang.hitu@gmail.com

# 1 Introduction

Although it has been a few decades since global environmental problems such as climate change are recognized as one of the important issues to be worked on in the international arena, local environmental problems are still significant, especially in emerging and developing economies. In comparison with developed countries, developing countries are still reluctant to implement stringent environmental policies, claiming the right to pursue economic growth at the expense of environmental protection. In addition, property rights to natural and environmental resources are often less well-defined in these countries. For these reasons, local-scale environmental problems, such as air and water pollution, soil contamination, desertification, and deforestation, remain substantial in many developing countries.[1]

Such local environmental problems vary by countries. In emerging economies, such as China and India, pollution problems accompanied by rapid industrialization are standing out, while less developed countries in Sub-Saharan Africa and Latin America rather face the problem of overexploitation of natural resources, bringing about resource depletion as well as pollution and loss of biodiversity from traditional agriculture (such as slash-and-burn shifting cultivation). The causes of each environmental problem also vary. Take, for example, deforestation, which is a result of excessive harvest of timber, but could also arise from persistent acid precipitation caused by sulfur or nitrogen oxide emissions. Featuring a wide range of the problem's nature and causes, an economically significant consequence of these problems is that, in addition to the possible risk to human health, primary-good industries that produce food and resource goods are vulnerable to environmental deterioration.

In this study, we theoretically address the issue of trade and the environment with an eye on developing countries.[2] These countries have increased their presence in the

[1]Take air pollution for instance. According to 2018 World Air Quality Report by IQAir AirVisual (available on https://www.airvisual.com/world-most-polluted-cities) Asian locations dominate the highest 100 average PM2.5 (particulate matter less than 2.5 microns in diameter) levels, with cities in India, China, Pakistan and Bangladesh occupying the top 50 cities. The Report also notes that although sources of pollution vary by region and city, common contributors include vehicle exhaust, open crop and biomass burning, industrial emissions and coal burning.

[2]The sensitivity of results on the relationship between trade openness and environmental quality to differences between developing countries and OECD countries has been empirically analyzed by Managi et al. (2009) and Tsurumi and Managi (2012). Managi et al. (2009) used panel data of sulfur dioxide (SO2) and carbon dioxide (CO2) emissions of 88 countries from 1973 to 2000, and showed that trade openness increases SO2 and CO2 emissions in non-OECD countries, while it decreases them in OECD countries. Using data on the annual rate of deforestation for 142 countries from 1990 to 2003, Tsurumi

world economy, as shown by the rapid growth in exports from developing countries and trade between them.[3] In view of the increased presence of developing countries in world trade and the fact that the implementation of their environmental policies can be often deficient, it is of great interest to investigate the extent to which trade liberalization affects the environment and economic welfare in countries where governments fail to implement appropriate environmental policies and environmental degradation harms the productivity of sectors that produce environmentally sensitive, primary commodities.[4]

For the purpose, we develop a two-sector model in which both production sectors are environmentally harmful by hampering the recovery of the environment, measured by a stock variable (environmental stock) that evolves over time. In addition, one sector among the two is environmentally sensitive in the sense that its productivity depends on the environmental stock. This sector can be interpreted as an agricultural- or resource-good sector, which can damage the environment via resource extraction or biomass burning and can also be affected by the environmental quality. The other sector can be considered as a manufacturing sector, the production process of which emits pollution that has a negative effect on the accumulation of environmental stock. We explicitly formulate the idea that both resource extraction and industrial pollution can damage the environment, and make an elaborate analysis that captures trade between, in particular, emerging and less developed countries. Our study contributes to the literature because existing studies on trade and the environment with an environmentally-sensitive, resource-good sector have considered resource use and pollution in different economic models.

The interaction between trade and renewable natural resources was explored by Brander and Taylor (1997b, 1998) in a dynamic Ricardian general equilibrium model with open-access renewable resources. In their model, there are two final goods, the resource good and the other good. Brander and Taylor (1997b, 1998) showed that while a resource importer gains from trade, a diversified resource-exporting country necessarily suffers a

and Managi (2012) found that an increase in trade openness increases deforestation for non-OECD countries while slowing down deforestation for OECD countries.

[3]According to UNCTAD (2013), the share of South–South trade in total world trade increased from slightly less than 30 per cent in 1995 to slightly more than 40 per cent in 2012. Furthermore, the share and value added of manufacturing in a developing country's exports to other developing countries are usually much higher than exports to developed countries.

[4]Pointing out that in many developed countries, the consumption of imported coffee, tea, sugar, textiles, fish, and other commodities causes a biodiversity footprint that is larger abroad (i.e., in developing countries) than at home, Lenzen et al. (2012) found that 30 per cent of global species threats are due to international trade.

decline in the steady-state utility level.[5] The Brander–Taylor model can also help understand the effects of trade involving economic activities both being environmentally sensitive and generating environmental impacts, such as timber industry that can lead to deforestation and agriculture that uses slash-and-burn cultivation.[6]

To address the issue on trade and the environment with pollution emitted in an environmentally insensitive sector, Copeland and Taylor (1999) developed a two-sector Ricardian model in which the production of "smokestack" manufacturing generates pollution, which lowers the productivity of an environmentally sensitive sector. Assuming a laissez-faire economy with pollution unregulated, they showed the extent to which international trade may benefit both countries by spatially separating dirty and clean industries and thereby raising the world's production possibilities.[7]

In terms of the model structure, we consider a hybrid model of the Brander–Taylor and Copeland–Taylor models. In a study closely related to ours, Rus (2016) combined the Brander–Taylor model with the Copeland–Taylor model to analyze international trade patterns and the effects of trade liberalization. His analysis is, however, confined to a small-open economy. By contrast, we analyze a two-country model, which is a profound step in the complexity and in providing a formal tool to consider trade between countries of different types. The first type is what we call the BT country where resource extraction is the main source of environmental deterioration. The second type is what we call the CT country where industrial pollution is the main source. Trade between BT and CT countries in our model thus represents typical contemporary South–South trade in reality; our hybrid model sheds light on the environmental and welfare effects of trade

[5]The Brander–Taylor model has been extended in a number of directions. Brander and Taylor (1997b) extended their original model by introducing countries that differ in their resource-management regimes (open access in one country and optimally regulated in the other country). Jinji (2007) analyzed the case in which two countries may differ in both their resource-management regime and relative resource abundance. By introducing land as another input and focusing on forest resources, Jinji (2006) examined a model with an endogenous carrying capacity of the resource. Copeland and Taylor (2009) developed a theory of resource management with an endogenous property-rights regime by combining the original model with a simple model of moral hazard. Takarada et al. (2013) considered a model of fishery resources shared by two countries.

[6]Using a balanced panel of 732 municipalities in the Brazilian Amazon from 2000 to 2010, Faria and Almeida (2016) empirically found that as openness to trade in the Amazon increases, deforestation also increases, and that the production of soybeans and beef cattle drives deforestation.

[7]There are a number of applications of the Copeland–Taylor model. Copeland and Taylor (1997) considered government policy that controls pollution. Focusing on transboundary pollution, Unterober-doerster (2001), Benarroch and Thille (2001), and Suga (2007) re-examined the spatial separation result. In another direction, Kondoh (2006) and Beladi et al. (2000) applied the Copeland–Taylor model to international migration and international capital movement, respectively.

between emerging economies, in which pollution emission is the often the main cause of environmental problem, and less developed, resource-rich economies, in which extraction or the inappropriate use of natural resources is the main culprit.

Having set up the model, we begin with charactering the properties of supply side, particularly those in the long run. The long-run (steady-state) supply curve is upward-sloping in a country of the BT type, yet downward-sloping in a country of the CT type. It is this distinction that generates the contrasting dynamic responses of the respective economies to trade liberalization.

In the analysis of trade between the two countries, we obtain a complete description of the dynamic system in the trading equilibrium. For this purpose, we first derive how *world specialization patterns* (which country specializes on which good) are distributed along the *comparative advantage index* (depending on environmental stocks and technologies in the two countries) and the *relative effective size* (depending on labor endowments and technologies).

In the long run, environmental stocks evolve over time, and so does the comparative advantage index. We scrutinize world specialization pattern, labor allocations, and environmental and welfare consequences of trade in the long run. Regarding welfare effects of trade, at least one country gains from trade if two countries are of the same type. However, if the two trading countries are of different types, a pessimistic scenario emerges as a possibility; trade could result in environmental degradation in both countries, and consequently may harm the welfare of both countries. This scenario is of special interest since it captures trade between emerging industrial nations and less developed resource-rich countries, and highlights that in the absence of appropriate regulation on environmentally harmful economic activities, mutually harmful international trade might occur.

The possibility of mutually harmful trade in the presence of environmental resource dynamics has been addressed in the literature. Karp et al. (2001) extended the North–South trade model developed by Chichilnisky (1994) in which differences in property rights for environmental resources create a motive for trade among otherwise identical countries. They showed that trade may aggravate the common property problem to the extent that both countries lose from trade. The mechanisms driving the results in their model, however, are different from ours. First, in their model, environmental distortion arises from imperfect property rights, and environmental change matters by affecting extraction cost of environmental services. Here, environmental distortion arises from market failure (among self-interest firms), and environmental change affects directly the productivity of final-good sector. Second, in their model, partial unemployment serves a key role in causing welfare losses under trade. In our model, full employment prevails; the driving force behind welfare losses in both trading countries is the combination of a productivity decline due to environmental degradation, and the possibility that both countries export their respective "dirtier" goods to one another.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the supply side. Sections 4 and 5 deal with the autarkic equilibrium and small open economy. Section 6 investigates two-country trade. Section 7 illustrates the results with numerical examples. Section 8 concludes.

# 2 The basic model

There are two production sectors, resource-good and manufacturing, producing under perfect competition with technologies:

$$X_f = A(S) L_f, \quad X_m = aL_m, \tag{1}$$

where the subscripts $f$ and $m$ denote respectively resource goods (food, forest or fishery resources) and manufacturing goods; $X_i$ and $L_i$ $(i = f, m)$ are the output from and labor allocated to the corresponding sector. Environmental stock $S$ measures the capacity of the environment, which is interpreted as the stock of renewable resources in Brander and Taylor (1997a, 1998), and as the quality of the environment in Copeland and Taylor (1999). The resource-good sector is environmentally sensitive in the sense that its productivity increases with environmental stock: $A'(S) > 0$. The productivity of the manufacturing sector is fixed. The labor market is perfectly competitive with flexible wages so that full employment prevails all the time:

$$L_f + L_m = L, \tag{2}$$

where $L$ is labor endowment and is assumed to be constant over time. Environmental stock evolves according to

$$\dot{S} = G(S) - E, \tag{3}$$

where $G(S)$ is the natural growth of environmental stock and $E$ the economic usage of the environment (the environmental impact of economic activities). To formulate the idea that the maximum capacity of the environment is finite, impose the following assumption:

**Assumption 1.** *There exists $K > 0$ such that $G(K) = 0$ and $G'(K) < 0$.*

If no economic activity occurs, $E = 0$ and the capacity of the environment at the steady state is $K$, which is usually called the carrying capacity of the environment. The economic usage of the environment depends on the scales of two sectors and can be expressed by

$$E = l_f X_f + l_m X_m, \tag{4}$$

where non-negative parameters $l_f$ and $l_m$ measure environmental impacts from per unit output in the two sectors. We exclude the trivial case of $l_f = l_m = 0$ to prevent the model from degenerating into a Ricardian model in the long run. Note that our model turns into the Brander–Taylor model by letting $l_f = 1$ and $l_m = 0$, and becomes the Copeland–Taylor model if $l_f = 0$ and $l_m > 0$.

The economic usage of the environment per unit labor is then $l_f A(S)$ in the resource-good sector and $l_m a$ in the manufacturing sector. Their relative magnitude proves crucial in determining the dynamic behavior of the economy.

**Definition 1** (Sector type)**.** A more (less) environmentally harmful, or "dirtier (cleaner)", sector is a sector with a higher (lower) economic usage of the environment per unit labor.

That is, the resource-good sector is "dirtier (cleaner)" if $l_f A(S) > l_m a$ ($< l_m a$). Letting $S_c$ defined by $l_f A(S_c) = l_m a$, the resource-good (manufacturing) sector is "dirtier" for any $S > S_c$ ($< S_c$). Figure 1 clearly illustrates this. Note that the Brander–Taylor model, by assuming $l_m = 0$, focuses on the special case that the resource-good sector is "dirtier" for any level of $S$. By contrast, the Copeland–Taylor model, by assuming $l_f = 0$, focuses on the special case of the manufacturing sector being always "dirtier".

Assume finally the preference described by a representative household with the in-

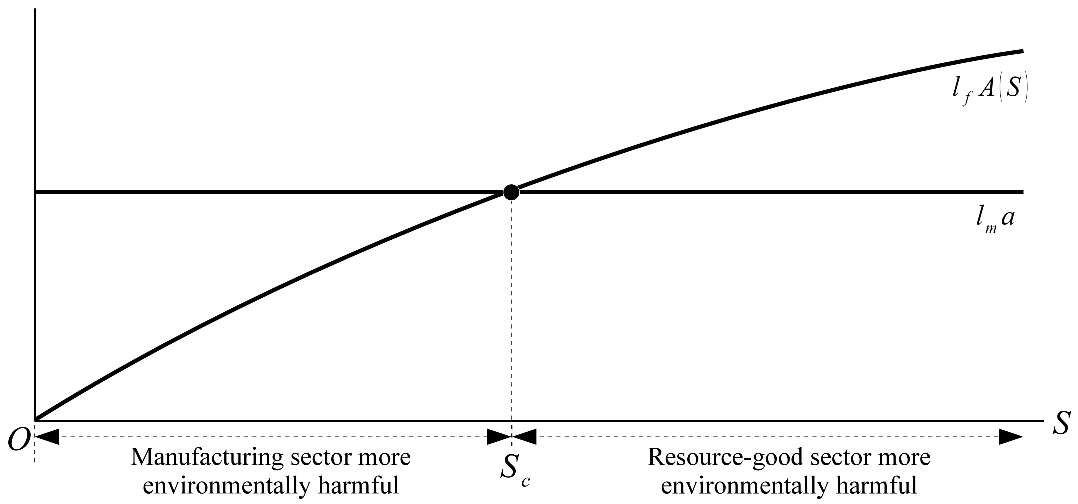



Figure 1: Environmental usage per unit labor. The resource (manufacturing) sector is more environmentally harmful for any $S > S_c$ ($< S_c$).

stantaneous utility satisfying

$$u(C_f, C_m) = b \ln C_f + (1-b) \ln C_m, \tag{5}$$

where parameter $0 < b < 1$ indicates the share of income spent on resource goods.

# 3 Properties of the supply side

As a preliminary analysis, this section formally defines two types of countries, and derives some properties of the supply side, especially the supply curve and the production possibility frontier (PPF) in the long run.

## 3.1 Short-run properties

It follows from (1) and (2) that at each point in time

$$\frac{X_f}{A(S)} + \frac{X_m}{a} = L. \tag{6}$$

Firms under perfect competition make decisions by taking environmental stock $S$ as given. The opportunity cost of producing manufacturing goods measured in resource goods is given by the marginal rate of transformation (MRT):

$$\text{MRT} = \frac{A(S)}{a}. \tag{7}$$

Note that (6) gives the expression of the short-run PPF, which is a straight line on the $(X_m, X_f)$ plane passing through point $(aL, 0)$ with the absolute value of its slope equal to the MRT. Clearly, the model behaves like a Ricardian economy in the short run.

In the long run, however, environmental stock $S$ can change over time. A production schedule in the short run is not necessarily feasible in the long run. To understand the long-run properties of the supply side, it is convenient to define $\beta \equiv L_f/L$ as the share of labor allocated to the resource-good sector. The production functions can be rewritten into

$$X_f = A(S)\beta L, \quad X_m = a(1-\beta)L. \tag{8}$$

Clearly, $\beta$ will be endogenously determined in equilibrium. For the moment, however, we focus on the supply side and treat $\beta$ as exogenously given.

Let $P$ denote the relative price of manufacturing goods to resource goods. Without loss of generality, let the resource good be the numeraire. (Thus $P$ also gives the price of manufacturing goods.) Wages are equalized within the country since labor is freely mobile across sectors. The necessary condition for both sectors to be active is therefore $w = A(S) = aP$, or equivalently,

$$\frac{A(S)}{a} = P. \tag{9}$$

It follows directly from (4) and (8) that

$$E = (l_f A(S)\beta + l_m a(1-\beta))L, \tag{10}$$

which is increasing with $S$ and bound by $l_f A(S) L$ and $l_m a L$ (since it is the linear combination of the two). Figure 2 shows that the locus of $E$ lies in the shadowed area between $l_f A(S) L$ and $l_m a L$, and rotates counter-clockwise on $(S_c, l_m a L)$ as $\beta$ increases.

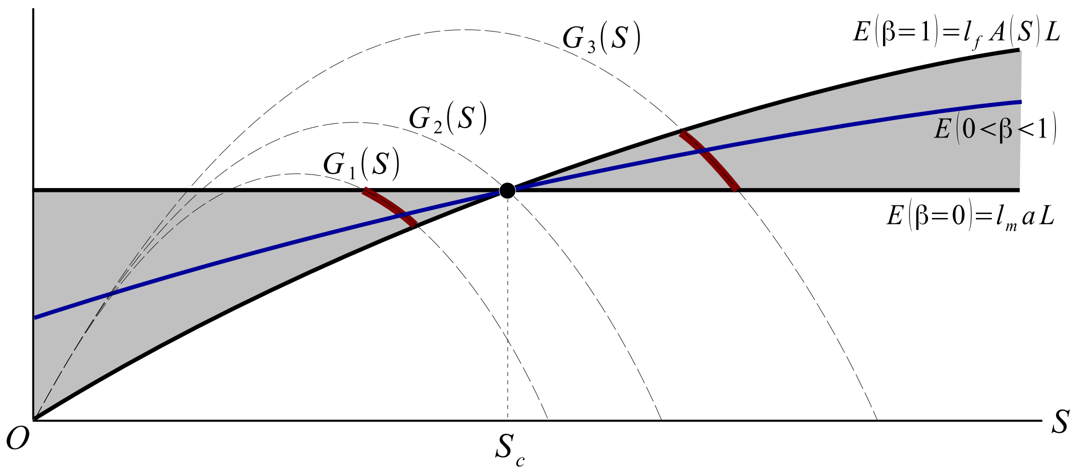



Figure 2: Economic usage of the environment. Given three different environmental growth functions $G_1(S)$, $G_2(S)$, and $G_2(S)$, environmental stocks at all steady states are less than, equal to, and greater than $S_c$.

## 3.2 Country type

Given labor allocation $\beta$, it follows from (3) and (10) that at the steady state

$$G(S) = (l_f A(S)\beta + l_m a(1-\beta))L. \tag{11}$$

The stability requires that $G(S)$ intersects the $E$ schedule from above, namely

$$G'(S) < l_f A'(S)\beta L. \tag{12}$$

Let $S_\infty(\beta)$ denote the solution(s) of $S$ to (11) which satisfies (12). For arbitrary parameters and functional forms, $S_\infty(\beta)$, namely the set of environmental stocks at the steady state corresponding to labor allocation $\beta$, can be empty, multi-valued, or discontinuous. For simplicity, we impose the following assumption.

**Assumption 2.** $S_\infty(\cdot)$ *is a positive-valued and continuous function in* $[0,1]$.

The assumption is not as restrictive as it seems. It holds in the Copeland–Taylor model, which assumes $G(0) > l_m a$ and $G'(S) < 0$. It also holds in the Brander–Taylor model, which assumes a logistic form of $G(S)$ with the maximum greater than $l_m a$.

Taking the total differential of (11) yields

$$S'_\infty(\beta) = \frac{(l_f A(S_\infty(\beta)) - l_m a)L}{G'(S_\infty(\beta)) - l_f A'(S_\infty(\beta))\beta L}. \tag{13}$$

The stability requires the denominator to be negative. Therefore, if the resource-good (manufacturing) sector is more environmentally harmful, a marginal increase in $\beta$ lowers (raises) the steady-state level of environmental stock. In other words, an expansion (contraction) of the "dirtier" sector harms (enhances) the environment.

A question naturally arising is whether a change in labor allocation, $\beta$, can change the steady-state level of environment stock to such an extent that the "dirtier" sector becomes the "cleaner" one. The following lemma provides the answer.[8]

**Lemma 1.** *The type of a sector remains unchanged at all steady states.*

This suggests that which sector is "dirtier" at the steady state is an intrinsic nature of the supply side, only depending on parameter values and functional forms. Note as well that Assumption 2 is the requirement for this result; otherwise a sector may be the "dirtier" one at some steady states while the "cleaner" one at others. Figure 2 illustrates three different environmental growth functions. Endowed with $G_1(S)$, we have $S_\infty(\beta) < S_c$ for all $\beta \in [0, 1]$ in the country, meaning that the manufacturing sector is "dirtier" at all steady states. By contrast, $S_\infty(\beta) > S_c$ holds for all $\beta \in [0, 1]$ in a country endowed with $G_3(S)$, ensuring that the resource-good sector is "dirtier" at all steady states. If a country is endowed with $G_2(S)$, which passes through $(S_c, l_m aL)$, $S_\infty(\beta) = S_c$ holds for all $\beta \in [0, 1]$. This knife-edge case is of no special interest since it resembles the classic Ricardian model at the steady state, and is excluded from the analysis. Lemma 1 suggests that countries can be categorized into two types as follows.

**Definition 2** (Country type). A country is of the BT (CT) type if the resource-good (manufacturing) sector is "dirtier" at all steady states.

It follows immediately from the discussion above that

**Proposition 1.** *A country is either of the BT type or the CT type. In a country of the BT (CT) type,* $S'_\infty(\beta) < 0$ *(*$> 0$*) holds for all* $\beta \in [0, 1]$.[9]

Figure 2 helps infer that a country is more likely to be of the BT type if it has: smaller labor endowment (lower $L$), faster growth of the environment (greater $G(S)$ for given $S$), higher productivity in the resource-good sector (greater $A(S)$ for given $S$), lower

[8]Appendix A provides formal proofs to lemmas and propositions.

[9]In the knife-edge case, $S'_\infty(\beta) = 0$ holds for all $\beta \in [0, 1]$.

manufacturing productivity (lower $a$), higher per unit output environmental impact in the resource-good sector (greater $l_f$), or lower per unit output environmental impact in the manufacturing sector (greater $l_m$). Otherwise, it is more likely to be of the CT type.

## 3.3 The long-run supply curve

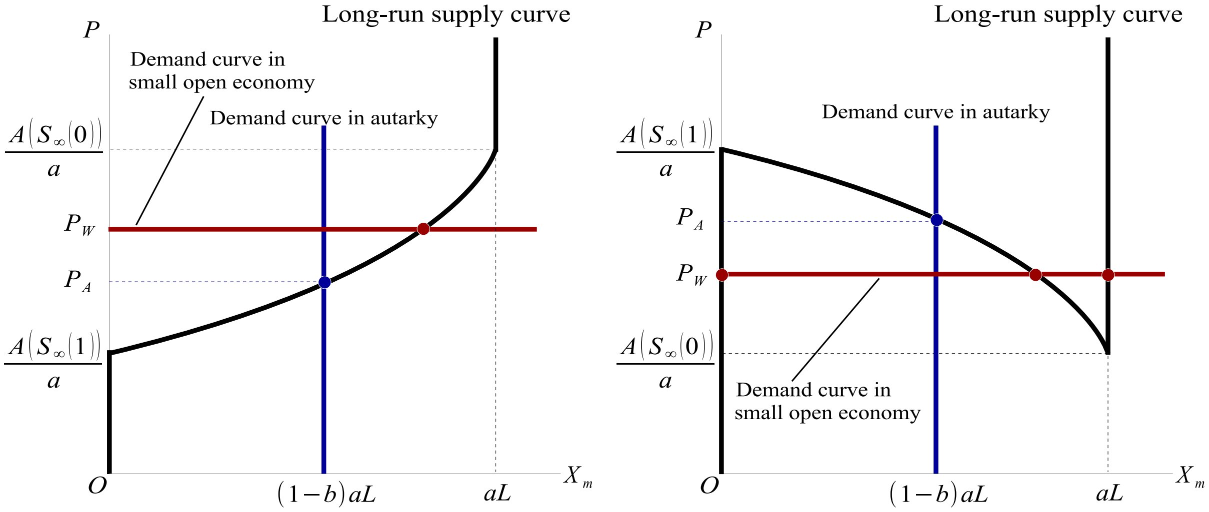
Consider the long-run supply curve of manufacturing goods, which draws the relationship between the output of manufacturing goods and its (relative) price at the steady state. We focus on the manufacturing good because its productivity remains constant, which produces a clear-cut distinction in the shape of the long-run supply curve between countries of different types.

The following result is useful regarding how the (relative) price of manufacturing goods varies with the output.

**Proposition 2.** *In a country of the BT (CT) type, the relative price of manufacturing goods rises (falls) with the output at the steady state.*

In a country of the BT type, a shift of labor into the "cleaner" manufacturing sector means a shift of labor out of the "dirtier" resource-good sector, which enhances the environment and consequently the productivity in the resource-good sector, resulting in a higher relative price of manufacturing goods. In a country of the CT type, the manufacturing sector becomes the "cleaner" one, and a shift of labor into the manufacturing sector leads to the opposite.

Proposition 2 provides how the price varies with the output. To draw the long-run supply curve, it is also required to characterize two endpoints: $X_m = 0$ (where $\beta = 1$) and $X_m = aL$ (where $\beta = 0$). When $X_m = 0$, the steady-state wage in the resource-good sector is $A\left(S_\infty\left(1\right)\right)$ and the potential wage in the manufacturing sector is $aP$. A necessary condition for all labor working in the resource-good sector is $A\left(S_\infty\left(1\right)\right) \geq aP$, which yields $P \leq A\left(S_\infty\left(1\right)\right)/a$. Similarly, we can obtain $P \geq A\left(S_\infty\left(0\right)\right)/a$ as a necessary condition for $X_m = aL$. Figure 3 applies the arguments above and draws the long-run supply curves (of manufacturing goods) for two types of countries.




(a) BT type. (b) CT type.

Figure 3: The long-run supply curve. The demand curve in autarky, which is a vertical line given the Cobb-Douglas preference, and the demand curve in a small open economy, which is a horizontal line, are also drawn.

## 3.4 The long-run PPF

The long-run PPF is useful for two reasons. First, it illustrates the deviation in the production cost between private firms and the whole economy. Firms take environmental stock as given and the opportunity cost of producing manufacturing goods (measured in resource goods) equals the MRT in (7). This deviates from the social opportunity cost, or the social MRT (SMRT), which takes into account environmental changes, and can be measured by the slope of the long-run PPF. Second, it provides an intuitive tool for the analysis of welfare effects of trade.

To derive the long-run PPF, substitute $S_\infty(\beta)$ into the first equation in (8) to obtain $X_f = A(S_\infty(\beta))\beta L$. Noting that the second equation in (8) gives $\beta = 1 - X_m/aL$, the long-run PPF can be then written as

$$X_f = T(X_m) = A\left(S_\infty\left(1 - \frac{X_m}{aL}\right)\right)\left(L - \frac{X_m}{a}\right). \tag{14}$$

The SMRT can be expressed by

$$\text{SMRT} = -\frac{dT(X_m)}{dX_m} = A'(S_\infty(\beta))\,S'_\infty(\beta)\,\frac{\beta}{a} + \text{MRT}, \tag{15}$$

where $\beta$ instead of $X_m$ is used to save notation. It follows immediately that

**Proposition 3.** *In a BT (CT) type country, the MRT is greater (less) than the SMRT for all* $X_m > 0$.

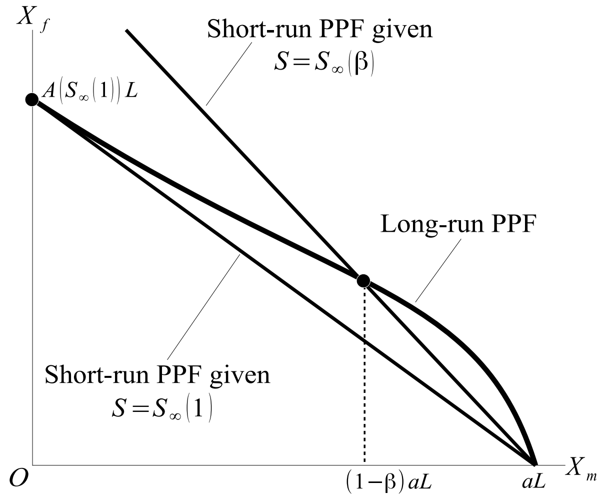
Intuitively, in a BT (CT) type country, the resource-good (manufacturing) sector is "dirtier" so that the cost of producing the good is, from the perspective of the whole economy, underestimated by private firms (which have no concern over such sectoral distinctions in environmental impacts).

To correct externalities, in a BT type country, a tax on resource goods (or a subsidy on manufacturing goods) can be imposed to reduce the MRT that faces private firms. For example, if the tax rate on the production of resource goods is $\tau$, the MRT becomes $\text{MRT}(\tau) = (1-\tau)\, A\left(S_\infty\left(\beta\right)\right)/a$. The optimal tax, if focusing on the steady state, can be achieved by equalizing $\text{MRT}(\tau)$ to the SMRT. The opposite holds in a CT type country.

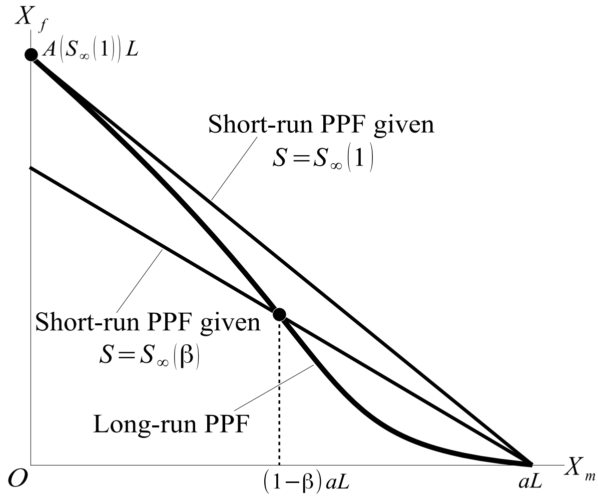
It follows from (6) that, on the $(X_m, X_f)$ plane, the straight line connecting $(aL, 0)$ with a certain point on the long-run PPF is actually the short-run PPF corresponding to the level of environmental stock that allows the economy to sustainably produce at that point. The following summarizes two features of the long-run PPF.

**Proposition 4.** *In a BT (CT) type country, the long-run PPF is strictly concave (convex) around the* $X_m$ *axis; a straight line passing through* $(aL, 0)$ *intersects, if any, the long-run PPF from above (below).*

Panagariya (1981) considered a two-sector model with increasing returns to scales (IRS) in one industry and decreasing returns to scales (DRS) in the other. He showed that the economy's PPF is strictly concave to the origin near the IRS axis and strictly convex to the origin near the DRS axis, and that welfare maximization requires a permanent tax/subsidy scheme encouraging the expansion of the IRS industry and the contraction of the DRS industry. Our findings summarized in Proposition 4 can be interpreted as a variant of Panagariya's argument. In our model, only the resource-good sector is subject to external diseconomies. However, depending on the relative magnitude of environmental impacts (per unit labor) of two sectors, the resource-good sector can either be an "IRS" sector or a "DRS" sector, which drives the difference in the shape of the long-run PPF.




(a) BT type.




(b) CT type.

Figure 4: The long-run PPF. Panel (a) draws the long-run PPF of a BT type country, which is strictly concave around $(aL, 0)$. Panel (b) draws that of a CT type country, which is strictly convex around $(aL, 0)$. In either case, the long-run PPF is not necessarily entirely concave or entirely convex, as illustrated in both panels. Two short-run PPFs corresponding to $S = S_\infty(1)$ and $S = S_\infty(\beta)$ (where $0 < \beta < 1$) are also drawn.

# 4 Autarkic equilibrium

Having characterized the supply side, the demand side can be introduced to close the model. In this section, we consider autarky as the benchmark to compare with trade.

In autarky, demand is fulfilled by domestic supply: $C_i = X_i$ $(i = f, m)$. The Cobb-Douglas preference ensures that both goods are produced in autarky, which requires the (relative) price of manufacturing goods satisfying $P = A(S)/a$ and the wage satisfying $w = A(S) = aP$. The income is then $A(S)L$. The maximization of utility requires that the share of income spent on the resource good is $b$, which gives

$$C_f = bA(S)L, \quad C_m = (1-b)aL. \tag{16}$$

Hence the demand for manufacturing goods is a vertical line on the $(X_m, P)$ plane, as illustrated in Figure 3. It follows from (1) that, in autarkic equilibrium,

$$\beta = b. \tag{17}$$

Environmental stock and the (relative) price of manufacturing goods at the autarkic steady state, denoted by $S_A$ and $P_A$ respectively, satisfy $S_A = S_\infty(b)$ and $P_A =$

$A\left(S_{\infty}\left(b\right)\right)/a$. Assumption 2 assures the uniqueness and stability of the steady state.[10]

Finally, the utility level at the autarkic steady state can be expressed by, using (16),

$$V_A = B + \ln L + b \ln A\left(S_{\infty}\left(b\right)\right) + (1-b)\ln a, \tag{18}$$

where $B \equiv b \ln b + (1-b)\ln(1-b)$.

# 5 Small open economy

The main purpose of the present paper is to investigate free trade between two countries and its long-run environmental and welfare consequences. In the section, however, we analyze trade in a small economy to highlight the difference between two types of countries in dynamic responses to trade liberalization, and illustrate the difference intuitively with the long-run supply curve and the long-run PPF.

Let $P_W$ denote the world (relative) price of manufacturing goods. The comparative advantage can be revealed by comparing the MRT with the world price: if $A\left(S\right)/a > P_W$ ($< P_W$), the economy has a comparative advantage in resource (manufacturing) goods. Firms take environmental stock as given, and the economy completely specializes in the good in which it has a comparative advantage.

In the long run, environmental stock evolves over time, thereby affecting the MRT and consequently trade pattern and specialization pattern. Geometrically, the trade steady state in a small open economy can be obtained by the intersection of the long-run (steady-state) supply curve and the demand curve of manufacturing goods, as illustrated in Figure 3.

The expression of the utility level at the trade steady state, denoted by $V_T$, varies with specialization patterns. To see this, note that when the economy specializes in the resource good ($\beta = 1$), we have $w = A\left(S_{\infty}\left(1\right)\right)$ and therefore, $C_f = bA\left(S_{\infty}\left(1\right)\right)L$ and $C_m = (1-b)\,A\left(S_{\infty}\left(1\right)\right)L/P_W$. When the economy remains diversified ($0 < \beta < 1$), we have $w = A\left(S_{\infty}\left(\beta\right)\right) = aP_W$ and therefore, $C_f = bA\left(S_{\infty}\left(\beta\right)\right)L = baP_WL$ and $C_m = (1-b)\,A\left(S_{\infty}\left(\beta\right)\right)L/P_W = (1-b)\,aL$. When the economy specializes in the manufacturing good ($\beta = 0$), we have $w = aP_W$ and therefore, $C_f = baP_WL$ and

[10] Proposition 1 in Brander and Taylor (1997a) and Copeland and Taylor (1999) gives similar results.

$C_m = (1-b)\, aL$. It then follows that, using (5),

$$V_T = \begin{cases} B + \ln L + \ln A\left(S_\infty(1)\right) - (1-b)\ln P_W & \text{if } \beta = 1, \\ B + \ln L + b \ln A\left(S_\infty(\beta)\right) + (1-b)\ln a & \text{if } 0 < \beta < 1, \\ B + \ln L + b\ln P_W + \ln a & \text{if } \beta = 0. \end{cases} \tag{19}$$

Focusing on the steady-state level of utility, we can compare $V_T$ with $V_A$ to assess whether the small economy gains or loses from trade liberalization. The following proposition summarizes the results regarding an economy of the BT type.

**Proposition 5.** *A small open economy of the BT type at the trade steady state can be characterized as follows.*

*(i) It exports manufacturing (resource) goods if the world relative price is higher (lower) than the autarkic price, and specializes if the world price is sufficiently high (low) in the sense that* $P_W \geq A\left(S_\infty(0)\right)/a$ $(\leq A\left(S_\infty(1)\right)/a)$*; otherwise, it remains diversified.*

*(ii) The environment improves (deteriorates) if exporting manufacturing (resource) goods.*

*(iii) It gains from trade if exporting manufacturing goods, or if exporting resource goods with the world relative price low enough:*

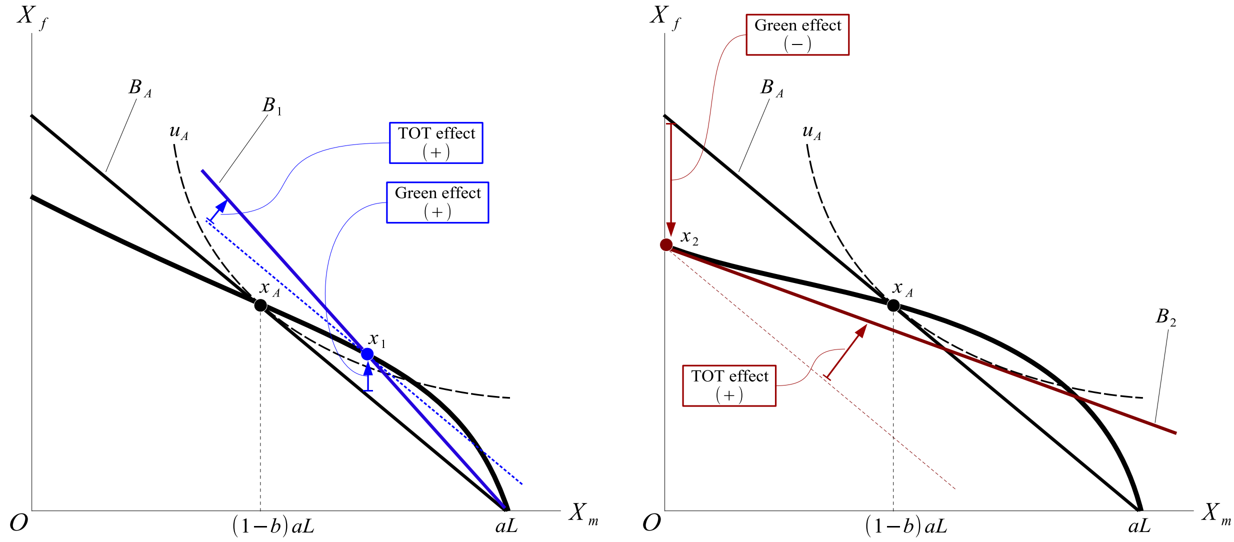
$$P_W < P' \equiv \frac{A\left(S_\infty(1)\right)}{a}\left(\frac{A\left(S_\infty(1)\right)}{A\left(S_\infty(b)\right)}\right)^{\frac{b}{1-b}}. \tag{20}$$

*It loses from trade if exporting resource goods with* $P_W > P'$.[11]

Figure 5 illustrates welfare effects of trade in a small economy of the BT type. In both panels, $B_A$ is the budget line at the autarkic steady state (which is also the short-run PPF given $S = S_\infty(b)$). In panel (a), the economy faces a world price higher than the autarkic price and exports manufacturing goods when opened to trade (yet not high enough for the economy to specialize). The resulting budget line at the trade steady state, $B_1$, can be obtained by rotating $B_A$ outward on $(aL, 0)$, implying a higher steady-state utility level compared to autarky.

In panel (b), the world price is lower than the autarkic price and the economy exports resource goods under free trade (and low enough for the economy to specialize). The

[11]Note that $P' < A\left(S_\infty(1)\right)/a$. Given $P_W > P'$, the small economy may specializes in resource goods (if $P_W \leq A\left(S_\infty(1)\right)/a$) or remain diversified (if $P_W > A\left(S_\infty(1)\right)/a$)..




(a) Exports manufacturing goods and gains.

(b) Specializes in resource goods and beak-even.

Figure 5: Welfare effects of trade in a small BT type economy. In both panels, $x_A$ and $B_A$ are the production (consumption) bundle and budget line at the autarkic steady state; $u_A$ is the corresponding indifference curve; the dashed line is parallel to $B_A$. In panel (a), the economy exports manufacturing goods, with the production bundle $x_1$ and budget line $B_1$. In panel (b), the economy exports and specializes in resource goods, with the production bundle $x_2$ and budget line $B_2$ (which is tangent to $u_A$).

resulting budget line at the trade steady state, $B_2$, happens to be tangent to the autarkic indifference curve, $u_A$. Hence, panel (b) illustrates the threshold case in which the resulting steady-state utility level is the same as in autarky. If the world price falls further, the budget line rotates outward on $(0, A(S_\infty(1))L)$ and therefore, the small economy gains from trade; if the opposite occurs, it loses from trade.

One may have noticed that these results are similar to those in Brander and Taylor (1997a) and Rus (2016). This should not be a surprise as our model shares the similar structure with Rus (2016) and, less obviously, an economy of the BT type in our model is indeed isomorphic to the economy formulated in Brander and Taylor (1997a). The present paper, however, contributes an intuitive exposition of welfare effects of trade by using the long-run PPF. Welfare effects of trade come from two sources: changes in the terms of trade, called the TOT effect, and changes in the productivity of the resource-good sector, called the green effect.[12] In a small open economy, the terms of trade are exogenously

[12] A specific explanation of the TOT and green effects is as follows. Let production (consumption) bundle at the autarkic steady state denoted by $\left(X_m^A, X_f^A\right)$, and labor allocation, production bundle, and consumption bundle at the trade steady state denoted respectively by $\beta_T$, $\left(X_m^T, X_f^T\right)$, and $\left(C_m^T, C_f^T\right)$.

given and thus, the TOT effect is a static effect and unambiguously positive.[13] By contrast, the green effect is a dynamic effect, which comes from environmental changes, and can be positive or negative. When the small BT type economy exports manufacturing goods, the expansion of the "cleaner" manufacturing sector enhances gradually the environment, which raises the productivity in the resource-good sector. This produces a positive green effect, as illustrated by an outward shift of $B_A$ to the parallel dashed line in panel (a) of Figure 5. The total welfare effect is therefore unambiguously positive. When the economy exports resource goods, which is "dirtier" in the BT type economy, however, there arises a negative green effect. The total welfare effect then depends on which effect, the positive TOT effect or the negative green effect, dominates. Panel (b) of Figure 5 shows the threshold case, in which the two opposing forces cancel each other out and the economy remains unchanged in terms of the steady-state utility level.

A small economy of the CT type presents quit different responses to trade liberalization as well as long-run welfare gains from trade.

**Proposition 6.** *A small open economy of the CT type at the trade steady state can be characterized as follows.*

*(i) It exports and specializes in manufacturing (resource) goods if the world relative price is higher (lower) than the autarkic price.*

*(ii) The environment deteriorates (improves) if exporting manufacturing (resource) goods.*

*(iii) It gains unambiguously from trade.*

The intuition about a small CT type economy always gaining from trade in the long run comes by realizing two facts. First, a small CT type economy always specializes

The green effect arises from an income gap between trade and autarky (measured in autarkic price):

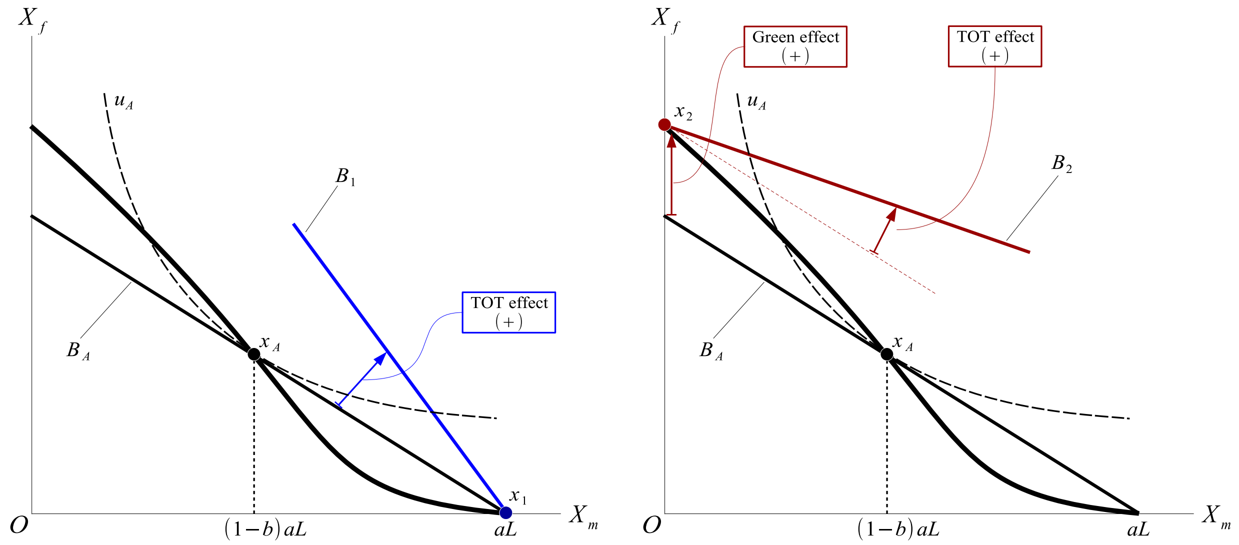
$$\Delta_{Green} = P_A X_m^T + X_f^T - \left(P_A X_m^A + X_f^A\right) = \left(A\left(S_\infty\left(\beta_T\right)\right) - A\left(S_\infty\left(b\right)\right)\right)\beta_T L,$$

where the second equality follows from $P_A = A\left(S_\infty\left(b\right)\right)/a$, $X_m^T = a\left(1-\beta_T\right)L$, $X_f^T = A\left(S_\infty\left(\beta_T\right)\right)\beta_T L$, $X_m^A = a\left(1-b\right)L$, and $X_f^A = A\left(S_\infty\left(b\right)\right)bL$. This shows that the source of the green effect is productivity change in the resource-good sector from $A\left(S_\infty\left(b\right)\right)$ to $A\left(S_\infty\left(\beta_T\right)\right)$. The TOT effect, on the other hand, arises from the gap between expenditure and income under trade (also measured in autarkic price):

$$\Delta_{TOT} = P_A C_m^T + C_f^T - \left(P_A X_m^T + X_f^T\right) = \left(P_W - P_A\right)\left(X_m^T - C_m^T\right),$$

where the second equality follows from trade balance condition $P_W C_m^T + C_f^T = \left(P_W X_m^T + X_f^T\right)$.

[13]Note that $\Delta_{TOT} = \left(P_W - P_A\right)\left(X_m^T - C_m^T\right)$ is always positive in a small open economy. In a two-country world, however, the TOT effect becomes a dynamic effect and can be negative.




(a) Specializes in manufacturing goods and gains.

(b) Specializes in resource goods and gains.

Figure 6: Welfare effects of trade in a small CT type economy. In both panels, $x_A$ and $B_A$ are the production (consumption) bundle and budget line at the autarkic steady state; $u_A$ is the corresponding indifference curve. In panel (a), the economy specializes in manufacturing goods, with the production bundle $x_1$ and budget line $B_1$. In panel (b), the economy specializes in resource goods, with the production bundle $x_2$ and budget line $B_2$; the dashed line is parallel to $B_A$.

at the trade steady state (since the diversified steady state is unstable). Second, the resource-good sector is the "cleaner" sector in a CT type economy. If the economy faces a world relative price higher than the autarkic price, it specializes in the production of manufacturing goods when opened to trade, which implies that (i) there is no green effect (since it produces no resource goods), and (ii) there is a positive TOT effect (by selling manufacturing good at higher price). The total effect is therefore positive, as illustrated in panel (a) of Figure 6, where the production bundle at the trade steady state is $x_1$ and the corresponding budget line is $B_1$ (with the absolute value of the slope equal to the world relative price) is steeper than the autarkic budget line $B_A$.

By contrast, if the economy faces a world relative price lower than the autarkic price, it specializes in resource goods and there is a positive green effect on top of the positive TOT effect. The total welfare effect is still positive, as illustrated in panel (b) of Figure 6, where the economy produces at $x_2$ and the resulting budget line is $B_2$, which lies outward against $B_A$. The positive green effect comes from an upward shift of the budget line; the positive TOT effect comes from a further counter-clockwise rotation. In either case, a small economy of the CT type enjoys an outward shift of the budget line, resulting in a higher steady-state utility level compared to autarky.

An interesting phenomenon featured in a small CT type economy is that, if trade pattern is determined by some other forces instead of comparative advantage, the economy could be locked in the "wrong" specialization pattern such that it loses from trade. To see this, suppose the world relative price is lower than autarkic price: $P_W < A(S_\infty(b))/a$. According to comparative advantage, when opened to trade, the economy would export resource goods and gain from trade. However, the government intends to stimulate manufacturing by providing subsidies so that the economy exports manufacturing goods. At the trade steady state, the economy specializes and the MRT facing private firms becomes $A(S_\infty(0))/a$. As long as the world relative price satisfies $P_W \geq A(S_\infty(0))/a$, the economy is better at producing manufacturing goods and remains specializing even without subsidies from the government. The resulting steady-state utility level, however, is lower than autarky. This can be illustrated in panel (a) of Figure 6 by drawing a budget line passing through $x_1$ (the production bundle) with the absolute value of the slope equal to the world relative price (thus lying below the autarkic budget line).

# 6 Two-country trade

This section deals with our major concern in trade between two countries, called Home and Foreign. Both are described by equations (1) through (5) with Foreign's variables and functions denoted by an asterisk in the superscript. For simplicity, assume identical preference among two countries.

Home's environmental stock $S$ and Foreign's environmental stock $S^*$ evolve over time, the dynamics of which can be described by, using (3) and (10),

$$\dot{S} = G(S) - (l_f A(S)\beta + l_m a(1-\beta))L, \tag{21}$$

$$\dot{S}^* = G^*(S^*) - \left(l_f^* A^*(S^*)\beta^* + l_m^* a^*(1-\beta^*)\right)L^*. \tag{22}$$

To obtain a complete description of the dynamic system, it remains to reveal how labor allocations $\beta$ and $\beta^*$ are dependent on environmental stocks $S$ and $S^*$. The rest of this section proceeds in two steps. First, we derive the correspondence between labor allocations and environmental stocks. We then move on to the characterization of trade steady state, transition dynamics, and environmental and welfare consequences of trade.

## 6.1 Labor allocations and environmental stocks

Consider how labor allocations are determined in the short run (given environmental stocks $S$ and $S^*$). For the purpose, noting that labor allocation is bound from one side if a country specializes (only able to shift out of the specialized sector), we shall discuss trade patterns and specialization patterns as well.

Featured with the short-run Ricardian structure (namely the MRT not varying with labor allocation given environmental stock), trade pattern can be revealed by comparing the MRTs, namely $A(S)/a$ in Home and $A^*(S^*)/a^*$ in Foreign. It is convenient to define the *comparative advantage index* by

$$v \equiv \frac{A^*(S^*)\,a}{A(S)\,a^*}. \tag{23}$$

If $v < 1$ ($> 1$), Home (Foreign) has a comparative advantage in producing resource goods, and exports the good under free trade. As long as $v \neq 1$, the short-run Ricardian structure also ensures that at least one country completely specializes.[14] If $v = 1$, neither country has comparative advantage, and trade pattern is indeterminate in the short run.

The characterization of labor allocations is related to which country specializes in which good, called *world specialization pattern*. There are seven of them in total:[15]

(f,d) Home produces only resource goods; Foreign produces both.

(f,m) Home produces only resource goods; Foreign produces only manufactures.

(d,m) Home produces both goods; Foreign produces only manufactures.

(d,d) Both countries produce both goods.

(d,f) Home produces both goods; Foreign produces only resource goods.

(m,f) Home produces only manufactures; Foreign produces only resource goods.

(m,d) Home produces only manufactures; Foreign produces both.

[14]Suppose for example $v < 1$. Under constant-returns technology and perfect competition, this means that when trade is opened, manufacturing firms in Foreign offer higher wages than resource-good firms can, and the opposite occurs in Home. Consequently, the manufacturing sector in Foreign and the resource-good sector in Home expand. This continues until Foreign, or Home, or both specialize.

[15]Neither pattern (m,m) nor pattern (f,f) arises for $0 < b < 1$.

It is then of interest which world specialization pattern arises given parameters and environmental stocks at the moment. Three factors prove crucial: the comparative advantage index $v$ defined by (23), preference parameter $b$, and the *relative effective size* defined by

$$z \equiv \frac{aL}{a^* L^*}. \tag{24}$$

The following proposition summarizes the results regarding the determination of world specialization pattern and corresponding labor allocations.

**Proposition 7.** *In the short run (given environmental stocks), labor allocations and world specialization pattern can be characterized as follows.*

*(i) Pattern (f,d) arises if* $(1-b)\, z/b < v < 1$*, where* $\beta = 1$ *and* $\beta^* = b - (1-b)\, z/v$*.*

*(ii) Pattern (f,m) arises if* $v \leq (1-b)\, z/b \leq 1$ *and* $v \neq 1$*, where* $\beta = 1$ *and* $\beta^* = 0$*.*

*(iii) Pattern (d,m) arises if* $v < 1 < (1-b)\, z/b$*, where* $\beta = b\,(1 + 1/z)$ *and* $\beta^* = 0$*.*

*(iv) Pattern (d,f) arises if* $1 < v < bz/(1-b)$*, where* $\beta = b - (1-b)\, v/z$ *and* $\beta^* = 1$*.*

*(v) Pattern (m,f) arises if* $1 \leq bz/(1-b) \leq v$ *and* $v \neq 1$*, where* $\beta = 0$ *and* $\beta^* = 1$*.*

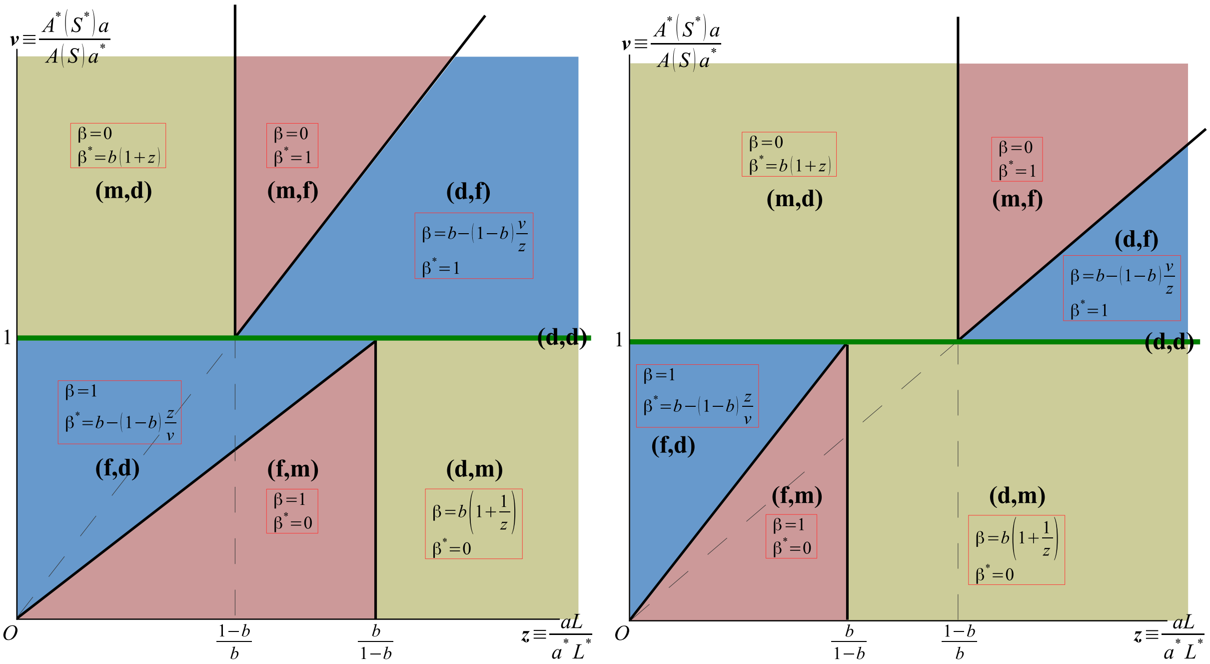
*(vi) Pattern (m,d) arises if* $bz/(1-b) < 1 < v$*, where* $\beta = 0$ *and* $\beta^* = b\,(1+z)$*.*

*(vii) Pattern (d,d) arises only if* $v = 1$*. Given* $v = 1$*, however, other pattern may also arise since labor allocations are indeterminate by satisfying*

$$\beta z + \beta^* = b\,(z+1)\,. \tag{25}$$

Intuitively, comparative advantage index $v$ matters as it determines trade pattern and indicates productivity difference between two countries. A greater $v$ thus means that Foreign (Home) becomes better (worse), in a relative sense, at producing resource goods, which shifts labor into (out of) the resource-good sector in Foreign (Home).[16] Preference parameter $b$ matters since it measures the share of income spent on resource goods. A greater $b$ thus requires more labor to be allocated to the resource-good sector, resulting in

[16] Both $\beta$ and $\beta^*$ are increasing with preference parameter $b$, which is straightforward since a stronger demand for resource goods induces more labor into the sector. On the other hand, both $\beta$ and $\beta^*$ are decreasing with the effective relative size $z$ for $v < 1$, and increasing with $z$ for $v > 1$. Intuitively, when $v < 1$, Home exports resource goods to Foreign; the portion of labor in Home for the production of resource-good exports (which is actually equal to $b/z$) falls when Foreign becomes smaller compared to Home (namely greater $z$). This implies a smaller $\beta$. At the same time, Foreign exports manufacturing goods to Home, and when Home becomes bigger compared to Foreign (greater $z$), the portion of labor in Foreign for the production of manufacturing exports (which is actually equal to $(1-b)\, z/v$) rises, resulting in a smaller $\beta^*$. The similar argument applies to $v > 1$.




(a) $b > 1/2$. (b) $b < 1/2$.

Figure 7: Distribution of world specialization patterns. Panel (a) illustrates the distribution for $b > 1/2$ and panel (b) for $b < 1/2$.

greater $\beta$ and $\beta^*$. Effective relative size $z$ matters by indicating the relative size of Home with respect to Foreign in terms of the capacity of manufacturing production. Suppose for example that Home is much smaller than Foreign in the effective size. Home's response to trade can be well described by a small open economy: it completely specializes ($\beta = 1$ or 0) as long as $v \neq 1$. By contrast, Foreign will observe little change in its labor allocation after trade liberalization ($\beta^* \approx b$). From this thought experiment, we can infer that a smaller (greater) $z$ implies a greater likelihood for Home (Foreign) to specialize and for Foreign (Home) to remain diversified under free trade.

When there exists no comparative advantage among two countries ($v = 1$), labor allocations $\beta$ and $\beta^*$ are indeterminate in the short run (taking environmental stocks as given) with one degree of freedom. As long as $0 < \beta < 1$ and $0 < \beta^* < 1$, we have pattern (d,d); otherwise pattern (f,d), (f,m), (d,m), (d,f), (m,f), or (m,d) arises. In the long run, however, labor allocations are determinate as the steady-state condition comes into effect in the long run as well.

Figure 7 provides a geometric illustration of Proposition 7 by showing how the seven world specialization patterns are distributed on the $(z, v)$ plane. The distribution appears somehow different for $b > 1/2$ and $b < 1/2$. In particular, when two countries are of similar size (around $z = 1$) and $v \neq 1$, one country must specialize in resource goods if $b > 1/2$, but neither will specialize in resource goods if $b < 1/2$.[17]

Figure 7 is also useful for comparative static exercises regarding how world specialization pattern varies with labor endowments, production technologies, and environmental stocks. For example, an increase in Home's labor endowment $L$ leads to an increase in $z$ but no change in $v$. The resulting world specialization pattern can be obtained by moving right horizontally on the plane. For another example, an increase in Home's manufacturing productivity $a$ has two effects. First, it raises Home's effective size and thus $z$, implying a horizontal movement to the right. Second, it reduces Home's MRT and thus raises $v$, implying an upward vertical movement. The total effect is an upper-right movement on the plane, with $v/z$ remaining unchanged.

## 6.2 Closing the dynamic system

Having derived how world specialization pattern and labor allocations are determined by given environmental stocks, we can substitute these results into (21) and (22) to close the dynamic system that governs this two-country world. Noting that labor allocations are determined in different manners for $v \neq 1$ and for $v = 1$, we proceed by considering the two cases one by one.

**The dynamic system for $v \neq 1$.** For any $S$ and $S^*$ satisfying $v \neq 1$, labor allocations are uniquely determined as given in Proposition 7. The following summarizes the results

[17] Intuitively, given $b > 1/2$, the world demand for resource goods is relatively strong such that the supply would not be able to match the demand if no country were devoted to the production of resource goods. If $b < 1/2$, however, the demand for resource goods is relatively weak, and there is no need for the resource-exporting country to completely specialize.

for easy reference:[18]

$$\beta(v)=\begin{cases}1 & \text{if } v<1 \text{ and } b\left(1+\frac{1}{z}\right)\geq 1,\\ b\left(1+\frac{1}{z}\right) & \text{if } v<1 \text{ and } b\left(1+\frac{1}{z}\right)<1,\\ b-(1-b)\frac{v}{z} & \text{if } v>1 \text{ and } b-(1-b)\frac{v}{z}>0,\\ 0 & \text{if } v>1 \text{ and } b-(1-b)\frac{v}{z}\leq 0,\end{cases} \tag{26}$$

$$\beta^*(v)=\begin{cases}0 & \text{if } v<1 \text{ and } b-(1-b)\frac{z}{v}\leq 0,\\ b-(1-b)\frac{z}{v} & \text{if } v<1 \text{ and } b-(1-b)\frac{z}{v}>0,\\ b(1+z) & \text{if } v>1 \text{ and } b(1+z)<1,\\ 1 & \text{if } v>1 \text{ and } b(1+z)\geq 1.\end{cases} \tag{27}$$

We write $\beta$ and $\beta^*$ explicitly as the functions of $v$ since $b$ and $z$ are exogenous parameters. Plugging $\beta(v)$ and $\beta^*(v)$ into (21) and (22) for $\beta$ and $\beta^*$ yields, using the definition of $v$,

$$\dot{S}=G(S)-\left[l_f A(S)\beta\left(\frac{A^*(S^*)a}{A(S)a^*}\right)+l_m a\left(1-\beta\left(\frac{A^*(S^*)a}{A(S)a^*}\right)\right)\right]L, \tag{28}$$

$$\dot{S}^*=G^*(S^*)-\left[l_f^* A^*(S^*)\beta^*\left(\frac{A^*(S^*)a}{A(S)a^*}\right)+l_m^* a^*\left(1-\beta^*\left(\frac{A^*(S^*)a}{A(S)a^*}\right)\right)\right]L^*, \tag{29}$$

which gives a complete description of the dynamic system for $v\neq 1$. Noting that $\beta(v)$ and $\beta^*(v)$ are piecewise functions of $v$, the dynamic system (28) and (29) has multiple regimes, which differ from each other in the rule that governs the dynamics.

**The dynamic system for $v=1$.** The argument above, however, cannot apply to the case of $v=1$ (in which pattern (d,d) may arise) where labor allocations are indeterminate by satisfying the world market-clearing condition (25). It follows from (26) and (27) that $\beta(v)$ and $\beta^*(v)$ jumps when crossing $v=1$, meaning that the dynamic system is discontinuous around $v=1$.

In summary, the dynamic system has two features: (i) multiple dynamic regimes for $S$ and $S^*$ satisfying $v\neq 1$, and (ii) discontinuity at $v=1$. The next two subsections

[18] The expressions of $\beta(v)$ and $\beta^*(v)$ can be written more specifically given specific range of preference $b$ and effective relative size $z$, since the set of possible world specialization patterns (patterns that may arise as $v$ changes) varies with the domain of $b$ and $z$, as shown in Figure 7. For example, given $b=2/3$ and $z=3$, patterns (d,m), (d,d), (d,f), and (m,f) arise as $v$ increases. By contrast, if $b=1/3$ and $z=1$, patterns (d,m), (d,d), and (m,d) arise as $v$ increases. Appendix B discusses this in detail.

examine steady state and transition dynamics while dealing with the two difficulties.

### 6.3 Trade steady state

This subsection characterizes the trade steady state in this two-country world, at which environmental stocks are endogenously determined. We proceed by considering (i) the trade steady state satisfying $v \neq 1$ and (ii) that satisfying $v = 1$, as the dynamic system behaves differently in the two situations.

**The steady state satisfying $v \neq 1$.** In this case, the steady state can be derived by letting $\dot{S} = \dot{S}^* = 0$ in (28) and (29) and solving for $S$ and $S^*$ from the two equations. Equivalently, notice that at the steady state

$$S = S_\infty \left( \beta \left( \frac{A^*(S^*)\, a}{A(S)\, a^*} \right) \right), \quad S^* = S^*_\infty \left( \beta^* \left( \frac{A^*(S^*)\, a}{A(S)\, a^*} \right) \right), \tag{30}$$

from which we can solve for $S$ and $S^*$ at the trade steady state.

Instead of solving for environmental stocks directly, an alternative approach turns out convenient by exploiting comparative advantage index. It follows from (23) that, at the trade steady state,

$$\frac{A^*(S^*_\infty(\beta^*(v)))\, a}{A(S_\infty(\beta(v)))\, a^*} = v, \tag{31}$$

from which we can solve for the steady-state level of $v$. Labor allocations then follow by plugging the solution into $\beta(v)$ and $\beta^*(v)$, and environmental stocks can be obtained by substituting labor allocations into $S_\infty(\beta)$ and $S^*_\infty(\beta^*)$. For simple notation, define

$$g(v) \equiv \frac{A^*(S^*_\infty(\beta^*(v)))\, a}{A(S_\infty(\beta(v)))\, a^*}, \tag{32}$$

then (31) can be written as

$$g(v) = v. \tag{33}$$

Technically, characterizing the trade steady state in such a manner facilitates the analysis of under what condition a certain world specialization pattern arises at the trade steady state by translating the problem into a relatively easier one: under what condition the solution to (33) lies in the range corresponding to that world specialization pattern.

**The steady state satisfying** $v = 1$**.** In this case, we need to obtain first labor allocations at the trade steady state, and then derive the steady-state environmental stocks. Specifically, at the trade steady state satisfying $v \neq 1$, it holds that

$$\frac{A^*\left(S_\infty^*\left(\beta^*\right)\right)a}{A\left(S_\infty\left(\beta\right)\right)a^*} = 1, \tag{34}$$

which together with the world market-clearing condition (25), namely $\beta z + \beta^* = b\left(z+1\right)$, can be solved for labor allocations at the trade steady state. The steady-state environmental stocks follow immediately by plugging them into $S = S_\infty\left(\beta\right)$ and $S^* = S_\infty^*\left(\beta^*\right)$.

In both situations, we can obtain environmental stocks, labor allocations, and world specialization pattern at the trade steady state. The following proposition summarizes the results regarding the existence and uniqueness of trade steady state with a specific world specialization pattern arising. For simple notation, define

$$\begin{aligned}
\Delta_{fd}\left(v\right) &\equiv \frac{A^*\left(S_\infty^*\left(b-\left(1-b\right)\frac{z}{v}\right)\right)a}{A\left(S_\infty\left(1\right)\right)a^*}, \\
\Delta_{df}\left(v\right) &\equiv \frac{A^*\left(S_\infty^*\left(1\right)\right)a}{A\left(S_\infty\left(b-\left(1-b\right)\frac{v}{z}\right)\right)a^*}\frac{1}{v}, \\
\Delta_{dd}\left(\beta,\beta^*\right) &\equiv \frac{A^*\left(S_\infty^*\left(\beta^*\right)\right)a}{A\left(S_\infty\left(\beta\right)\right)a^*},
\end{aligned}$$

and let

$$\begin{aligned}
\Delta_{fd}^{inf} &= \inf\left\{\Delta_{fd}\left(v\right) : \frac{1-b}{b}z < v \leq 1\right\}, \quad \Delta_{fd}^{sup} = \sup\left\{\Delta_{fd}\left(v\right) : \frac{1-b}{b}z < v \leq 1\right\}, \\
\Delta_{df}^{inf} &= \inf\left\{\Delta_{df}\left(v\right) : 1 \leq v < \frac{b}{1-b}z\right\}, \quad \Delta_{df}^{sup} = \sup\left\{\Delta_{df}\left(v\right) : 1 \leq v < \frac{b}{1-b}z\right\}, \\
\Delta_{dd}^{inf} &= \inf\left\{\Delta_{dd}\left(\beta,\beta^*\right) : \beta z + \beta^* = b\left(z+1\right)\right\}, \quad \Delta_{dd}^{sup} = \sup\left\{\Delta_{dd}\left(\beta,\beta^*\right) : \beta z + \beta^* = b\left(z+1\right)\right\}.
\end{aligned}$$

**Proposition 8.** *The conditions for the existence and uniqueness of trade steady state(s) with a certain world specialization pattern arising can be characterized as follows.*

*(i) There exists trade steady states(s) with pattern (f,d) if and only if*

$$\left(1-b\right)z/b < 1 \text{ and } \Delta_{fd}^{inf} \leq 1 \leq \Delta_{fd}^{sup}, \tag{35}$$

*and, if* $\Delta_{fd}^{inf} = 1$ *or* $\Delta_{fd}^{sup} = 1$*, there exists* $\left(1-b\right)z/b < v_1 \leq 1$ *such that* $\Delta_{fd}\left(v_1\right) = 1$*.*

*(ii) There exists a unique trade steady state with pattern (f,m) if and only if*

$$\frac{A^*\left(S^*_\infty\left(0\right)\right)a}{A\left(S_\infty\left(1\right)\right)a^*}\leq\frac{1-b}{b}z\leq 1. \tag{36}$$

*(iii) There exists a unique trade steady states with pattern (d,m) if and only if*

$$\frac{A^*\left(S^*_\infty\left(0\right)\right)a}{A\left(S_\infty\left(b\left(1+\frac{1}{z}\right)\right)\right)a^*}\leq 1<\frac{1-b}{b}z. \tag{37}$$

*(iv) There exists trade steady state(s) with pattern (d,f) if and only if*

$$bz/\left(1-b\right)>1 \text{ and } \Delta^{inf}_{df}\leq 1\leq\Delta^{sup}_{df}, \tag{38}$$

*and, if* $\Delta^{inf}_{df}=1$ *or* $\Delta^{sup}_{df}=1$*, there exists* $1\leq v<bz/\left(1-b\right)$ *such that* $\Delta_{df}\left(v_1\right)=1$*.*

*(v) There exists a unique trade steady state with pattern (m,f) if and only if*

$$1\leq\frac{b}{1-b}z\leq\frac{A^*\left(S^*_\infty\left(1\right)\right)a}{A\left(S_\infty\left(0\right)\right)a^*}. \tag{39}$$

*(vi) There exists a unique trade steady states with pattern (m,d) if and only if*

$$\frac{b}{1-b}z<1\leq\frac{A^*\left(S^*_\infty\left(b\left(1+z\right)\right)\right)a}{A\left(S_\infty\left(0\right)\right)a^*}. \tag{40}$$

*(vii) There exists trade steady state(s) with pattern (d,d) if and only if*

$$\Delta^{inf}_{dd}\leq 1\leq\Delta^{sup}_{dd}, \tag{41}$$

*and, if* $\Delta^{inf}_{dd}=1$ *or* $\Delta^{sup}_{dd}=1$*, there exist* $\beta_1,\beta_1^*\neq 0,1$ *satisfying (25) and* $\Delta_{dd}\left(\beta_1,\beta_1^*\right)=1$*.*

The basic idea to prove these results is to check (i) for those steady states satisfying $v\neq 1$, whether there exists solution(s) to (33) that lies in the range required by the corresponding world specialization pattern, and (ii) for those steady states satisfying $v=1$, whether there exists feasible solution(s) of $\beta$ and $\beta^*$ to (25) and (34).

Several conditions in Proposition 8 can be further specified.

**Corollary 1.** *If Foreign is of the BT type, there exists a unique trade steady state with* *pattern (f,d) if and only if* $(1-b)\,z/b < 1$ *and*

$$\frac{A^*\left(S^*_\infty\left(b-(1-b)\,z\right)\right)a}{A\left(S_\infty\left(1\right)\right)a^*} \le 1 < \frac{A^*\left(S^*_\infty\left(0\right)\right)a}{A\left(S_\infty\left(1\right)\right)a^*}\left(\frac{1-b}{b}z\right)^{-1}. \tag{42}$$

*If Home is of the BT type, there exists a unique trade steady state with pattern (d,f) if and only if* $bz/\left(1-b\right) > 1$ *and*

$$\frac{A^*\left(S^*_\infty\left(1\right)\right)a}{A\left(S_\infty\left(0\right)\right)a^*}\left(\frac{b}{1-b}z\right)^{-1} < 1 \le \frac{A^*\left(S^*_\infty\left(1\right)\right)a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*}. \tag{43}$$

*If both countries are of the same type, the trade steady state with pattern (d,d), if existing, is unique. The sufficient and necessary condition for the existence is given in Table 1.*

Table 1: Conditions for pattern (d,d) to arise at the trade steady state.

| | | |
|---|---|---|
| $z < \min\left\{\frac{1-b}{b}, \frac{b}{1-b}\right\}$ | Both BT | $\frac{A^*(S^*_\infty(b(1+z)))a}{A(S_\infty(0))a^*} < 1 < \frac{A^*(S^*_\infty(b-(1-b)z))a}{A(S_\infty(1))a^*}$ |
| | Both CT | $\frac{A^*(S^*_\infty(b(1+z)))a}{A(S_\infty(0))a^*} > 1 > \frac{A^*(S^*_\infty(b-(1-b)z))a}{A(S_\infty(1))a^*}$ |
| $\min\left\{\frac{1-b}{b}, \frac{b}{1-b}\right\} < z < \max\left\{\frac{1-b}{b}, \frac{b}{1-b}\right\}$ | Both BT | $\frac{A^*(S^*_\infty(1))a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*} < 1 < \frac{A^*(S^*_\infty(b-(1-b)z))a}{A(S_\infty(1))a^*}$ |
| | Both CT | $\frac{A^*(S^*_\infty(1))a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*} > 1 > \frac{A^*(S^*_\infty(b-(1-b)z))a}{A(S_\infty(1))a^*}$ |
| $z > \max\left\{\frac{1-b}{b}, \frac{b}{1-b}\right\}$ | Both BT | $\frac{A^*(S^*_\infty(1))a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*} < 1 < \frac{A^*(S^*_\infty(0))a}{A\left(S_\infty\left(b\left(1+\frac{1}{z}\right)\right)\right)a^*}$ |
| | Both CT | $\frac{A^*(S^*_\infty(1))a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*} > 1 > \frac{A^*(S^*_\infty(0))a}{A\left(S_\infty\left(b\left(1+\frac{1}{z}\right)\right)\right)a^*}$ |

*Notes*: There are three intervals of $z$, with which the condition varies.

The following gives two examples of the application of Proposition 8 and Corollary 1.

**Example 1.** If both countries are of the BT type and $A^*\left(S^*_\infty\left(0\right)\right)a/A\left(S_\infty\left(0\right)\right)a^* = 1$, neither country can specialize in manufacturing at the trade steady state.[19]

To see why, note that $S_\infty\left(1\right) < S_\infty\left(b\left(1+\frac{1}{z}\right)\right) < S_\infty\left(0\right)$ holds since Home is of the BT type and $S^*_\infty\left(1\right) < S^*_\infty\left(b\left(1+z\right)\right) < S^*_\infty\left(0\right)$ holds since Foreign is of the BT type.

[19]Example 1 suggests that part (i) of Proposition 4 in Brander and Taylor (1998), which says that neither country can specialize in manufacturing, comes mainly from their assumption that $A\left(S_\infty\left(0\right)\right) = A^*\left(S^*_\infty\left(0\right)\right) = \alpha K$ and $a = a^* = 1$ (which actually implies $A^*\left(S^*_\infty\left(0\right)\right)a/A\left(S_\infty\left(0\right)\right)a^* = 1$).

This together with $A^*(S^*_\infty(0))\,a/A(S_\infty(0))\,a^* = 1$ yields

$$\frac{A^*(S^*_\infty(0))\,a}{A(S_\infty(1))\,a^*} > \frac{A^*(S^*_\infty(0))\,a}{A\left(S_\infty\left(b\left(1+\frac{1}{z}\right)\right)\right)a^*} > \frac{A^*(S^*_\infty(0))\,a}{A(S_\infty(0))\,a^*} = 1,$$
$$\frac{A^*(S^*_\infty(1))\,a}{A(S_\infty(0))\,a^*} < \frac{A^*(S^*_\infty(b(1+z)))\,a}{A(S_\infty(0))\,a^*} < \frac{A^*(S^*_\infty(0))\,a}{A(S_\infty(0))\,a^*} = 1.$$

Note that inequalities in the first line violate (36) and (37) in Proposition 8, implying that patterns (f,m) and (d,m) cannot arise. Those in the second line violate (39) and (40), implying that (m,f) and (m,d) cannot arise.

**Example 2.** If both countries are of the BT type and $A^*(S^*_\infty(1))\,a/A(S_\infty(1))\,a^* = 1$, pattern (d,d) arises at the trade steady state.

Given the constraint above, one can verify that the conditions in Table 1 necessarily hold for any level of $z$, which ensures that pattern (d,d) arises at the trade steady state.

The following proposition focuses on the stability of trade steady state given that a specific world specialization pattern has arisen.

**Proposition 9.** *The stability of trade steady state can be characterized as follows.*

*(i) The steady state with one country specialized in manufacturing, either pattern (f,m), (d,m), (m,f), or (m,d), if existing, is locally stable.*

*(ii) The steady state with pattern (f,d) or (d,f), if existing, is locally stable if*

$$g'(v) < 1. \tag{44}$$

*(iii) The steady state with pattern (d,d), if existing, is locally stable if both countries are of the BT type, and unstable if both countries are of the CT type. If the two countries are of different types, the steady state(s) with pattern (d,d) is locally stable if*

$$S^{*\prime}_\infty z\frac{\partial v}{\partial S^*}\left(G^{*\prime} - l^*_f A^{*\prime}\beta^* L^*\right) - S'_\infty\frac{\partial v}{\partial S}\left(G' - l_f A'\beta L\right) > 0, \tag{45}$$

$$S^{*\prime}_\infty z\frac{\partial v}{\partial S^*} - S'_\infty\frac{\partial v}{\partial S} < 0. \tag{46}$$

The following corollary is also useful.

**Corollary 2.** *If both countries are of the BT type, there exists a unique, stable steady state. If both countries are of the CT type, there may exist multiple stable steady states* *(with the same or different world specialization patterns arising), at which at least one country completely specializes.*

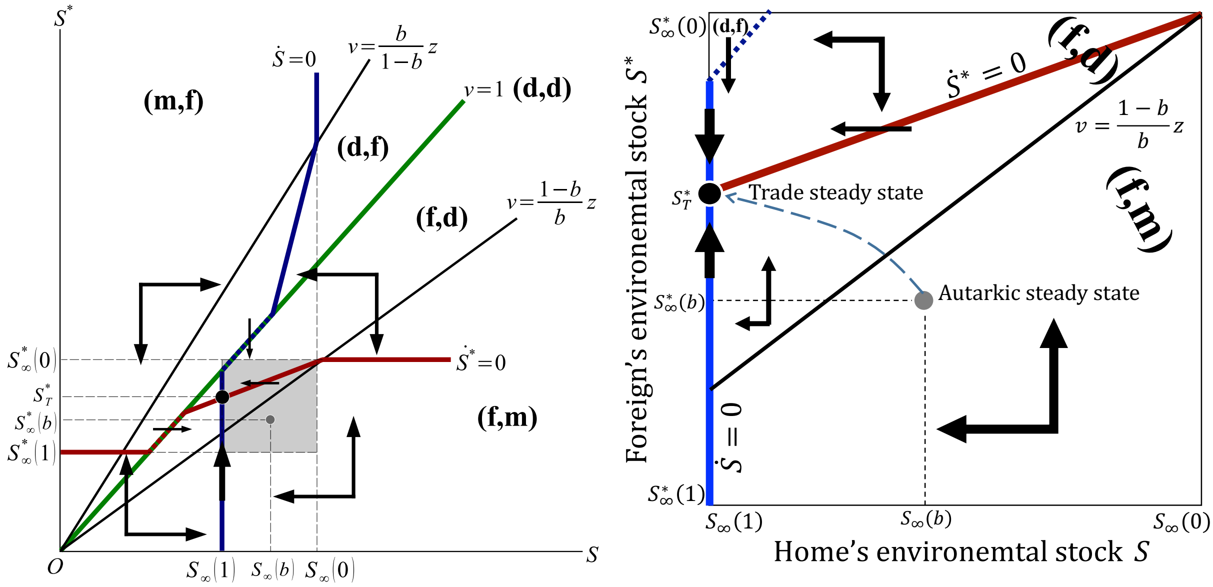
The corollary suggests that there exists no stable steady state with pattern (d,d) arising when two countries of the CT type are trading with each other. Intuitively, the comparative advantage in a CT type economy is self-reinforcing in the sense that an expansion of the exporting sector makes the CT type economy better at producing its exports. This self-reinforcing process amplifies the short-run Ricardian structure so that at least one country specializes at the steady state.

## 6.4 Transition dynamics

Transition from autarky toward the trade steady state can be analyzed intuitively with phase diagram, which draws the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves on the $(S, S^*)$ plane. Appendix C explains technical details about how to draw phase diagram. Depending on parameters and functional forms, a variety of steady-state world specialization patterns and transition dynamics can arise. This subsection illustrates three example phase diagrams, all satisfying: (i) demand for resource goods is relatively strong, namely $b > 1/2$, and (ii) two countries are similarly sized, namely $(1 - b)/b < z < b/(1 - b)$. Given this, the $(S, S^*)$ plane can be then divided into four regions, corresponding with patterns (f,m), (f,d), (d,f), and (m,f), respectively (see Appendix B for detail).

Figure 8 illustrates the first example phase diagram, in which both countries are of the BT type. Panel (a) gives the whole picture of the dynamic system, and panel (b) provides an enlarged view of the shadowed area $[S_\infty(1), S_\infty(0)] \times [S^*_\infty(1), S^*_\infty(0)]$ in panel (a) where all possible steady states lie. At the autarkic steady state, environmental stocks are $(S_\infty(b), S^*_\infty(b))$, which lies in the region corresponding with pattern (f,m). This means that pattern (f,m) arises on the spot as two countries begin to trade with each other. Since Home exports the "dirtier" resource goods and Foreign the "cleaner" manufacturing goods, Home's environment deteriorates gradually and Foreign's improves. This yields an upper-left trajectory from the autarkic steady state $(S_\infty(b), S^*_\infty(b))$ converging to the trade steady state $(S_\infty(1), S^*_T)$, where pattern (f,d) arises.

It is worth noting that in this example the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves are positively (yet not necessarily strictly) sloped. This is because, as shown in Appendix C, both countries are of the BT type. The uniqueness of trade steady state then follows immediately.



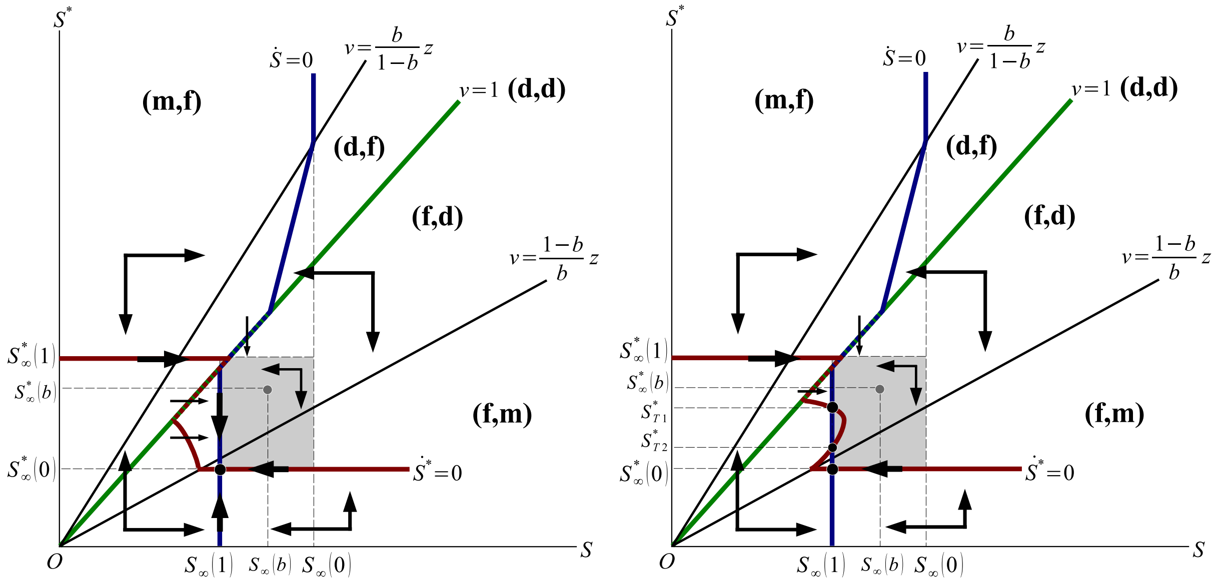

(a) Phase diagram on the $(S, S^*)$ plane. (b) An enlarged view of the shadowed area.

Figure 8: An example phase diagram with both countries of the BT type and pattern (f,d) arising at the trade steady state. Point $(S_\infty(b), S^*_\infty(b))$ indicates the levels of environmental stocks at the autarkic steady state, $(S_\infty(1), S^*_T)$ gives the levels of environmental stocks at the trade steady state. Panel (b) enlarges the shadowed area in panel (a).

Moreover, world specialization pattern changes from (f,m) right after trade to (f,d) at the steady state, suggesting that specialization pattern or even trade pattern could vary during the transition.

Figure 9 gives other two example phase diagrams, in both of which Home is of the BT type and Foreign of the CT type. The autarkic steady-state levels of environmental stocks are located in the region corresponding to pattern (f,d), meaning that pattern (f,d) arises right after trade is opened. By exporting resource goods, which are "dirtier" to a BT type country, Home's environment deteriorates gradually. By exporting manufacturing goods, which are "dirtier" to a CT type country, Foreign's environment deteriorates gradually, too. Therefore, two countries of different types could export their respective "dirtier" goods to each other, which worsens the environment in both countries.

In panel (a) of Figure 9, the $\dot{S}^* = 0$ curve intersects with the $\dot{S} = 0$ curve only once in the (f,m) region, yielding a unique trade steady state with pattern (f,m) arising. In panel (b), the $\dot{S}^* = 0$ curve in the (f,d) region is not negatively sloped everywhere such that it intersects with the $\dot{S} = 0$ curve three times, meaning that there exist three trade steady states: one with pattern (f,m) and the other two with pattern (f,d) arising. This verifies




(a) Unique trade steady state.

(b) Multiple trade steady states.

Figure 9: Two example phase diagrams with Home of the BT type and Foreign of the CT type. In panel (a), there exists a unique trade steady state with pattern (f,m). In panel (b), three trade steady states exist, one with pattern (f,m) and two with pattern (f,d).

the existence of multiple trade steady states claimed in Proposition 8 and Corollary 1. Moreover, it is easy to see in panel (b) that, among the two trade steady states with pattern (f,d), only $(S_\infty(1), S^*_{T1})$ is stable. It then depends on other conditions to which stable trade steady state (among the remaining two) the two-country world converges.

## 6.5 Environmental and welfare consequences of trade

Consider now environmental impacts and welfare gains from trade in the long run by comparing the steady-state levels of environmental stocks and utility in autarky with those under free trade. The following is concerned with environmental impacts of trade.

**Proposition 10.** *Environmental impacts of trade can be characterized as follows.*

*(i) If two countries are of the same type, the environment improves in one country (by exporting "cleaner" goods) and deteriorates in the other (by exporting "dirtier" goods).*

*(ii) If two countries are of different types, the environment either improves in both countries (by both exporting their respective "cleaner" goods), or deteriorates in both countries (by both exporting their respective "dirtier" goods).*

Intuitively, if two countries are of the same type and trading with each other, the same good is either the "dirtier", or the "cleaner" good in both countries. Hence, if one country exports the "dirtier" good, the other must export the "cleaner" good. Suppose for concreteness that both countries are of the BT type. Then the country exporting resource goods are exporting the 'dirtier" good. The other country then exports the "cleaner" manufacturing good. Driving the expansion of the exporting sectors, trade harms the environment in the resource-exporting country but enhances that in the other.

By contrast, in a world with two countries of different types, the same good is "dirtier" in one country yet "cleaner" in the other. Therefore, either both countries export their respective "dirtier" goods, or their respective "cleaner" goods to each other. In the former case, trade improves the environment in both countries, and in the latter, as illustrated in Figure 9, trade harms the environment in both countries.

As for welfare effects of trade in the long run, Home's utility level at the trade steady state and that of Foreign can be expressed by

$$V_T = \begin{cases} B + \ln L + \ln A\left(S_\infty(1)\right) - (1-b)\ln P_T & \text{if } \beta_T = 1, \\ B + \ln L + b\ln A\left(S_\infty(\beta_T)\right) + (1-b)\ln a & \text{if } 0 < \beta_T < 1, \\ B + \ln L + \ln a + b\ln P_T & \text{if } \beta_T = 0, \end{cases}$$

$$V_T^* = \begin{cases} B + \ln L^* + \ln A^*\left(S_\infty^*(1)\right) - (1-b)\ln P_T & \text{if } \beta_T^* = 1, \\ B + \ln L^* + b\ln A^*\left(S_\infty^*(\beta_T^*)\right) + (1-b)\ln a^* & \text{if } 0 < \beta_T^* < 1, \\ B + \ln L^* + \ln a^* + b\ln P_T & \text{if } \beta_T^* = 0, \end{cases}$$

where $\beta_T$ and $\beta_T^*$ denote labor allocations and $P_T$ the world relative price at the trade steady state. Note that the world price at the steady state is endogenously determined in this two-country world by general equilibrium and steady state conditions. In essence, if there exists a country remaining diversified at the trade steady state, the world price is determined by technological condition (the MRT) in the diversified country. If both countries specialize, the world price is then determined by technological conditions in both countries as well as the balance of trade.

Based on the observation above, the following proposition summarizes the results regarding welfare gains from trade between two countries of the BT type.

**Proposition 11.** *If both countries are of the BT type, welfare effects of trade in the long run are as follows.*

*(i) The country exporting "cleaner" manufactures gains unambiguously from trade.*

*(ii) The country exporting "dirtier" resource goods may gain or lose. It gains from trade if the world relative price is low enough in the sense that*[20]

$$P_T < P'' \equiv \begin{cases} \frac{A(S_\infty(1))}{a}\left(\frac{A(S_\infty(1))}{A(S_\infty(b))}\right)^{\frac{b}{1-b}} & \text{if Home exports resource goods,} \\ \frac{A^*(S^*_\infty(1))}{a^*}\left(\frac{A^*(S^*_\infty(1))}{A^*(S^*_\infty(b))}\right)^{\frac{b}{1-b}} & \text{if Foreign exports resource goods.} \end{cases} \tag{47}$$

*It loses if $P_T > P''$.*

The following result is concerned with welfare effects of trade if two trading countries are of the CT type.

**Proposition 12.** *If both countries are of the CT type, welfare effects of trade in the long run are as follows.*

*(i) The country exporting "cleaner' resource goods gains unambiguously from trade.*

*(ii) The country exporting "dirtier" manufactures may gain or lose. It gains from trade if it specializes and the other country remains diversified, or if both countries specialize with the world relative price high enough in the sense that*[21]

$$P_T > P''' \equiv \begin{cases} \frac{A^*(S^*_\infty(b))}{a^*} & \text{if Foreign exports manufactures,} \\ \frac{A(S_\infty(b))}{a} & \text{if Home exports manufactures.} \end{cases} \tag{48}$$

*It loses from trade if both countries specialize with $P_T < P'''$, or if it remains diversified.*

Deviating from a small open economy of the CT type (which necessarily specialize at the trade steady state), a country of the CT type in a two-country world may remain diversified at the steady state because of the limited size of the world market. This together with the fact that the world price is endogenously determined produces a possibility of a negative TOT effect in the two-country world.

To see this concretely, suppose Foreign exports manufacturing goods and remains diversified at the steady state. Compared to autarky, environmental stock in Foreign decreases, implying a productivity decline in the resource-good sector and consequently a negative green effect (since Foreign also produces resource goods). This also produces a

[20]With (47) holding, it must specialize in the production of resource goods.

[21]The former includes patterns (m,d) and (d,m), and the latter includes patterns (m,f) and (f,m).

(dynamic) negative TOT effect (since the world price of manufacturing goods, Foreign's exports, falls over time). The total welfare effect is therefore unambiguously negative.

It is worth emphasizing that the country exporting manufacturing goods may loss from trade even if completely specializing. This is because that the CT type country can specialize in manufacturing goods at the trade steady state while facing a lower relative price compared to autarky. If this is the case, there is no green effect but a negative TOT effect, and the resulting budget line lies below that in autarky, implying a welfare loss.

Thus far we consider two trading countries of the same type and show that at least one country (the country exporting the "dirtier" good) gains from trade in the long run. The results mimic those in Brander and Taylor (1998) and Copeland and Taylor (1999).[22] The following proposition suggests that, however, if two countries are of different types, it is possible for both to lose from trade in the long run.

**Proposition 13.** *If two countries are of different types, welfare effects of trade in the long run are as follows.*

*(i) Both countries gain unambiguously from trade if they export their respective "cleaner" goods to one another (namely the BT type country exports manufactures and the CT type country exports resource goods), or if trade pattern is reversed with (47) and (48) holding.*

*(ii) Both countries lose from trade if they export their respective "dirtier" goods to one another with (47) and (48) holding reversely.*

Part (ii) of the proposition highlights the possibility that trade can harm both trading countries. For such a pessimistic scenario to arise it requires that trade liberalization harms the environment in both countries, which can be satisfied only if (i) two countries are of different types, and (ii) the BT type country exports resource goods and the CT type country exports manufacturing goods. If this is the case, the openness of trade shifts more labor into the "dirtier" sector in both countries, resulting in environmental deterioration and a productivity decline of the resource-good sector in both countries.

In such a circumstance, it suffices for the CT type country to lose from trade by remaining diversified at the trade steady state, as illustrated in panel (a) of Figure 10.

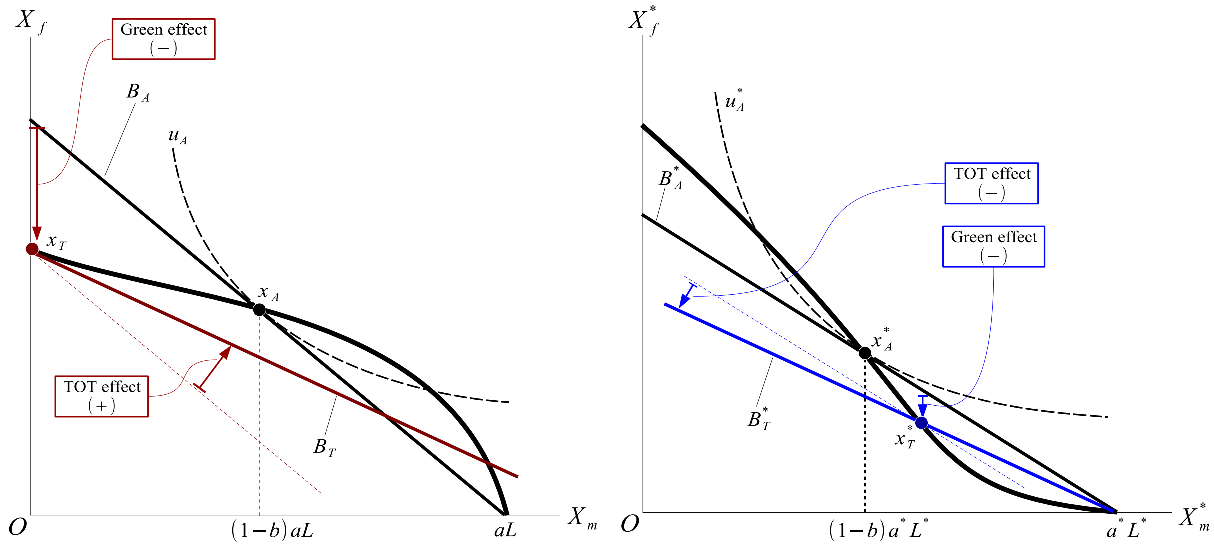
[22] As mentioned in the analysis of small open economy, a BT type country in our model is isomorphic to the economy formulated in Brander and Taylor (1998), whereas a CT type country here is isomorphic to the economy formulated in Copeland and Taylor (1999). Therefore, the two-country trade in Brander and Taylor (1998) is presented in our model by a trade between two countries of the BT type, whereas that in Copeland and Taylor (1999) is presented by a trade between two countries of the CT type.

First, it faces a negative green effect. Staying diversified, the resource-good sector still contributes to the economy and a productivity decline in the sector reduces the total income to the country. Second, it faces a negative TOT effect. Remaining diversified means that the world price is determined by technological condition (the MRT) in the diversified country and thus, a productivity decline in the resource-good sector raises the relative price of imports to exports (recalling that the CT type country are now exporting manufactures). The two negative effects together bring about a long-run welfare loss.

As mentioned above in the discussion of trade between two CT type countries, the CT type country could lose from trade even if specializing in manufacturing goods. Then there is no green effect as it produces no resource goods. The total effect comes only from the TOT effect, which is not necessarily positive since the CT type country can specialize in manufacturing while facing a relative price lower than autarky.

In the BT type country, a productivity decline in the resource-good sector (which is the exporting sector now) also produces a negative green effect. On the other hand, the BT type country enjoys a positive TOT effect by facing a higher relative price of the country's exports to imports compared to autarky (otherwise it will not export resource goods). The BT type country loses from trade in the long run only if the negative green effect dominates the positive TOT effect, which is the case illustrated in panel (b) of Figure 10.[23]

[23]The following concrete example may provide further intuition. Suppose energy is produced with coal in one country and with hydro power in the other, so that energy-intensive manufacturing pollutes in the coal country but not in the hydro-power country. Both countries have renewable resource stocks whereas the hydro-power country is more vulnerable to excessive harvesting than the coal country is. Then, if trade stimulates manufacturing in the coal country, two effects emerge. First, the resource-good sector becomes less productive owing to environmental damage (which we call a negative green effect); second, resource goods become relatively more expensive compared to manufacturing goods, worsening the trade condition (which we call a negative TOT effect). With the two negative effects, trade openness unambiguously harms the coal country. On the other hand, by exporting resource goods, resource stock is depleted as well in the hydro-power country (a negative green effect). If for some reason, say, the hydro-power country is relatively small, the prices are determined according to the coal country's technological conditions, the hydro-power country would benefit from a rise in the relative price of resource goods (a positive TOT effect). Facing these two counter-effects, the hydro-power country may gain or lose from trade, and loses if the negative green effect dominates. We are grateful to an anonymous referee for kindly suggesting this intuitive example.




(a) Home (BT type).

(b) Foreign (CT type).

Figure 10: Both countries lose from trade with pattern (f,d) arising at the steady state. Panel (a) draws the long-run PPF in Home, with $x_A$ and $B_A$ denoting the production (consumption) bundle and budget line at the autarkic steady state, $u_A$ the corresponding indifference curve, $x_T$ and $B_T$ the production bundle and budget line at the trade steady state; the dashed line parallel to $B_A$. Panel (b) draws Foreign counterparts. Note that $B_T$ and $B_T^*$ are identically sloped (the absolute value equal to the world price $P_T$).

# 7 Numerical example

For illustration purpose, specify the general model analyzed in previous sections by letting

$$A(S) = \alpha S, \quad A^*(S^*) = \alpha S^*, \tag{49a}$$

$$a = a^* = 1, \tag{49b}$$

$$l_f = l_f^* = 1, \tag{49c}$$

$$G(S) = \begin{cases} \delta S & \text{if } S \le \frac{g}{\delta+g}K, \\ g(K-S) & \text{if } S > \frac{g}{\delta+g}K, \end{cases} \quad G^*(S^*) = \begin{cases} \delta^* S^* & \text{if } S^* \le \frac{g^*}{\delta^*+g^*}K^*, \\ g^*(K^*-S^*) & \text{if } S^* > \frac{g^*}{\delta^*+g^*}K^*, \end{cases} \tag{49d}$$

where $\alpha$, $\delta$, $g$, $\delta^*$, and $g^*$ are parameters. Note that we assume that the growth functions of the environment are "tent-shaped."[24] Assumption 2 can be replaced by

[24] This tent-shaped environmental growth function approximates the widely used logistic function while remaining highly tractable. See Benchekroun (2008) and Benchekroun and Long (2016) for applications.

Table 2: Numerical specification.

| Specification | $b$ | $\alpha$ | $\delta$ | $g$ | $l_m$ | $K$ | $\delta^*$ | $g^*$ | $l_m^*$ | $K^*$ |
|---|---|---|---|---|---|---|---|---|---|---|
| I | 0.55 | 1 | 9 | 1 | 1 | 2 | 9 | 1 | 1 | 2 |
| II | 0.55 | 1 | 9 | 1 | 1 | 2 | 9 | 1 | 1.5 | 3 |

**Assumption 3.** *Home's labor endowment and parameters satisfy*

$$L \leq \frac{\delta}{\alpha} \text{ and } L \leq \frac{\delta g}{\delta + g} \frac{K}{l_m}.$$

*Foreign's labor endowment and parameters satisfy*

$$L^* \leq \frac{\delta^*}{\alpha} \text{ and } L^* \leq \frac{\delta^* g^*}{\delta^* + g^*} \frac{K^*}{l_m^*}.$$

## 7.1 Country type

Given (49), the steady-state environmental stocks given labor allocations, namely $S_\infty(\beta)$ in Home and $S_\infty^*(\beta^*)$ in Foreign, can be expressed by

$$S_\infty(\beta) = \frac{gK - (1-\beta) l_m L}{g + \alpha\beta L}, \quad S_\infty^*(\beta^*) = \frac{g^* K^* - (1-\beta^*) l_m^* L^*}{g^* + \alpha\beta^* L^*}. \tag{50}$$

It then follows that

$$S'_\infty(\beta) \lesseqgtr 0 \text{ if } \frac{L}{g} \lesseqgtr \frac{K}{l_m} - \frac{1}{\alpha}, \quad S^{*\prime}_\infty(\beta^*) \lesseqgtr 0 \text{ if } \frac{L^*}{g^*} \lesseqgtr \frac{K^*}{l_m^*} - \frac{1}{\alpha}. \tag{51}$$

Recalling that Home is of the BT (CT) type if $S'_\infty < 0$ ($> 0$), and Foreign is of the BT (CT) type if $S^{*\prime}_\infty < 0$ ($> 0$), equation (51) gives the condition for country types.

## 7.2 World specialization pattern at the steady state

Applying Propositions 7 and 8, we can derive the condition for a certain world specialization pattern to arise at the trade steady state, and Proposition 9 can be applied to check the stability of the steady state. To illustrate these results in a more specific manner, in what follows we present two numerical examples to visualize how world specialization patterns, and welfare gains from trade are distributed on the $(L, L^*)$ plane.

Table 2 summarizes the numerical specifications. Specification I focuses on two symmetric countries (except for labor endowments) and specification II allows two countries differing in environmental impacts from the manufacturing sector and in the carrying capacity of the environment. For both specifications, Home is of the BT (CT) type if $L < 1$ ($> 1$) and Foreign is of the BT (CT) type if $L^* < 1$ ($> 1$).

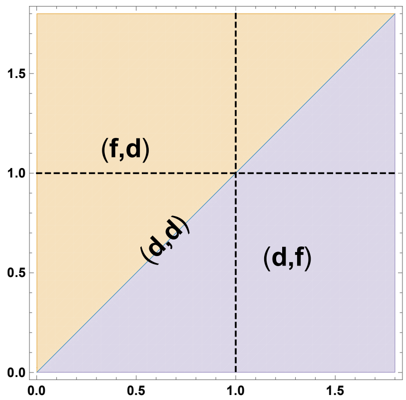
Figure 11 draws the distribution of world specialization patterns for specification I, and Figure 12 for specification II. In both figures, when there exists a country being of the CT type (either $L > 1$ or $L^* > 1$), there are areas corresponding to different world specialization patterns overlapping, which means that, given any $(L, L^*)$ within the area, there exist multiple trade steady states, which correspond to these different patterns. This verifies the existence of multiple trade steady states suggested by Proposition 8.

## 7.3 Welfare effects of trade in the long run

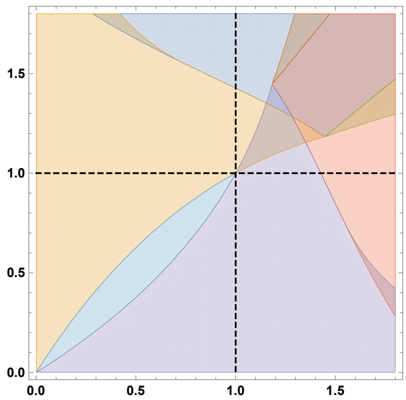
With the same numerical specification, we can assess the long-run welfare effects of trade by showing which area on the $(L, L^*)$ plane corresponds to (i) both countries gain from trade, (ii) Home gains and Foreign loses, (iii) Home loses and Foreign gains, (iv) both lose from trade, and (v) both remain unchanged. Figure 13 illustrates the results for specification I, and Figure 14 for specification II. It only arises in the latter specification that both countries lose from trade, where Foreign's environment has greater carrying capacity and thus, Foreign is more likely to remain diversified under free trade when it is of the CT type. The existence of the area of both losing verifies part (ii) of Proposition 13. Other areas in both figures are also consistent with Propositions 10 to 13.

## 7.4 Regime change

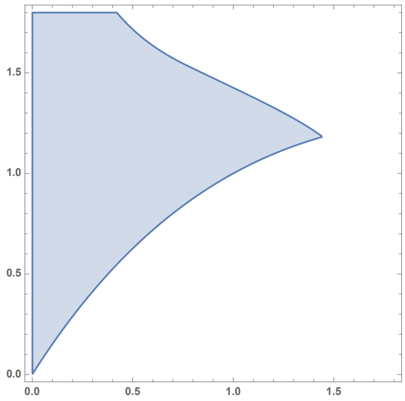
The discussion above also suggests that trade pattern is crucially dependent on the types of countries involved in trade. A change in parameters may alter country types, thus resulting in a dramatic change in trade pattern. Consider two ex-ante identical countries described by (49) with $L = L^*$, $K = K^*$, and $l_m = l_m^*$. Suppose that initially labor endowments are less than $g\left(K/l_m - 1/\alpha\right)$, thereby both countries being of the BT type and there is no trade between the two identical BT type countries even when trade is liberalized between them. Suppose then labor endowments in both countries increase gradually



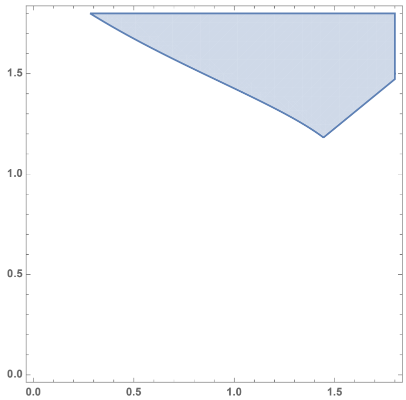

(a) WSP right after trade opened.



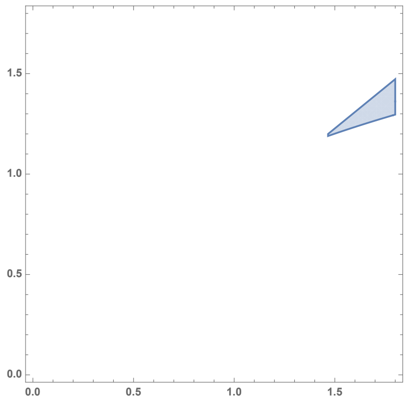
(b) WSP at trade steady state.



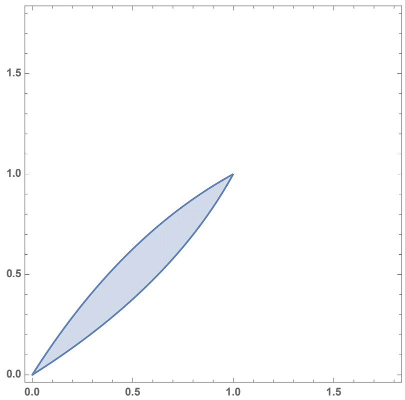
(c) Pattern (f,d).



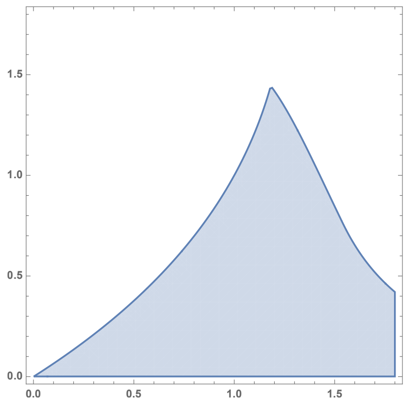
(d) Pattern (f,m).



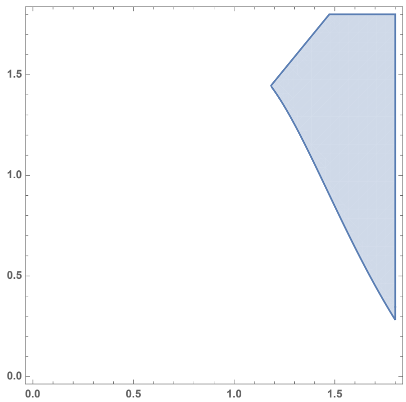
(e) Pattern (d,m).



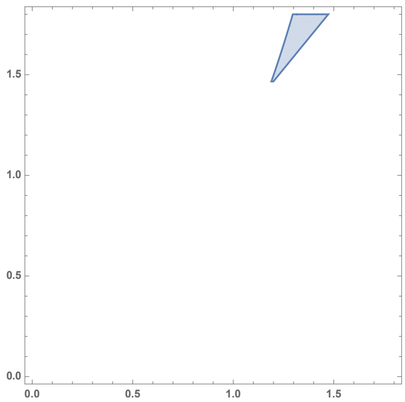
(f) Pattern (d,d).



(g) Pattern (d,f).

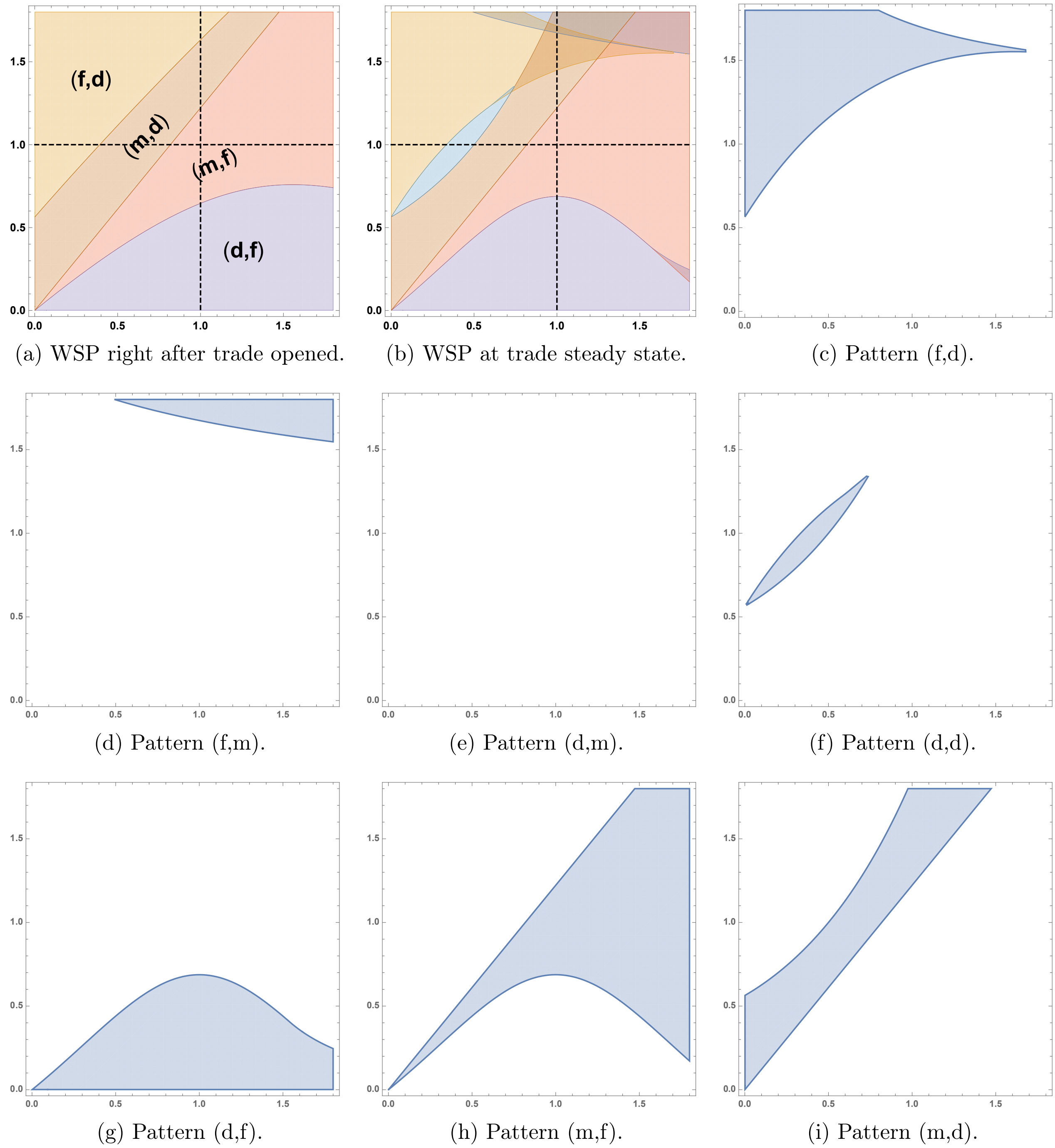


(h) Pattern (m,f).



(i) Pattern (m,d).

Figure 11: Distribution of world specialization pattern (WSP) with specification I in Table 2. The horizontal axis represents Home's labor endowment and the vertical Foreign's. Home is of the BT (CT) type when its labor endowment less (greater) than one. The same holds for Foreign. Panel (a) gives WSP distribution right after the openness of trade, and panel (b) gives that at stable trade steady states. Panels (c) to (k) illustrate respectively the distribution of each WSP at the trade steady state.

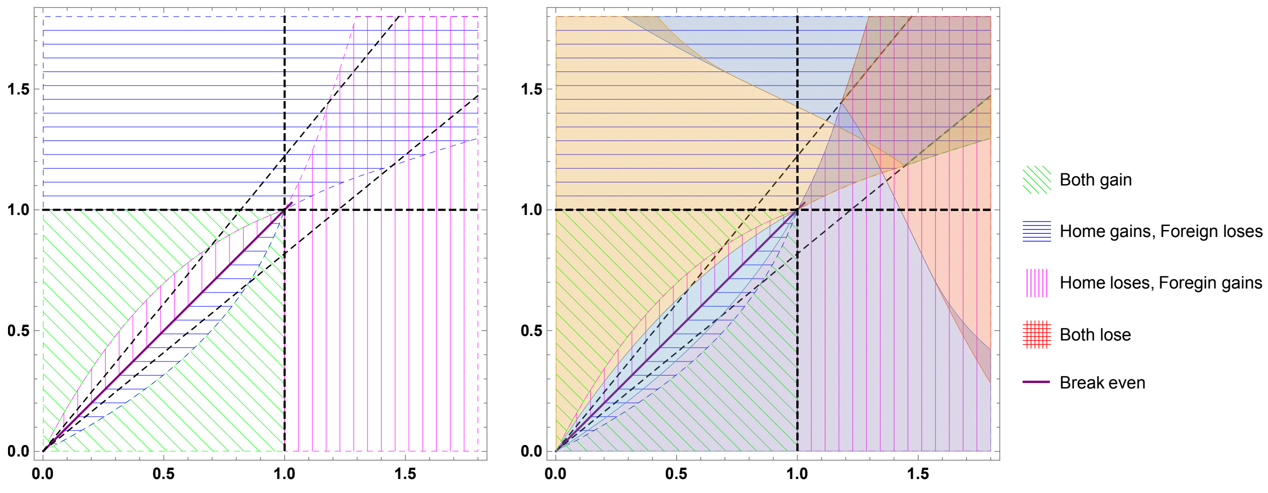



(a) WSP right after trade opened. (b) WSP at trade steady state. (c) Pattern (f,d).

(d) Pattern (f,m). (e) Pattern (d,m). (f) Pattern (d,d).

(g) Pattern (d,f). (h) Pattern (m,f). (i) Pattern (m,d).

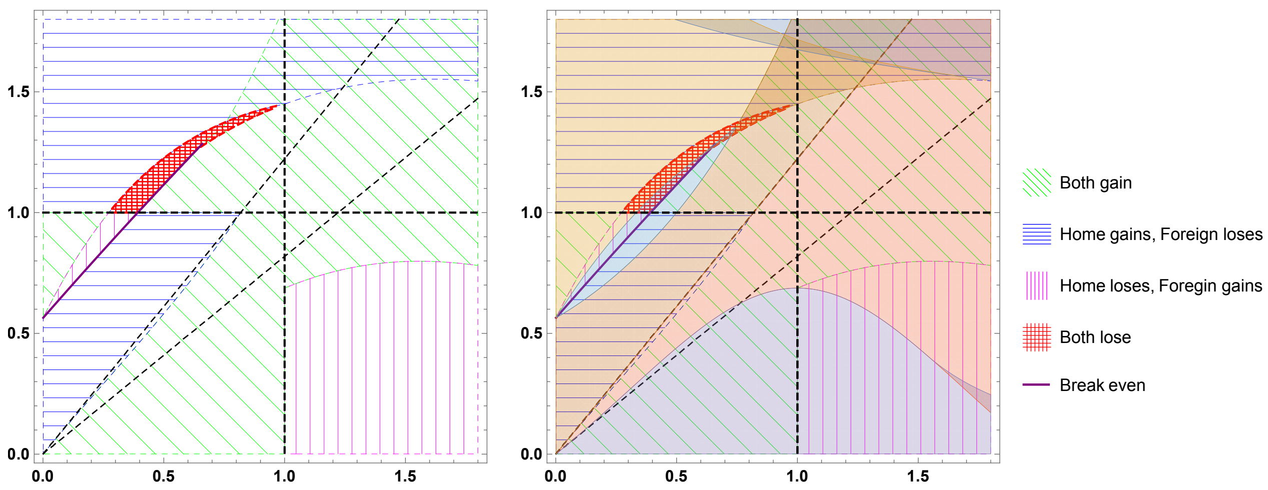
Figure 12: Distribution of world specialization pattern (WSP) with specification II in Table 2. The horizontal axis represents Home's labor endowment and the vertical Foreign's. Home is of the BT (CT) type when its labor endowment less (greater) than one. The same holds for Foreign. Panel (a) gives WSP distribution right after the openness of trade, and panel (b) gives WSP distribution at stable trade steady states. Panels (c) to (k) illustrate respectively the distribution of each WSP at the trade steady state. Note that there exists no steady state with pattern (d,m).




(a) Welfare effects of trade.

(b) Welfare effects and WPS.

Figure 13: The long-run welfare effects of trade with specification I in Table 2. Panel (a) draws only the distribution of welfare effects; panel (b) draws the distribution of welfare effects together with the distribution of world specialization patterns. Given specification I, there is no labor endowments such that both countries lose from trade.




(a) Welfare effects of trade.

(b) Welfare effects and WPS.

Figure 14: The long-run welfare effects of trade with specification II in Table 2. Panel (a) draws only the distribution of welfare effects; panel (b) draws the distribution of welfare effects together with the distribution of world specialization patterns.

by the same amount, which represents an upper-right movement along the ray $L/L^* = 1$ in panel (b) of Figure 11. As long as labor endowment is less than $g\,(K/l_m - 1/\alpha)$ (equal to unity in the numerical examples), there remains no trade between the two countries. When $L$ and $L^*$ exceed that level, however, both countries turn into the CT type and consequently trade volume between them soars. At the trade steady state, world specialization pattern is (f,d) or (d,f) as shown in the figure.

# 8 Conclusion

This study synthesized two kinds of models in the literature on trade and environment: the model regarding renewable natural resources developed by Brander and Taylor (1997a, 1998), and that formulating inter-industrial externalities of industrial pollution developed by Copeland and Taylor (1999). Our hybrid model captured trade between countries with a variety of sources of environmental deterioration. Yet involving regime changes and discontinuity in the dynamic system, a full treatment of the model provides insights into specialization patterns among trading countries, and sheds light on environmental and welfare consequences of trade between emerging economies (where industrial pollution is the main source of environmental deterioration) and less developed economies (where inappropriate use of natural resources is the primary factor of such damage).

A distinct feature of our model is the capability of formulating two types of countries (categorized by whether the resource-good sector, namely the environmental sensitive sector, is more environmentally harmful). This provides a framework to analyze trade between countries of different types, which in our opinion represents typical contemporary trades between emerging economies and less developed (yet resource-rich) economies. We showed that free trade between a country exporting manufacturing goods and a country exporting resource goods may result in both countries losing from trade in the long run. No formal analysis has thus far been undertaken regarding such a possibility in the context of South–South trade without appropriate government policies to control economic activities that harm the environment. This finding highlights the significance of policy interventions for environmental preservation in developing countries, which tend to be less stringent compared to developed countries.

In reality, various policies regulating pollution emissions and/or resource extraction are implemented in developing economies. The analysis of trade and the environment with explicit considerations of policy interventions is of great importance. Moreover, we assumed in the present paper that factor endowment (labor) and manufacturing productivities are exogenous. Endogenising these factors would also have important implications for considering the interaction between economic growth and the environment in open economies. Extensions of our model in these directions are left for future research.

# A Proofs

## A.1 Proof of Lemma 1

Assume to the contrary that the sign of $l_f A(S_\infty(\beta)) - l_m a$ changes as $\beta$ varies. From the continuity of $S_\infty(\beta)$ (Assumption 2), there exists $\beta_0 \in [0,1]$ satisfying $l_f A(S_\infty(\beta_0)) = l_m a$. Two results follow: (i) $S_\infty(\beta_0) = S_c$ since $S_c$ is defined by $l_f A(S_c) = l_m a$, and (ii) $G(S_c) = (l_f A(S_c)\beta_0 + l_m a(1-\beta_0)) L = l_m a L$. The two results together imply that $S_\infty(\beta) = S_c$ for all $\beta \in [0,1]$, and thus $l_f A(S_\infty(\beta)) - l_m a = 0$ holds for all $\beta \in [0,1]$, which leads to a contradiction.

To see why, assume to the contrary that there exists $\beta_1 \neq \beta_0$ such that $S_\infty(\beta_1) \neq S_c$. It follows immediately that $G'(S_c) \geq l_f A'(S_c)\beta_1 L$; otherwise, we would rather have $S_\infty(\beta_1) = S_c$. On the contrary, the stability at $\beta = \beta_0$ requires $G'(S_c) < l_f A'(S_c)\beta_0 L$, which together with $G'(S_c) \geq l_f A'(S_c)\beta_1 L$ gives $\beta_1 < \beta_0$. Clearly, there exists $\beta_2 \in [\beta_1, \beta_0)$ such that $G'(S_c) = l_f A'(S_c)\beta_2 L$. For any $\beta \in (\beta_2, 1]$, the stability condition $G'(S_c) < l_f A'(S_c)\beta L$ holds and we have $S_\infty(\beta) = S_c$. For any $\beta \in [0, \beta_2]$, however, the stability condition fails to hold and we have $S_\infty(\beta) \neq S_c$. This gives $\lim_{\beta\to\beta_2^+} S_\infty(\beta) = S_c \neq S_\infty(\beta_2)$, meaning that $S_\infty(\beta)$ is discontinuous at $\beta_2$. This contradicts the continuity of $S_\infty(\beta)$. Hence, $S_\infty(\beta) = S_c$ for all $\beta \in [0,1]$.

## A.2 Proof of Proposition 1

Since we have ignored the knife-edge case in which $l_f A(S_\infty(\beta)) = l_m a$ for all $\beta \in [0,1]$, according to Lemma 1, either $l_f A(S_\infty(\beta)) > l_m a$ holds for all $\beta \in [0,1]$, or $l_f A(S_\infty(\beta)) < l_m a$ holds for all $\beta \in [0,1]$. By the definition of country type, in the former case the country is of the BT type and in the latter case it is of the CT type. From (12) and (13), it follows that $S'_\infty(\beta) < 0$ ($> 0$) holds for all $\beta \in [0,1]$ in the country of the BT (CT) type.

## A.3 Proof of Proposition 2

Plugging $S_\infty(\beta)$ into (9) gives $A(S_\infty(\beta))/a = P$. Taking the total differential yields, using (8),

$$\frac{dP}{dX_m} = -\frac{A'(S_\infty(\beta))\, S'_\infty(\beta)}{a^2 L}.$$

Recall that $A'(\cdot) > 0$ and, according to Proposition 1, $S'_\infty(\cdot) < 0$ ($> 0$) holds in a BT (CT) type country. It then follows that $dP/dX_m > 0$ ($< 0$) holds in a BT (CT) type country.

## A.4 Proof of Proposition 3

According to Proposition 1, $S'_\infty(\cdot) < 0$ $(> 0)$ in a country of the BT (CT) type, which together with (15) gives MRT $>$ SMRT ($<$ SMRT) for all $\beta \in (0,1]$.

## A.5 Proof of Proposition 4

Consider a certain point $(X_{m1}, X_{f1})$ on the long-run PPF, which corresponds to the steady-state environmental stock $S_\infty(\beta_1)$. From (6), the straight line connecting $(aL, 0)$ with $(X_{m1}, X_{f1})$ is the short-run PPF given $S = S_\infty(\beta_1)$. If the country is of the BT (CT) type, from Proposition 3, the MRT is greater (less) than the SMRT at $(X_{m1}, X_{f1})$ and the short-run PPF intersects the long-run PPF from above (below), which implies that the long-run PPF is necessarily strictly concave (convex) around $(aL, 0)$.

To verify this, use (15) and $\beta = 1 - X_m/aL$ to obtain

$$\begin{aligned} T''(X_m) &= \frac{d}{dX_m}\left(\frac{dT(X_m)}{dX_m}\right) = \frac{d}{d\beta}\left(-A'(S_\infty(\beta))\,S'_\infty(\beta)\frac{\beta}{a} - \frac{A(S_\infty(\beta))}{a}\right) \\ &= \frac{\Delta}{a^2 L}, \end{aligned}$$

where $\Delta \equiv A''(S_\infty(\beta))\left(S'_\infty(\beta)\right)^2\beta + A'(S_\infty(\beta))\,S''_\infty(\beta)\,\beta + 2A'(S_\infty(\beta))\,S'_\infty(\beta)$. For the sake of notation, henceforth use, say, $A'$ instead of $A'(S_\infty(\beta))$. It follows from (13) that

$$\begin{aligned} S''_\infty(\beta) &= \frac{l_f A' S'_\infty L \Phi_2 - \Phi_1 L\left(G'' S'_\infty - l_f A'' S'_\infty \beta L - l_f A' L\right)}{\Phi_2^2} \\ &= \frac{\Phi_1 L^2}{\Phi_2^2}\left(2 l_f A' + \frac{\Phi_1}{\Phi_2}\left(l_f A'' \beta L - G''\right)\right), \end{aligned}$$

where $\Phi_1 \equiv l_f A - l_m a$, $\Phi_2 \equiv G' - l_f A' \beta L$. Thus, we have

$$A' S''_\infty \beta = \frac{\Phi_1 L}{\Phi_2^2} A' \beta L \left(2 l_f A' + \frac{\Phi_1}{\Phi_2}\left(l_f A'' \beta L - G''\right)\right).$$

Similarly, we can obtain $A'' S'^2_\infty \beta = \left(\Phi_1 L/\Phi_2^2\right)\Phi_1 A'' \beta L$ and $2A' S'_\infty = \left(\Phi_1 L/\Phi_2^2\right) 2\Phi_2 A'$. Thus,

$$\begin{aligned} \Delta &= \frac{\Phi_1 L}{\Phi_2^2}\left(A' \beta L\left(2 l_f A' + \frac{\Phi_1}{\Phi_2}\left(l_f A'' \beta L - G''\right)\right) + \Phi_1 A'' \beta L + 2\Phi_2 A'\right) \\ &= \frac{\Phi_1 L}{\Phi_2^2} A'^2 \left(2\frac{G'}{A'} - \frac{\Phi_1}{\Phi_2}\beta L \frac{d}{dS}\left(\frac{G'}{A'}\right)\right). \end{aligned}$$

It then follows that

$$T''(X_m) = \frac{\Delta}{a^2 L} = \frac{\Phi_1 A'^2}{a^2 \Phi_2^2}\left(2\frac{G'}{A'} - \frac{\Phi_1}{\Phi_2}\beta L \frac{d}{dS}\left(\frac{G'}{A'}\right)\right).$$

It then follows that, noting that $\beta \to 0$ as $X_m \to aL$,

$$\lim_{X_m \to aL} T''(X_m) = \frac{2\Phi_1 A' G'}{a^2 \Phi_2^2}.$$

The stability requires $G' - l_f A'\beta L < 0$ holds for all $\beta \in [0,1]$, which implies $G' < 0$ (since $G' - l_f A'\beta L = G'$ as $\beta = 0$). This means that $T''(X_m)$ has the opposite sign to $\Phi_1$ as $X_m \to aL$, which is negative (positive) in a country of the BT (CT) type.

## A.6 Proof of Proposition 5

We consider specialization pattern, then environmental consequences, and finally welfare effects of trade, all focusing on the steady state.

**Specialization pattern.** Given that the world relative price satisfies

$$\frac{A(S_\infty(1))}{a} < P_W < \frac{A(S_\infty(0))}{a},$$

there exists a unique steady state in a small economy of the BT type, as illustrated in panel (a) of Figure 3, at which the economy remain diversified. To see why, suppose that the economy specializes in the resource good. The environmental stock then approaches $S_\infty(1)$ over time. With $P_W > A(S_\infty(1))/a$, $P_W > A(S)/a$ will hold at some point in time, meaning that the economy loses its comparative advantage in the resource good and thus, is unable to specialize in it. A similar argument applies if the economy specialized in the manufacturing good. Clearly, if the world price is sufficiently high (low) such that $P_W \geq A(S_\infty(0))/a$ ($\leq A(S_\infty(1))/a$), the economy necessarily specialize in the manufacturing (resource) good at the steady state.

**Environmental consequences.** Let $\beta_T$ denote labor allocation at the trade steady state. When exporting manufacturing (resource) good, either diversified or specialized, it holds that $\beta_T < b$ ($> b$). Since $S'_\infty(\cdot) < 0$ holds in an economy of the BT type, the trade steady-state level of environmental stock, which can be expressed by $S_\infty(\beta_T)$, satisfies $S_\infty(\beta_T) > S_\infty(b)$ ($< S_\infty(b)$), implying an increase (decline) in environmental stock compared to autarky.

**Welfare effects.** At the autarkic steady state, the utility level can be expressed by, according to (18) and using $P_A = A(S_\infty(b))/a$,

$$V_A = B + \ln L + \ln a + b \ln P_A.$$

At the trade steady state, if the economy remains diversified, the utility level can be expressed by, according to (19),

$$V_T = B + \ln L + b \ln A(S_\infty(\beta_T)) + (1-b)\ln a,$$

which can be rewritten into, noting that $P_W = A(S_\infty(\beta_T))/a$ when remaining diversified,

$$V_T = B + \ln L + b \ln P_W + \ln a.$$

If the economy specializes in the manufacturing good at the trade steady state, we have the same expression as above. Compared with the autarkic state, the utility level changes by

$$V_T - V_A = b \ln \frac{P_W}{P_A}. \tag{52}$$

This has two implications. First, the economy gains from trade ($V_T > V_A$) if it exports manufacturing goods at the trade steady state since exporting manufacturing goods requires that $P_W > P_A$ holds. Second, the economy loses from trade ($V_T < V_A$) if it exports resource goods while remaining diversified, since this requires that $P_W < P_A$ hold.

When the economy specializes in the resource good, from (19), we have

$$V_T = B + \ln L + \ln A\left(S_\infty(1)\right) - (1-b)\ln P_W.$$

On the other hand, $V_A$ can be rewritten into

$$V_A = B + \ln L + \ln A\left(S_\infty(b)\right) - (1-b)\ln P_A.$$

Therefore, when the economy specializes in the resource good at the trade steady state,

$$V_T - V_A = \ln \frac{A\left(S_\infty(1)\right)}{A\left(S_\infty(b)\right)} - (1-b)\ln \frac{P_W}{P_A}. \tag{53}$$

Note that, in an economy of the BT type, $S_\infty(1) < S_\infty(b)$ and therefore, the first term on the RHS of (53) is negative. On the other hand, specializing in the resource good requires $P_W \leq A\left(S_\infty(1)\right)/a$. Recalling that $P_A = A\left(S_\infty(b)\right)/a$, we have $P_W < P_A$. That is, the second term on the RHS of (53) is positive. Clearly, there exists a threshold such that $V_T = V_A$ if the world relative price take the threshold value. Letting $V_T = V_A$ in (53) gives the threshold

$$P' = \frac{A\left(S_\infty(1)\right)}{a}\left(\frac{A\left(S_\infty(1)\right)}{A\left(S_\infty(b)\right)}\right)^{\frac{b}{1-b}}.$$

That is, a small economy of the BT type gains from trade if $P_W < P'$.

## A.7 Proof of Proposition 6

We consider specialization pattern, then environmental consequences, and finally welfare effects of trade, all focusing on the steady state.

**Specialization pattern.** Given that the world relative price satisfies

$$\frac{A\left(S_\infty(1)\right)}{a} < P_W < \frac{A\left(S_\infty(0)\right)}{a},$$

there exist three steady states, as illustrated in panel (b) of Figure 3, among which the two specialized steady states are stable and the diversified one is unstable. However, if

$$P_W \geq \frac{A\left(S_\infty(0)\right)}{a},$$

the economy will stably specialize in manufacturing goods; if

$$P_W \leq \frac{A\left(S_\infty\left(1\right)\right)}{a},$$

it will stably specialize in resource goods. In either case, a small economy of the CT type always specializes at the steady state. Note that, right after trade liberalization, the economy exports and specializes in the good that is relatively cheaper compared to the world. This trade pattern and specialization pattern remain at the trade steady state. To see this, suppose for example that $P_W < P_A$ holds before trade liberalization. When opened to trade, the economy specializes in resource goods. Recalling that the resource good is less environmentally harmful in a CT economy, the environment improves gradually, which reinforces the economy's comparative advantage in the resource good. The self-reinforcing process ensures that, in a small economy of the CT type, trade pattern and specialization pattern right after trade liberalization remain unchanged during the transition.

**Environmental consequences.** Let $\beta_T$ denote labor allocation at the trade steady state. When specializing in the manufacturing (resource) good, $\beta_T = 0 < b$ ($\beta_T = 1 > b$). Since $S'_\infty\left(\cdot\right) > 0$ holds in an economy of the CT type, $S_\infty\left(\beta_T\right) < S_\infty\left(b\right)$ ($> S_\infty\left(b\right)$), implying a decline (increase) in environmental stock compared to autarky.

**Welfare effects.** If the economy specializes in manufacturing goods, we have (52) holds. Since $P_W > P_A$ is required for the economy to export manufacturing right after trade liberalization, it follows immediately that $V_T > V_A$ holds.

If the economy specializes in resource goods, (53) holds. Noting that $S'_\infty\left(\cdot\right) > 0$ in a CT economy, $A\left(S_\infty\left(1\right)\right) > A\left(S_\infty\left(b\right)\right)$ holds. Moreover, $P_W < P_A$ is required for the economy to export resource goods right after trade liberalization. Hence, both terms on the RHS of (53) are positive, implying that $V_T > V_A$ holds. In either case, the small economy enjoys a higher steady-state level of utility under free trade compared to autarky.

## A.8 Proof of Proposition 7

Here, we derive the conditions for respective world specialization patterns to arise in the short run, in which environmental stocks $S$ and $S^*$ are taken as given. As by-products, we also obtain labor allocations $\beta$ and $\beta^*$, and the world relative price $P$.

**Pattern (f,d) in the short run.** In pattern (f,d), Home produces only the resource good, $X_f = A\left(S\right)L$, and Foreign produces both goods, $X_f^* = A^*\left(S^*\right)\beta^* L^*$ and $X_m^* = a^*\left(1-\beta^*\right)L^*$. The world relative price of the manufacturing good with respect to the resource good satisfies

$$P = \frac{A^*\left(S^*\right)}{a^*}, \tag{54}$$

which depends only on Foreign's condition. Recalling that the resource good is the numeraire, $P$ is also the world price of manufacturing good. Since the resource good is produced in both countries in pattern (f,d), the wage in Home is $w = A\left(S\right)$ and that in Foreign is $w^* = A^*\left(S^*\right)$.

Since the manufacturing good is supplied only by Foreign, the market-clearing condition requires, given the same preference in the two countries, $(1-b)(wL + w^*L^*) = PX_m^*$. This gives Foreign's labor allocation:

$$\beta^* = b - (1-b)\frac{z}{v}, \tag{55}$$

where as defined in (23) and (24),

$$v \equiv \frac{A^*(S^*)\,a}{A(S)\,a^*}, \quad z \equiv \frac{aL}{a^*L^*}.$$

Since Foreign produces both goods, we have $\beta^* > 0$, which is equivalent to $v > (1-b)\,z/b$. Recalling that a necessary condition for (f,d), (f,m), and (d,m) is $v \leq 1$, the necessary condition for pattern (f,d) to arise is

$$\frac{1-b}{b}z < v \leq 1. \tag{56}$$

A sufficient condition can be obtained by excluding $v = 1$:

$$\frac{1-b}{b}z < v < 1. \tag{57}$$

**Pattern (f,m) in the short run.** In pattern (f,m), Home produces only the resource good, $X_f = A(S)\,L$, and Foreign produces only the manufacturing good, $X_m^* = a^*L^*$. The wage in Home is $w = A(S)$, whereas that in Foreign is $w^* = a^*P$. Since both countries completely specialize, the world (relative) price of the manufacturing good, $P$, is determined such that world supply equals world demand, $b(wL + w^*L^*) = X_f$, which gives

$$P = \frac{(1-b)\,A(S)\,L}{ba^*L^*}. \tag{58}$$

For the two countries to completely specialize and Home to export the resource good, the necessary condition is $A^*(S^*)/a^* \leq P \leq A(S)/a$, which yields

$$v \leq \frac{1-b}{b}z \leq 1. \tag{59}$$

A sufficient condition can be obtained by excluding $v = 1$:

$$v \leq \frac{1-b}{b}z \leq 1 \text{ and } v \neq 1. \tag{60}$$

**Pattern (d,m) in the short run.** In pattern (d,m), Home produces both goods, $X_f = A(S)\,\beta L$ and $X_m = a(1-\beta)L$, and Foreign produces only the manufacturing good, $X_m^* = a^*L^*$. The world manufacturing price depends only on Home's condition:

$$P = \frac{A(S)}{a}. \tag{61}$$

The wage in Home is $w = A(S)$, whereas that in Foreign is $w^* = a^*P = A(S)\,a^*/a$. Since the resource good is supplied only by Home, the market-clearing condition requires $b(wL + w^*L^*) =$

$X_f$. This gives Home's labor allocation:

$$\beta = b\left(1 + \frac{1}{z}\right). \tag{62}$$

Since Home produces both goods, we have $\beta < 1$ and therefore, using (62), $(1-b)\,z/b > 1$. Recalling that a necessary condition for (f,d), (f,m), and (d,m) is $v \geq 1$, the necessary condition for pattern (d,m) is

$$v \leq 1 < \frac{1-b}{b}z. \tag{63}$$

A sufficient condition follows by excluding $v = 1$:

$$v < 1 < \frac{1-b}{b}z. \tag{64}$$

**Pattern (d,f) in the short run.** In pattern (d,f), we have $X_f = A(S)\,\beta L$, $X_m = a(1-\beta)\,L$, and $X_f^* = A^*(S^*)\,L^*$. The world price of the manufacturing good is given by (61). The wages are $w = A(S)$ and $w^* = A^*(S^*)$. The market-clearing condition requires $(1-b)(wL + w^*L^*) = PX_m$, which gives Home's labor allocation:

$$\beta = b - (1-b)\,\frac{v}{z}. \tag{65}$$

It follows from $\beta > 0$ that $v < bz/(1-b)$, which together with $v \geq 1$ gives the necessary condition for pattern (d,f):

$$1 \leq v < \frac{b}{1-b}z. \tag{66}$$

Again, excluding $v = 1$ yields a sufficient condition:

$$1 < v < \frac{b}{1-b}z. \tag{67}$$

**Pattern (m,f) in the short run.** In pattern (m,f), we have $X_m = aL$ and $X_f^* = A^*(S^*)\,L^*$. The wages are $w = aP$ and $w^* = A^*(S^*)$. The world price of the manufacturing good is determined by the market-clearing condition $b(wL + w^*L^*) = X_f$, which gives

$$P = \frac{(1-b)\,A^*(S^*)\,L^*}{baL}. \tag{68}$$

For the two countries to completely specialize and Foreign to export the resource good, the necessary condition is $A(S)/a \leq P \leq A^*(S^*)/a^*$, which yields

$$1 \leq \frac{b}{1-b}z \leq v. \tag{69}$$

A sufficient condition can be obtained by excluding $v = 1$:

$$1 \leq \frac{b}{1-b}z \leq v \text{ and } v \neq 1. \tag{70}$$

**Pattern (m,d) in the short run.** In pattern (m,d), we have $X_m = aL$, $X_f^* = A^*(S^*)\beta^* L^*$, and $X_m^* = a^*(1-\beta^*)L^*$. The world price of the manufacturing good is given by (54). The wages are $w = aP$ and $w^* = A^*(S^*)$. The market-clearing condition requires $b(wL + w^*L^*) = X_f^*$, which gives Foreign's labor allocation:

$$\beta^* = b(1+z). \tag{71}$$

It follows from $\beta^* < 1$ that $bz/(1-b) < 1$, which together with $v \geq 1$ gives the necessary condition for pattern (m,d):

$$\frac{b}{1-b}z < 1 \leq v. \tag{72}$$

A sufficient condition follows by excluding $v = 1$:

$$\frac{b}{1-b}z < 1 < v. \tag{73}$$

**Pattern (d,d) in the short run.** In pattern (d,d), both countries produce both goods. A necessary condition for pattern (d,d) to arise is $v = 1$. The wages are $w = A(S)$ and $w^* = A^*(S^*)$. The market-clearing condition requires $b(wL + w^*L^*) = A(S)\beta L + A^*(S^*)\beta^* L^*$, which gives (25), namely

$$\beta z + \beta^* = b(z+1).$$

With (25) as the only constraint on the two variables, $\beta$ and $\beta^*$ are indeterminate (and thus other patterns may arise).

## A.9 Proof of Proposition 8

We derive the conditions for the existence and uniqueness of steady state(s) with a certain world specialization pattern arising.

**Pattern (f,d) at the steady state.** Recall that a necessary condition for pattern (f,d) to arise is $(1-b)z/b < 1$. With this necessary condition holing, (i) pattern (f,d) arises at the steady state if $(1-b)z/b < v < 1$ holds at the trade steady state, and (ii) it may also arise even if $v = 1$ at the steady state.

Consider (i) first. In this case, the steady state is determined by $g(v)/v = 1$, where

$$g(v) \equiv \frac{A^*(S_\infty^*(\beta^*(v)))\,a}{A(S_\infty(\beta(v)))\,a^*}.$$

In pattern (f,d), recalling that $\beta^* = b - (1-b)z/v$ (and $\beta = 1$), we have

$$g(v) = \frac{A^*\left(S_\infty^*\left(b-(1-b)\frac{z}{v}\right)\right)a}{A(S_\infty(1))\,a^*}.$$

The existence of steady state(s) with pattern (f,d) requires that there exists solution(s) of $v$

(satisfying $(1-b)\,z/b < v < 1$) to

$$\frac{A^*\left(S^*_\infty\left(b-(1-b)\,\frac{z}{v}\right)\right)a}{A\left(S_\infty(1)\right)a^*}\frac{1}{v} = 1.$$

Consider (ii) then. The existence of steady state(s) with pattern (f,d) requires

$$\frac{A^*\left(S^*_\infty\left(b-(1-b)\,z\right)\right)a}{A\left(S_\infty(1)\right)a^*} = 1.$$

Therefore, if we define

$$\Delta_{fd}(v) \equiv \frac{A^*\left(S^*_\infty\left(b-(1-b)\,\frac{z}{v}\right)\right)a}{A\left(S_\infty(1)\right)a^*}\frac{1}{v},$$

the existence of trade steady state(s) with pattern (f,d) arising for both cases (i) and (ii) requires there exits solution(s) satisfying $(1-b)\,z/b < v \leq 1$ to

$$\Delta_{fd}(v) = 1. \tag{74}$$

Note that $\Delta_{fd}(v)$ is not necessarily monotonic. Specifically, if Foreign is of the BT type, $S^{*\prime}_\infty(\cdot) < 0$ and thus $\Delta_{fd}(v)$ is decreasing with $v$. However, if Foreign is of the CT type, $S^{*\prime}_\infty(\cdot) > 0$ and thus $\Delta_{fd}(v)$ is not necessarily monotonic. Let

$$\Delta^{inf}_{fd} = \inf\left\{\Delta_{fd}(v) : \frac{1-b}{b}z < v \leq 1\right\},$$
$$\Delta^{sup}_{fd} = \sup\left\{\Delta_{fd}(v) : \frac{1-b}{b}z < v \leq 1\right\},$$

then the existence of solution(s) to (74) satisfying $(1-b)\,z/b < v \leq 1$ requires

$$\Delta^{inf}_{fd} \leq 1 \leq \Delta^{sup}_{fd}. \tag{75}$$

Note that (75) is a necessary condition, and the polar case may take place in which $v = (1-b)\,z/b$ is the only solution to (74) (so that pattern (f,m) arises). The following condition helps to exclude this polar case and obtain the sufficient and necessary condition: if $\Delta^{inf}_{fd} = 1$ or $\Delta^{sup}_{fd} = 1$, there exists $v_1 \neq (1-b)\,z/b$ such that $\Delta_{fd}(v_1) = 1$.

Therefore, the sufficient and necessary condition for the existence of trade steady state(s) with pattern (f,d) is the combination of three conditions: $(1-b)\,z/b < 1$, (75), and if $\Delta^{inf}_{fd} = 1$ or $\Delta^{sup}_{fd} = 1$, there exists $v_1 \neq (1-b)\,z/b$ so that $\Delta_{fd}(v_1) = 1$.

Since $\Delta_{fd}(v)$ is not necessarily monotonic, there may exist multiple solutions to (74), meaning that there may exist multiple steady states with pattern (f,d).

**Pattern (f,m) at the steady state.** Recall that a necessary condition for pattern (f,m) to arise is $(1-b)\,z/b \leq 1$. With this necessary condition holing, (i) pattern (f,m) arises if $v \leq (1-b)\,z/b$ and $v \neq 1$ holds at the steady state, and (ii) it may also arise if $v = (1-b)\,z/b = 1$ hold at the steady state.

Consider (i) first. In this case, the steady state can be obtained by solving $g(v)/v = 1$ with

$$g(v) = \frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty (1)\right) a^*}.$$

Thus the existence of steady state(s) with pattern (f,m) arising implies

$$\frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty (1)\right) a^*} \leq \frac{1-b}{b} z \text{ and } \frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty (1)\right) a^*} \neq 1.$$

Consider (ii) then. In this case, the existence of steady state(s) with pattern (f,m) implies

$$\frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty (1)\right) a^*} = 1.$$

Therefore, the sufficient and necessary condition for the existence of trade steady state(s) with pattern (f,m) is

$$\frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty (1)\right) a^*} \leq \frac{1-b}{b} z \leq 1.$$

Note that the steady-state environmental stocks with pattern (f,m) can be expressed by

$$S = S_\infty (1), \quad S^* = S^*_\infty (0),$$

which are uniquely determined. This gives the uniqueness of the steady state.

**Pattern (d,m) at the steady state.** Recall that a necessary condition for pattern (d,m) to arise is $(1-b) z/b > 1$. With this necessary condition holing, (i) pattern (d,m) arises if $v < 1$ holds at the steady state, and (ii) it may also arise even if $v = 1$ at the steady state.

Consider (i) first. In this case, the steady state can be obtained by solving $g(v)/v = 1$ with

$$g(v) = \frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty \left(b \left(1 + \frac{1}{z}\right)\right)\right) a^*}.$$

Thus, the existence of steady state(s) with pattern (d,m) arising implies

$$\frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty \left(b \left(1 + \frac{1}{z}\right)\right)\right) a^*} < 1.$$

Consider (ii) then. The existence of steady state(s) with pattern (d,m) implies

$$\frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty \left(b \left(1 + \frac{1}{z}\right)\right)\right) a^*} = 1.$$

Therefore, the sufficient and necessary condition for the existence of trade steady state(s) with pattern (d,m) is

$$\frac{A^* \left(S^*_\infty (0)\right) a}{A \left(S_\infty \left(b \left(1 + 1/z\right)\right)\right) a^*} \leq 1 < \frac{1-b}{b} z.$$

Note that the steady-state environmental stocks with pattern (d,m) can be expressed by

$$S = S_\infty\left(b\left(1+\frac{1}{z}\right)\right), \quad S^* = S^*_\infty(0),$$

which are uniquely determined. This gives the uniqueness of the steady state.

**Pattern (d,f) at the steady state.** Recall that a necessary condition for pattern (d,f) to arise is $bz/(1-b) > 1$. With this necessary condition holing, (i) pattern (d,f) arises at the steady state if $1 < v < bz/(1-b)$ holds at the trade steady state, and (ii) it may also arise even if $v = 1$ at the steady state.

Consider (i) first. In this case, the steady state is determined by $g(v)/v = 1$, with

$$g(v) = \frac{A^*(S^*_\infty(1))\,a}{A\left(S_\infty\left(b-(1-b)\frac{v}{z}\right)\right)a^*}.$$

The existence of steady state(s) with pattern (d,f) requires that there exists solution(s) of $v$ (satisfying $1 < v < bz/(1-b)$) to

$$\frac{A^*(S^*_\infty(1))\,a}{A\left(S_\infty\left(b-(1-b)\frac{v}{z}\right)\right)a^*}\frac{1}{v} = 1.$$

Consider (ii) then. The existence of steady state(s) with pattern (d,f) requires

$$\frac{A^*(S^*_\infty(1))\,a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*} = 1.$$

Therefore, if we define

$$\Delta_{df}(v) \equiv \frac{A^*(S^*_\infty(1))\,a}{A\left(S_\infty\left(b-(1-b)\frac{v}{z}\right)\right)a^*}\frac{1}{v},$$

the existence of trade steady state(s) with pattern (d,f) arising for both cases (i) and (ii) requires there exits solution(s) satisfying $1 \le v < bz/(1-b)$ to

$$\Delta_{df}(v) = 1. \tag{76}$$

Note that $\Delta_{df}(v)$ is not necessarily monotonic. Specifically, if Home is of the BT type, $S'_\infty < 0$ and thus $\Delta_{df}(v)$ is decreasing with $v$. However, if Home is of the CT type, $S'_\infty > 0$ and thus $\Delta_{df}(v)$ is not necessarily monotonic. Let

$$\Delta^{inf}_{df} = \inf\left\{\Delta_{df}(v) : 1 \le v < \frac{b}{1-b}z\right\},$$
$$\Delta^{sup}_{df} = \sup\left\{\Delta_{df}(v) : 1 \le v < \frac{b}{1-b}z\right\},$$

then the existence of solution(s) to (76) satisfying $1 \le v < bz/(1-b)$ requires

$$\Delta^{inf}_{df} \le 1 \le \Delta^{sup}_{df}. \tag{77}$$

Note that (77) is a necessary condition, and the polar case may take place in which $v = bz/(1-b)$ is the only solution to (76) (so that pattern (m,f) arises). The following condition helps to exclude this polar case and obtain the sufficient and necessary condition: if $\Delta_{df}^{inf} = 1$ or $\Delta_{df}^{sup} = 1$, there exists $v_1 \neq bz/(1-b)$ such that $\Delta_{df}(v_1) = 1$.

Therefore, the sufficient and necessary condition for the existence of trade steady state(s) with pattern (d,f) is the combination of three conditions: $bz/(1-b) > 1$, (77), and if $\Delta_{df}^{inf} = 1$ or $\Delta_{df}^{sup} = 1$, there exists $v_1 \neq bz/(1-b)$ so that $\Delta_{df}(v_1) = 1$.

Since $\Delta_{df}(v)$ is not necessarily monotonic, there may exist multiple solutions to (76), meaning that there may exist multiple steady states with pattern (d,f).

**Pattern (m,f) at the steady state.** Recall that a necessary condition for pattern (m,f) to arise is $bz/(1-b) \geq 1$. With this necessary condition holing, (i) pattern (m,f) arises if $v \geq bz/(1-b)$ and $v \neq 1$ holds at the steady state, and (ii) it may also arise if $v = bz/(1-b) = 1$ hold at the steady state.

Consider (i) first. In this case, the steady state can be obtained by solving $g(v)/v = 1$ with

$$g(v) = \frac{A^*(S_\infty^*(1))\,a}{A(S_\infty(0))\,a^*}.$$

Thus the existence of steady state(s) with pattern (m,f) arising implies

$$\frac{A^*(S_\infty^*(1))\,a}{A(S_\infty(0))\,a^*} \geq \frac{b}{1-b}z \text{ and } \frac{A^*(S_\infty^*(1))\,a}{A(S_\infty(0))\,a^*} \neq 1.$$

Consider (ii) then. In this case, the existence of steady state(s) with pattern (m,f) implies

$$\frac{A^*(S_\infty^*(1))\,a}{A(S_\infty(0))\,a^*} = 1.$$

Therefore, the sufficient and necessary condition for the existence of trade steady state(s) with pattern (m,f) is

$$1 \leq \frac{b}{1-b}z \leq \frac{A^*(S_\infty^*(1))\,a}{A(S_\infty(0))\,a^*}.$$

Note that the steady-state environmental stocks with pattern (m,f) can be expressed by

$$S = S_\infty(0), \quad S^* = S_\infty^*(1),$$

which are uniquely determined. This gives the uniqueness of the steady state.

**Pattern (m,d) at the steady state.** Recall that a necessary condition for pattern (m,d) to arise is $bz/(1-b) < 1$. With this necessary condition holing, (i) pattern (m,d) arises if $v > 1$ holds at the steady state, and (ii) it may also arise even if $v = 1$ at the steady state.

Consider (i) first. In this case, the steady state can be obtained by solving $g(v)/v = 1$ with

$$g(v) = \frac{A^*(S_\infty^*(b(1+z)))\,a}{A(S_\infty(0))\,a^*}.$$

Thus, the existence of steady state(s) with pattern (m,d) arising implies

$$\frac{A^*\left(S_\infty^*\left(b\left(1+z\right)\right)\right)a}{A\left(S_\infty\left(0\right)\right)a^*}>1.$$

Consider (ii) then. The existence of steady state(s) with pattern (m,d) implies

$$\frac{A^*\left(S_\infty^*\left(b\left(1+z\right)\right)\right)a}{A\left(S_\infty\left(0\right)\right)a^*}=1.$$

Therefore, the sufficient and necessary condition for the existence of trade steady state(s) with pattern (d,m) is

$$\frac{b}{1-b}z<1\leq\frac{A^*\left(S_\infty^*\left(b\left(1+z\right)\right)\right)a}{A\left(S_\infty\left(0\right)\right)a^*}.$$

Note that the steady-state environmental stocks with pattern (m,d) can be expressed by

$$S=S_\infty\left(0\right),\quad S^*=S_\infty^*\left(b\left(1+z\right)\right),$$

which are uniquely determined. This gives the uniqueness of the steady state.

**Pattern (d,d) at the steady state.** Recall that a necessary condition for pattern (d,d) to arise is $v=1$. Thus, the existence of trade steady state(s) with pattern (d,d) is equivalent to the existence of $0<\beta<1$ and $0<\beta^*<1$ satisfying

$$\begin{aligned}\frac{A^*\left(S_\infty^*\left(\beta^*\right)\right)a}{A\left(S_\infty\left(\beta\right)\right)a^*}&=1,\\ \beta z+\beta^*&=b\left(z+1\right),\end{aligned}$$

where the second equation comes from the world market-clearing condition (or, equivalently, the balance of trade). Therefore, if we define

$$\Delta_{dd}\left(\beta,\beta^*\right)\equiv\frac{A^*\left(S_\infty^*\left(\beta^*\right)\right)a}{A\left(S_\infty\left(\beta\right)\right)a^*},$$

and let

$$\begin{aligned}\Delta_{dd}^{inf}&=\inf\left\{\Delta_{dd}\left(\beta,\beta^*\right):\beta z+\beta^*=b\left(z+1\right)\right\},\\ \Delta_{dd}^{sup}&=\sup\left\{\Delta_{dd}\left(\beta,\beta^*\right):\beta z+\beta^*=b\left(z+1\right)\right\},\end{aligned}$$

then the existence of trade steady state(s) with pattern (d,d) arising requires

$$\Delta_{dd}^{inf}\leq1\leq\Delta_{dd}^{sup}.\tag{78}$$

Note that (78) is a necessary condition since it cannot exclude the polar case in which either $\beta=0$, $\beta=1$, $\beta^*=0$, or $\beta^*=1$ holds in all solution(s). The sufficient and necessary condition can be obtained by imposing the following constraint: if $\Delta_{dd}^{inf}=1$ or $\Delta_{dd}^{sup}=1$, there exist $\beta_1,\beta_1^*\neq0,1$ so that $\beta_1z+\beta_1^*=b\left(z+1\right)$ and $\Delta_{dd}\left(\beta_1,\beta_1^*\right)=1$.

Note that, given that $\beta$ and $\beta^*$ satisfy $\beta z+\beta^*=b\left(z+1\right)$, $\Delta_{dd}\left(\beta,\beta^*\right)$ is monotonic as $\beta$

(equivalently, $\beta^*$) varies if both countries are of the same type, and therefore, the steady state with pattern (d,d), if existing, is unique; otherwise, there may be multiple ones.

## A.10 Proof of Corollary 1

If Foreign is of the BT type, $\Delta_{fd}(v)$ is decreasing with $v$. It follows that

$$\Delta_{fd}^{inf} = \frac{A^*\left(S_\infty^*\left(b-(1-b)z\right)\right)a}{A\left(S_\infty(1)\right)a^*},$$
$$\Delta_{fd}^{sup} = \frac{A^*\left(S_\infty^*(0)\right)a}{A\left(S_\infty(1)\right)a^*}\left(\frac{1-b}{b}z\right)^{-1},$$

which can be substituted into part (i) of Proposition 8 to obtain (42). Clearly, the solution is unique, and so is the steady state with pattern (f,d).

If Home is of the BT type, $\Delta_{df}(v)$ is decreasing with $v$. It follows that

$$\Delta_{df}^{inf} = \frac{A^*\left(S_\infty^*(1)\right)a}{A\left(S_\infty(0)\right)a^*}\left(\frac{b}{1-b}z\right)^{-1},$$
$$\Delta_{df}^{sup} = \frac{A^*\left(S_\infty^*(1)\right)a}{A\left(S_\infty\left(b-\frac{1-b}{z}\right)\right)a^*},$$

which can be substituted into part (iv) of Proposition 8 to obtain (43).

If both countries are of the same type, $S_\infty'$ and $S_\infty^{*\prime}$ are of the same sign and therefore, $\Delta_{dd}(\beta,\beta^*)$ is monotonic as $\beta$ (or $\beta^*$) varies (as long as $\beta$ and $\beta^*$ satisfy $\beta z+\beta^* = b(z+1)$). We can then obtain $\Delta_{dd}^{inf}$ and $\Delta_{dd}^{sup}$ using the corner values of $\beta$ and $\beta^*$. Specifically, if

$$z < \min\left\{\frac{1-b}{b},\frac{b}{1-b}\right\},$$

the corner values are $(\beta,\beta^*) = (0, b(1+z))$ and $(\beta,\beta^*) = (1, b-(1-b)z)$. If

$$\min\left\{\frac{1-b}{b},\frac{b}{1-b}\right\} < z < \max\left\{\frac{1-b}{b},\frac{b}{1-b}\right\},$$

the corner values are $(\beta,\beta^*) = (b-(1-b)/z, 1)$ and $(\beta,\beta^*) = (1, b-(1-b)z)$. If

$$z > \max\left\{\frac{1-b}{b},\frac{b}{1-b}\right\},$$

the corner values are $(\beta,\beta^*) = (b-(1-b)/z, 1)$ and $(\beta,\beta^*) = (b(1+1/z), 0)$.

Moreover, given that $\beta z+\beta^* = b(z+1)$, $\Delta_{dd}(\beta,\beta^*)$ is monotonic as $\beta$ varies, and thus

$$\Delta_{dd}^{inf} < 1 < \Delta_{dd}^{sup}$$

is the sufficient and necessary condition. Summarizing these results gives Table 1.

As for the uniqueness, note that the world market-clearing condition $\beta z+\beta^* = b(z+1)$

represents a downward-sloping curve on the $(\beta, \beta^*)$ plane. On the other hand,

$$\frac{A^*\left(S^*_\infty\left(\beta^*\right)\right)a}{A\left(S_\infty\left(\beta\right)\right)a^*}=1,$$

gives an upward-sloping curve if both countries are of the same type (since then $S'_\infty$ and $S^{*\prime}_\infty$ are of the same sign). This implies a unique solution to the two equations above, and thus a unique steady state with pattern (d,d). However, if the two countries are of different types, the later equation represents a downward-sloping curve (since then $S'_\infty$ and $S^{*\prime}_\infty$ have the opposite signs). The possibility remains of multiple solutions and thus multiple steady states.

## A.11 Proof of Proposition 9

We first consider the stability of the steady state satisfying $v \neq 1$ and that satisfying $v = 1$.

**The stability of the steady state satisfying $v \neq 1$.** Given $v \neq 1$ at the trade steady state, any pattern except for (d,d) may arise. As long as $v \neq 1$, the dynamic system is governed by (28) and (29), the Jacobian of which can be expressed by

$$\begin{aligned}J&=\begin{bmatrix}\frac{\partial\dot{S}}{\partial S} & \frac{\partial\dot{S}}{\partial S^*}\\ \frac{\partial\dot{S}^*}{\partial S} & \frac{\partial\dot{S}^*}{\partial S^*}\end{bmatrix}\\ &=\begin{bmatrix}G'-l_fA'\beta L-\left(l_fA-l_ma\right)L\beta'\frac{\partial v}{\partial S} & -\left(l_fA-l_ma\right)L\beta'\frac{\partial v}{\partial S^*}\\ -\left(l^*_fA^*-l^*_ma^*\right)L^*\beta^{*\prime}\frac{\partial v}{\partial S} & G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*-\left(l^*_fA^*-l^*_ma^*\right)L^*\beta^{*\prime}\frac{\partial v}{\partial S^*}\end{bmatrix}.\end{aligned}$$

The stability around the steady state holds if tr$J<0$ and det$J>0$, where tr$J$ is the trace and det$J$ is the determinant of $J$ evaluated at the steady state.

As for tr$J$, we have

$$\begin{aligned}\mathrm{tr}J&=G'-l_fA'\beta L-\left(l_fA\left(S\right)-l_ma\right)L\beta'\frac{\partial v}{\partial S}+G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*-\left(l^*_fA^*-l^*_ma^*\right)L^*\beta^{*\prime}\frac{\partial v}{\partial S^*}\\ &=\left(G'-l_fA'\beta L\right)\left(1-S'_\infty\beta'\frac{\partial v}{\partial S}\right)+\left(G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*\right)\left(1-S^{*\prime}_\infty\beta^{*\prime}\frac{\partial v}{\partial S^*}\right),\end{aligned}\tag{79}$$

where $\left(l_fA\left(S\right)-l_ma\right)L=\left(G'-l_fA'\beta L\right)S'_\infty$ and $\left(l^*_fA^*-l^*_ma^*\right)L^*=\left(G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*\right)S^{*\prime}_\infty$ can be obtained from (13) and its Foreign counterpart.

As for det$J$, it is convenient to calculate

$$\begin{aligned}\frac{\det J}{\left(G'-l_fA'\beta L\right)\left(G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*\right)}&=\begin{vmatrix}1-\frac{\left(l_fA-l_ma\right)L}{G'-l_fA'\beta L}\beta'\frac{\partial v}{\partial S} & -\frac{\left(l_fA-l_ma\right)L}{G'-l_fA'\beta L}\beta'\frac{\partial v}{\partial S^*}\\ -\frac{\left(l^*_fA^*-l^*_ma\right)L^*}{G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*}\beta^{*\prime}\frac{\partial v}{\partial S} & 1-\frac{\left(l^*_fA^*-l^*_ma^*\right)L^*}{G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*}\beta^{*\prime}\frac{\partial v}{\partial S^*}\end{vmatrix}\\ &=\begin{vmatrix}1-S'_\infty\beta'\frac{\partial v}{\partial S} & -S'_\infty\beta'\frac{\partial v}{\partial S^*}\\ -S^{*\prime}_\infty\beta^{*\prime}\frac{\partial v}{\partial S} & 1-S^{*\prime}_\infty\beta^{*\prime}\frac{\partial v}{\partial S^*}\end{vmatrix}\\ &=1-S'_\infty\beta'\frac{\partial v}{\partial S}-S^{*\prime}_\infty\beta^{*\prime}\frac{\partial v}{\partial S^*}.\end{aligned}\tag{80}$$

Note that $G'-l_fA'\beta L<0$ and $G^{*\prime}-l^*_fA^{*\prime}\beta^*L^*<0$, which are required for stability when labor allocations are given. Hence, $\det J > 0$ is equivalent to

$$S'_\infty \beta' \frac{\partial v}{\partial S} + S^{*\prime}_\infty \beta^{*\prime} \frac{\partial v}{\partial S^*} < 1. \tag{81}$$

Now, we shall show that $g'(v) < 1$ is equivalent to $\det J > 0$ and $\mathrm{tr} J < 0$. The equivalence between $g'(v) < 1$ and $\det J > 0$ can be obtained as follows. With the definition of $g(v)$:

$$g(v) \equiv \frac{A^*\left(S^*_\infty\left(\beta^*(v)\right)\right) a}{A\left(S_\infty\left(\beta(v)\right)\right) a^*},$$

we have

$$\begin{aligned} g'(v) &= \frac{A^{*\prime} S^{*\prime}_\infty \beta^{*\prime} a}{A a^*} - \frac{A^* a A' S'_\infty \beta'}{A^2 a^*} \\ &= S^{*\prime}_\infty \beta^{*\prime} v \frac{A^{*\prime}}{A^*} - S'_\infty \beta' v \frac{A'}{A} \\ &= S'_\infty \beta' \frac{\partial v}{\partial S} + S^{*\prime}_\infty \beta^{*\prime} \frac{\partial v}{\partial S^*}. \end{aligned}$$

It follows immediately from (81) that $g'(v) < 1$ is equivalent to $\det J > 0$

We then show that $g'(v) < 1$ is equivalent to $\mathrm{tr} J < 0$ given that (81) holds. Noting that for any steady state satisfying that $v \neq 1$, at least one country specializes. If only Home specializes, namely pattern (f,d) or (m,d) arises, we have $\beta' = 0$; if only Foreign specializes, namely pattern (d,f) or (d,m) arises, we have $\beta^{*\prime} = 0$; if both countries specialize, namely pattern (f,m) or (m,f) arises, we have $\beta' = \beta^{*\prime} = 0$. This together with $S'_\infty \beta' \partial v/\partial S + S^{*\prime}_\infty \beta^{*\prime} \partial v/\partial S^* < 1$ implies that $S^{*\prime}_\infty \beta^{*\prime} \partial v/\partial S^* < 1$ (if $\beta' = 0$), $S'_\infty \beta' \partial v/\partial S < 1$ (if $\beta^{*\prime} = 0$), or $S'_\infty \beta' \partial v/\partial S = S^{*\prime}_\infty \beta^{*\prime} \partial v/\partial S^* = 0$ (if $\beta' = \beta^{*\prime} = 0$). In either case, we have $1 - S'_\infty \beta' \partial v/\partial S > 0$ and $1 - S^{*\prime}_\infty \beta^{*\prime} \partial v/\partial S^* > 0$, which together with $G' - l_f A' \beta L < 0$ and $G^{*\prime} - l^*_f A^{*\prime} \beta^* L^* < 0$ yields $\mathrm{tr} J > 0$.

The arguments above suggest that the condition for the local stability of the steady state (satisfying $v \neq 1$), namely $\det J > 0$ and $\mathrm{tr} J < 0$, can be equivalently written as

$$g'(v) < 1.$$

In what follows, we apply the condition and check the stability for each specialization pattern.

(i) In a steady state with pattern (f,m) or (m,f), we have $\beta' = 0$ and $\beta^{*\prime} = 0$, which can be substituted into (79) and (80) to obtain

$$\begin{aligned} \mathrm{tr} J &= G' - l_f A' \beta L + G^{*\prime} - l^*_f A^{*\prime} \beta^* L^* < 0, \\ \det J &= \left(G' - l_f A' \beta L\right)\left(G^{*\prime} - l^*_f A^{*\prime} \beta^* L^*\right) > 0. \end{aligned}$$

This gives the local stability of a steady state with pattern (f,m) or (m,f).

(ii) In a steady state with pattern (f,d), $\beta' = 0$ and $\beta^{*\prime} > 0$. It follows that

$$\begin{aligned} \mathrm{tr} J &= G' - l_f A' \beta L + \left(G^{*\prime} - l^*_f A^{*\prime} \beta^* L^*\right)\left(1 - S^{*\prime}_\infty \beta^{*\prime} \frac{\partial v}{\partial S^*}\right), \\ \det J &= \left(G' - l_f A' \beta L\right)\left(G^{*\prime} - l^*_f A^{*\prime} \beta^* L^*\right)\left(1 - S^{*\prime}_\infty \beta^{*\prime} \frac{\partial v}{\partial S^*}\right). \end{aligned}$$

From the definition of $v$, we have $\partial v/\partial S^* > 0$. If Foreign is of the BT type, $S_\infty^{*\prime} < 0$ and thus, $1 - S_\infty^{*\prime}\beta^{*\prime}\partial v/\partial S^* > 0$, which implies $\mathrm{tr}J < 0$ and $\det J > 0$. Therefore, a steady state with pattern (f,d) is locally stable if Foreign is of the BT type; otherwise, it is locally stable if $g'(v) < 1$. A similar argument applies to pattern (d,f). It follows that a steady state with pattern (d,f) is locally stable if Home is of the BT type; otherwise, it is locally stable if $g'(v) < 1$.

(iii) In a steady state with pattern (d,m) or (m,d), $\beta' = 0$ and $\beta^{*\prime} = 0$. Therefore,

$$\begin{aligned}\mathrm{tr}J &= G' - l_f A'\beta L + G^{*\prime} - l_f^* A^{*\prime}\beta^* L^* < 0,\\ \det J &= \left(G' - l_f A'\beta L\right)\left(G^{*\prime} - l_f^* A^{*\prime}\beta^* L^*\right) > 0.\end{aligned}$$

This gives the local stability of a steady state with pattern (d,m) or (d,m).

**The stability of the steady state satisfying** $v = 1$**.** Given $v = 1$ at the trade steady state, pattern (d,d) may arise and, as knife-edge situations, other patterns may arise as well. For simplicity, here we ignore those knife-edge cases and focus on the stability of pattern (d,d).

Verifying the stability of the steady state with pattern (d,d) faces the difficulty that the dynamic system (28) and (29) is discontinuous at $v = 1$.[25] To get around this difficulty, redefine $\beta(v)$ and $\beta^*(v)$ such that both are continuous around $v = 1$, while letting the redefined $\beta(v)$ and $\beta^*(v)$ have fluctuations sufficiently large around $v = 1$, that is, $\beta' \ll -1$ and $\beta^{*\prime} \gg 1$. This gives a continuous dynamic system that approximates the original discontinuous dynamic system. Such an approximation allows us to use (79) and (80) to evaluate the stability.

Note that $\beta(v)$ and $\beta^*(v)$ are supposed to satisfy the equilibrium condition (25) around $v = 1$, which gives $zd\beta + d\beta^* = 0$ there. This implies, around $v = 1$,

$$\beta^{*\prime} = -z\beta'. \tag{82}$$

By substituting (82) into the expressions of $\mathrm{tr}J$ and $\det J$, we obtain

$$\begin{aligned}\mathrm{tr}J &= \left(G' - l_f A'\beta L\right)\left(1 - S'_\infty\beta'\frac{\partial v}{\partial S}\right) + \left(G^{*\prime} - l_f^* A^{*\prime}\beta^* L^*\right)\left(1 + S_\infty^{*\prime} z\beta'\frac{\partial v}{\partial S^*}\right)\\ &= -S'_\infty\beta'\frac{\partial v}{\partial S}\left(G' - l_f A'\beta L\right) + S_\infty^{*\prime} z\beta'\frac{\partial v}{\partial S^*}\left(G^{*\prime} - l_f^* A^{*\prime}\beta^* L^*\right)\\ &= \beta'\left(S_\infty^{*\prime} z\frac{\partial v}{\partial S^*}\left(G^{*\prime} - l_f^* A^{*\prime}\beta^* L^*\right) - S'_\infty\frac{\partial v}{\partial S}\left(G' - l_f A'\beta L\right)\right),\end{aligned}$$

[25]Note that $\beta$ drops and $\beta^*$ jumps as $v$ crosses one. The magnitude of discontinuity varies with exogenous variables $v$ and $b$, which can be categorized into four cases as in B. Specifically, in Case 1, world specialization pattern changes from (f,d) to (m,d), and according to (101) in Appendix B, $\beta$ drops from 1 to 0 and $\beta^*$ jumps from $b - (1-b)z$ to $b(1+z)$. In Case 2, the pattern changes from (d,m) to (d,f), and from (102), $\beta$ drops from $b\left(1+z^{-1}\right)$ to $b - (1-b)z^{-1}$ and $\beta^*$ jumps from 0 to 1. In Case 3, the pattern changes from (f,d) to (d,f), and from (103), $\beta$ drops from 1 to $b - (1-b)z^{-1}$ and $\beta^*$ jumps from $b - (1-b)z$ to 1. In Case 4, the pattern changes from (d,m) to (m,d), and according to (104), $\beta$ drops from $b\left(1+z^{-1}\right)$ to 0 and $\beta^*$ jumps from 0 to $b(1+z)$. In all cases, the dynamic system is discontinuous at $v = 1$.

and

$$\frac{\det J}{(G' - l_f A'\beta L)\left(G^{*\prime} - l_f^* A^{*\prime}\beta^* L^*\right)} = 1 - S'_\infty \beta' \frac{\partial v}{\partial S} + S^{*\prime}_\infty z \beta' \frac{\partial v}{\partial S^*}$$
$$= \beta' \left( S^{*\prime}_\infty z \frac{\partial v}{\partial S^*} - S'_\infty \frac{\partial v}{\partial S} \right).$$

During the algebra above, we keep the terms relating to $\beta'$ and drop others since $\beta' \ll -1$. The sufficient condition for the stability, tr$J < 0$ and det$J > 0$, can expressed by

$$S^{*\prime}_\infty z \frac{\partial v}{\partial S^*} \left(G^{*\prime} - l_f^* A^{*\prime}\beta^* L^*\right) - S'_\infty \frac{\partial v}{\partial S} \left(G' - l_f A'\beta L\right) > 0, \tag{83}$$

$$S^{*\prime}_\infty z \frac{\partial v}{\partial S^*} - S'_\infty \frac{\partial v}{\partial S} < 0, \tag{84}$$

which are exactly (45) and (46). Note that $\partial v/\partial S < 0$, $\partial v/\partial S^* > 0$, $G' - l_f A'\beta L < 0$, and $G^{*\prime} - l_f^* A^{*\prime}\beta^* L^* < 0$, whether the condition holds depends crucially on the signs of $S'_\infty$ and $S^{*\prime}_\infty$ (which are dependent on the type of Home and Foreign).

Now we shall apply the conditions above to check the stability of steady state with pattern (d,d) for various combinations of country types. (i) If both countries are of the BT type, $S'_\infty < 0$ and $S^{*\prime}_\infty < 0$. This together gives (83) and (84). That is, the steady state with pattern (d,d) is locally stable. (ii) If both countries are of the CT type, we have $S'_\infty > 0$ and $S^{*\prime}_\infty > 0$, which gives the opposite signs to (83) and (84). That is, the steady state with pattern (d,d) is unstable. (iii) If the two countries are of different types, $S'_\infty$ and $S^{*\prime}_\infty$ are differently signed. The stability holds if (83) and (84) hold.

## A.12 Proof of Corollary 2

The idea of the proof is to check whether the intersection of the parameter sets (corresponding to the condition required for each specialization pattern to arise) is empty. If the intersection is empty, then a unique world specialization pattern (among the seven) arises at the steady state; otherwise, there may exist multiple steady states which differ in specialization patterns. Hence, the proof proceeds by considering four cases (see Appendix B for detail), which differ in the set of possible world specialization patterns (obtained by varying environmental stocks $S$ and $S^*$).

**Both countries of the BT type** In Case 1, the set of possible world specialization patterns includes (f,m), (f,d), (d,d), and (m,d). If both countries are of the BT type, according to Proposition 8, the conditions for a certain pattern to arise at the steady state are (36) for pattern (f,m), (42) for pattern (f,d), (40) for pattern (m,d), and Table 1 for pattern (d,d). It is easy to verify that the intersection of the parameter sets corresponding to these conditions is empty, meaning that only one world specialization pattern among the four arises at the trade steady state. One can verify that this also holds for Cases 2 to 4.

Moreover, it follows from Corollary 1 that, if both countries are of the BT type, the steady state with a certain specialization pattern, if existing, is unique and, according to Proposition 9, locally stable. In summary, if both countries are of the BT type the trade steady state is unique and locally stable.

**Both countries of the CT type** If both countries are of the CT type, in Case 1, the corresponding conditions are, according to Proposition 8, (36) for pattern (f,m), (35) for pattern (f,d), (40) for pattern (m,d), and Table 1 for pattern (d,d). The intersection of the parameter sets corresponding to these conditions is, however, not necessarily empty, implying that there may exist multiple steady states corresponding to different specialization patterns. One can verify that this also holds for Cases 2 to 4.

Moreover, according to Corollary 1, if both countries are of the CT type, multiple stable steady states with pattern (f,d) or (d,f) might exist, and according to Proposition 9, the steady state with pattern (d,d), if any, is unstable. It follows from these results that, if both countries are of the CT type, multiple stable steady states may exist, either with the same world specialization pattern or with distinct ones, and at least one country completely specializes at the steady state.

## A.13 Proof of Proposition 10

Let $\beta_T$ and $\beta_T^*$ denote labor allocations in Home and Foreign at the trade steady state. Supposing Home exports resource goods and Foreign exports manufacturing goods at the trade steady state, we have

$$\beta_T > b, \quad \beta_T^* < b.$$

Consider two countries of the same type. If both countries are of the BT type,

$$S_\infty(\beta_T) < S_\infty(b), \quad S_\infty^*(\beta_T^*) > S_\infty^*(b).$$

If both countries are of the CT type, then

$$S_\infty(\beta_T) > S_\infty(b), \quad S_\infty^*(\beta_T^*) < S_\infty^*(b).$$

In either case, environmental stock in one country increases and that in the other decreases at the trade steady state (compared to the autarkic steady state). From similar arguments, the same conclusion holds if trade pattern is reversed.

Consider now two countries of different types. If Home is the BT type and Foreign is of the CT type, then

$$S_\infty(\beta_T) < S_\infty(b), \quad S_\infty^*(\beta_T^*) < S_\infty^*(b).$$

If Home is the CT type and Foreign is of the BT type, the opposite holds:

$$S_\infty(\beta_T) > S_\infty(b), \quad S_\infty^*(\beta_T^*) > S_\infty^*(b).$$

In either case, environmental stocks in two countries change in the same direction at the trade steady state (compared to the autarkic steady state). Similarly, the same conclusion holds if trade pattern is reversed.

## A.14 Proof of Proposition 11

We first consider the situation in which Home exports resource goods at the trade steady state, and then consider what if trade pattern is reversed.

**Home exports resource goods.** Given that Home exports resource goods at the trade steady state, Foreign must export manufacturing goods then, either remaining diversified ($\beta_T^* > 0$) or specializing in manufacturing goods ($\beta_T^* = 0$). If Foreign remains diversified, the world relative price at the trade steady state can be expressed by

$$P_T = \frac{A^*\left(S_\infty^*\left(\beta_T^*\right)\right)}{a^*}.$$

Since Foreign exports manufacturing goods ($\beta_T^* < b$) and is of the BT type, we have

$$P_T > P_A^* = \frac{A^*\left(S_\infty^*\left(b\right)\right)}{a^*}. \tag{85}$$

On the other hand, the argument for deriving (52) in the proof of Proposition 5 applies, yielding

$$V_T^* - V_A^* = b \ln \frac{P_T}{P_A^*}. \tag{86}$$

It follows from (85) and (86) that $V_T^* > V_A^*$ holds.

If Foreign specializes in manufacturing goods at the trade steady state, it holds that

$$P_T \geq \frac{A^*\left(S_\infty^*\left(0\right)\right)}{a^*}, \tag{87}$$

from which we also obtain (85). Noting that (86) still holds, we have, again, $V_T^* > V_A^*$. In either case, Foreign (the country exporting manufacturing goods) gains from trade in the long run.

Consider Home then. Home may remain diversified ($\beta_T < 1$) or specialize in the resource good ($\beta_T = 1$) at the trade steady state. If Home remains diversified, then

$$P_T = \frac{A\left(S_\infty\left(\beta_T\right)\right)}{a}. \tag{88}$$

Since Home exports resource goods ($\beta_T > b$) and is of the BT type,

$$P_T < \frac{A\left(S_\infty\left(b\right)\right)}{a} = P_A. \tag{89}$$

Similar as we obtain (52), it holds that

$$V_T - V_A = b \ln \frac{P_T}{P_A}. \tag{90}$$

It follows from (89) and (90) that Home (the country exporting resource goods) loses from trade in the long run.

If Home specializes in resource goods at the trade steady state, then

$$P_T \leq \frac{A\left(S_\infty\left(1\right)\right)}{a}. \tag{91}$$

Now the argument for deriving (53) applies, which gives

$$V_T - V_A = \ln \frac{A\left(S_\infty\left(1\right)\right)}{A\left(S_\infty\left(b\right)\right)} - (1-b) \ln \frac{P_T}{P_A}, \tag{92}$$

the sign of which is indefinite given (91). Letting $V_T > V_A$ in (92) yields

$$P_T < \frac{A(S_\infty(1))}{a}\left(\frac{A(S_\infty(1))}{A(S_\infty(b))}\right)^{\frac{b}{1-b}}. \tag{93}$$

**Foreign exports resource goods.** Given that Foreign exports resource goods at the trade steady state, Home must export manufacturing goods, and similar arguments as above apply.

Specifically, Home (the country exporting manufacturing goods) necessarily gains from trade in terms of the steady-state level of utility. Foreign (the country exporting resource goods) loses if it remains diversified at the trade steady state. If Foreign specializes, the argument for deriving (53) applies and we can obtain

$$V_T^* - V_A^* = \ln\frac{A^*(S_\infty^*(1))}{A^*(S_\infty^*(b))} - (1-b)\ln\frac{P_T}{P_A^*}. \tag{94}$$

Letting $V_T^* > V_A^*$ gives

$$P_T < \frac{A^*(S_\infty^*(1))}{a^*}\left(\frac{A^*(S_\infty^*(1))}{A^*(S_\infty^*(b))}\right)^{\frac{b}{1-b}}. \tag{95}$$

Combining (93) and (95) gives (47).

## A.15 Proof of Proposition 12

We first consider the situation in which Home exports resource goods at the trade steady state, and then consider what if trade pattern is reversed.

**Home exports resource goods.** Consider Home first. At the trade steady state, Home either remains diversified ($\beta_T < 1$) or specializes in resource goods ($\beta_T = 1$). If Home remains diversified, as shown in the proof of Proposition 11, we have (88) and (90). Since Home exports resource goods ($\beta_T > b$) and is of the CT type, we have $S_\infty(\beta_T) > S_\infty(b)$ and thus, by (88), $P_T > P_A$. This together with (90) yields $V_T > V_A$.

If Home specializes in resource goods at the trade steady state, we have (91) and (92). Note that

$$\ln\frac{A(S_\infty(1))}{A(S_\infty(b))} > (1-b)\ln\frac{A(S_\infty(1))}{A(S_\infty(b))} \geq (1-b)\ln\frac{P_T}{P_A},$$

where the first inequality comes from $S_\infty(1) > S_\infty(b)$ (since Home is of the CT type), and the second inequality from (91). This together with (92) gives $V_T > V_A$. In either case, Home (the country exporting resource goods) gains from trade in the long run.

Consider Foreign then. Since Home exports resource good, Foreign must export manufacturing goods at the trade steady state, either remaining diversified ($\beta_T^* > 0$) or specializing ($\beta_T^* = 0$). If Foreign remains diversified, (86) holds. Since Foreign exports manufacturing goods ($\beta_T^* < b$) and is of the CT type, we have

$$P_T = \frac{A^*(S_\infty^*(\beta_T^*))}{a^*} < \frac{A^*(S_\infty^*(b))}{a^*} = P_A^*, \tag{96}$$

which together with (86) gives $V_T^* < V_A^*$.

If Foreign specializes in manufacturing goods at the trade steady state, (86) holds as well. Moreover, we have

$$P_T \geq \frac{A^*\left(S_\infty^*\left(0\right)\right)}{a^*} < \frac{A^*\left(S_\infty^*\left(b\right)\right)}{a^*} = P_A^*, \tag{97}$$

from which the sign of (86) remains ambiguous. There are two possibilities.

The first possibility is that Home remains diversified at the trade steady state, which gives $P_T > P_A$ as obtained above. Note that trade pattern among two countries of the CT type is self-reinforced in the sense that trade pattern remains unchanged during dynamic transition (as long as there is no exogenous shocks).[26] This self-reinforcing feature ensures that, given that Home exports resource goods at the trade steady state, Home also exports resource goods right after the openness of trade, which requires

$$P_A \geq P_A^*.$$

It then follows that $P_T > P_A^*$ and thus $V_T^* > V_A^*$.

The second possibility is that Home specializes in resource goods at the trade steady state, which requires (91) to hold. However, with the constraints obtained above, the sign of (86) still remains ambiguous. Letting $V_T^* > V_A^*$ in (86) yields

$$P_T > P_A^* = \frac{A^*\left(S_\infty^*\left(b\right)\right)}{a^*}. \tag{98}$$

**Foreign exports resource goods.** Given that Foreign exports resource goods at the trade steady state, Home must export manufacturing goods then. Similar arguments as above apply.

Specifically, Foreign (the country exporting resource goods) necessarily gains from trade in the long run. Home (the country exporting manufacturing goods) loses if it remains diversified. If Home specializes, the argument for deriving (52) applies, which gives (90). Note that

$$P_T \geq \frac{A\left(S_\infty\left(0\right)\right)}{a} < \frac{A\left(S_\infty\left(b\right)\right)}{a} = P_A,$$

the sign of which remains ambiguous. There are also two possibilities. The first is that Foreign remains diversified. Since Foreign exports resource goods ($\beta_T^* > b$) and is of the CT type, we have

$$P_T = \frac{A^*\left(S_\infty^*\left(\beta_T^*\right)\right)}{a^*} > \frac{A^*\left(S_\infty^*\left(b\right)\right)}{a^*} = P_A^*.$$

The self-reinforcing trade pattern (since both countries are of the CT type) ensures that

$$P_A^* \geq P_A,$$

which gives $P_T > P_A$ and thus $V_T > V_A$. The second possibility is that Foreign specializes in manufacturing goods, which requires (87) to hold. Yet with these constraints, the sign of (90)

[26]The environment in the country exporting resource goods will be enhanced, making it better at producing its exports. By contrast, the country exporting manufacturing goods faces environment deterioration and thus gets better at producing the manufacturing good.

remains ambiguous. Letting $V_T > V_A$ in (90) yields

$$P_T > \frac{A(S_\infty(b))}{a}. \tag{99}$$

Combining (98) and (99) gives (48).

## A.16 Proof of Proposition 13

Consider first what happens if Home is of the BT type (thus Foreign is of the CT type since two countries are of different types), and consider then what if Foreign is of the BT type.

**Home is of the BT type.** As for part (i), consider first what if Home (the BT type country) exports manufacturing goods ($\beta_T < b$) at the trade steady state, which means that Foreign (the CT type country) exports resource goods ($\beta_T^* > b$). Home either remains diversified ($\beta_T > 0$) or specializes ($\beta_T = 0$). If Home remains diversified, then

$$P_T = \frac{A(S_\infty(\beta_T))}{a} > P_A,$$

where the inequality comes from $\beta_T < b$ and that Home is of the BT type. If Home specializes, then

$$P_T \geq \frac{A(S_\infty(0))}{a} > P_A.$$

In either case, $P_T > P_A$ holds, which together with the expression of $V_T$ given by (90) (noting that Home exports manufacturing goods) yields $V_T > V_A$. That is, Home (the BT type country that exports manufacturing goods) gains from trade in the long run.

Similarly, Foreign either remains diversified ($\beta_T^* < 1$) or specializes in the production of resource good ($\beta_T^* = 1$). If Foreign remains diversified, then

$$P_T = \frac{A^*(S_\infty^*(\beta_T^*))}{a^*} > P_A^*,$$

where the inequality comes from noting that $\beta_T^* > b$ and that Foreign is of the CT type. Staying diversified, the difference in Foreign's steady-state utility level between trade and autarky can be expressed by (86), which together with $P_T > P_A^*$ yields $V_T^* > V_A^*$. If Foreign specializes, then

$$P_T \leq \frac{A^*(S_\infty^*(1))}{a^*}. \tag{100}$$

The difference between the utility level at the trade steady state and that in autarky can be expressed by (94). Note that

$$\ln\frac{A^*(S_\infty^*(1))}{A^*(S_\infty^*(b))} > (1-b)\ln\frac{A^*(S_\infty^*(1))}{A^*(S_\infty^*(b))} \geq (1-b)\ln\frac{P_T}{P_A^*},$$

where the first inequality comes from noting that Foreign is of the CT type and thus $S_\infty^*(1) > S_\infty^*(b)$, and the second inequality from (100). This together with (94) yields, again, $V_T^* > V_A^*$. In either case, Foreign (the CT type country that exports resource goods) gains from trade.

Consider then what if trade pattern is reversed, with Home (the BT type country) specializing in resource goods ($\beta_T = 1$) and Foreign (the CT type country) specializing in manufacturing goods ($\beta_T^* = 0$) at the trade steady state. Similar arguments apply and we have (91) and (92) in Home, and (86) and (97) in Foreign. It then follows from Proposition 11 that Home gains from trade if (47) holds and from Proposition 12 that Foreign gains from trade if (48) holds.

As for part (ii). If Home (the BT type country) exports resource goods and (47) holds reversely, it may remain diversified or specialize, but necessarily loses from trade in the long run as shown in Proposition 11. Similarly, if Foreign (the CT type country) exports manufacturing goods and (48) holds reversely, it may remain diversified or specialize, but necessarily loses from trade in the long run as shown in Proposition 12.

**Foreign is of the BT type.** The arguments above remain true by switching Home (and Home related variables) for Foreign (and Foreign related variables).

# B Sets of possible world specialization patterns

Note that the sets of possible world specialization patterns, which can be obtained by varying environmental stocks $S$ and $S^*$, may change given different values of effective relative size $z$ and preference parameter $b$. Figure 7 suggests that there are four cases.[27]

**Case 1: Relatively small Home.** Consider the case that Home is relatively small:

$$z < \min\left\{\frac{1-b}{b}, \frac{b}{1-b}\right\}.$$

In this case, world specialization pattern changes from (f,m) to (f,d) and further to (d,d) and (m,d) as $v$ increases. According to (55), (71), (60), (57), and (73), labor allocation in Home, $\beta$, and that in Foreign, $\beta^*$, depend on $v$ and $z$ as follows:

$$\beta(v) = \begin{cases} 1 & \text{if } v \leq \frac{1-b}{b}z, \\ 1 & \text{if } \frac{1-b}{b}z < v < 1, \\ 0 & \text{if } v > 1, \end{cases} \quad \beta^*(v) = \begin{cases} 0 & \text{if } v \leq \frac{1-b}{b}z \\ b-(1-b)\frac{z}{v} & \text{if } \frac{1-b}{b}z < v < 1, \\ b(1+z) & \text{if } v > 1. \end{cases} \tag{101}$$

Note that $\beta(v)$ and $\beta^*(v)$ are continuous except at $v = 1$. When crossing $v = 1$, $\beta(v)$ drops from 1 to 0 and $\beta^*(v)$ jumps from $b-(1-b)z$ to $b(1+z)$. For $v = 1$, it is likely that both countries produce both goods, leaving two variables, $\beta$ and $\beta^*$, to be determined. However, only one constraint, (25), can be obtained from the world market-clearing condition. As a result, $\beta$ and $\beta^*$ are indeterminate at $v = 1$, with one degree of freedom.

[27]We omit two knife-edge cases: $z = (1-b)/b$ and $z = b/(1-b)$. Given that $b \in (1/2, 1)$, if $z = (1-b)/b$, world specialization pattern changes from (f,m) to (f,d) and further to (d,d) and (m,f); if $z = b/(1-b)$, the pattern changes from (f,m) to (d,d) and further to (d,f) and (m,f). Given that $b \in (0, 1/2)$, if $z = (1-b)/b$, world specialization pattern changes from (f,m) to (d,d) and further to (m,d). If $z = b/(1-b)$, the pattern changes from (d,m) to (d,d) and further to (m,f).

**Case 2: Relatively small Foreign.** Now consider the case that Foreign is relatively small:

$$z > \max\left\{\frac{1-b}{b}, \frac{b}{1-b}\right\}.$$

In this case, world specialization pattern changes from (d,m) to (d,d) and further to (d,f) and (m,f). According to (62), (65), (64), (67), and (70), we obtain

$$\beta(v) = \begin{cases} b\left(1+\frac{1}{z}\right) & \text{if } v < 1, \\ b-(1-b)\frac{v}{z} & \text{if } 1 < v < \frac{b}{1-b}z, \\ 0 & \text{if } v \geq \frac{b}{1-b}z, \end{cases} \quad \beta^*(v) = \begin{cases} 0 & \text{if } v < 1, \\ 1 & \text{if } 1 < v < \frac{b}{1-b}z, \\ 1 & \text{if } v \geq \frac{b}{1-b}z. \end{cases} \tag{102}$$

Again, $\beta(v)$ and $\beta^*(v)$ are continuous except at $v = 1$, where labor allocations become indeterminate: $\beta$ can vary from $b(1+1/z)$ to $b-(1-b)/z$ and $\beta^*$ from 0 to 1 as long as the world market-clearing condition (25) holds.

**Case 3: Similar sized with strong demand for resource goods.** Consider the case that the demand for resource goods is relatively strong, namely $b > 1/2$, and two countries are of similar size:

$$\frac{1-b}{b} \leq z \leq \frac{b}{1-b}.$$

In this case, world specialization pattern changes from (f,m) to (f,d) and further to (d,d), (d,f), and (m,f) as $v$ increases. By (55), (65), (60), (57), (67) and (70), we have

$$\beta(v) = \begin{cases} 1 & \text{if } v \leq \frac{1-b}{b}z, \\ 1 & \text{if } \frac{1-b}{b}z < v < 1, \\ b-(1-b)\frac{v}{z} & \text{if } 1 < v < \frac{b}{1-b}z, \\ 0 & \text{if } v \geq \frac{b}{1-b}z, \end{cases} \quad \beta^*(v) = \begin{cases} 0 & \text{if } v \leq \frac{1-b}{b}z, \\ b-(1-b)\frac{z}{v} & \text{if } \frac{1-b}{b}z < v < 1, \\ 1 & \text{if } 1 < v < \frac{b}{1-b}z, \\ 1 & \text{if } v \geq \frac{b}{1-b}z. \end{cases} \tag{103}$$

Both $\beta(v)$ and $\beta^*(v)$ are continuous except at $v = 1$. For $v = 1$, the two countries can be diversified and labor allocations become indeterminate: $\beta$ can be any number from 1 to $b-(1-b)/z$ and $\beta^*$ from $b-(1-b)z$ to 1 as long as they satisfy (25).

**Case 4: Similar size with strong demand for manufacturing goods.** Now consider the case that the demand for manufacturing goods is relatively weak, namely $b < 1/2$, and two countries are of similar size:

$$\frac{b}{1-b} \leq z \leq \frac{1-b}{b}.$$

In this case, world specialization pattern changes from (d,m) to (d,d) and to (m,d). From (62), (71), (73), and (64), labor allocations are

$$\beta(v) = \begin{cases} b\left(1+\frac{1}{z}\right) & \text{if } v < 1, \\ 0 & \text{if } v > 1, \end{cases} \quad \beta^*(v) = \begin{cases} 0 & \text{if } v < 1, \\ b(1+z) & \text{if } v > 1. \end{cases} \tag{104}$$

At $v = 1$, labor allocations become indeterminate: $\beta$ can be any number from $b(1 + 1/z)$ to 0 and $\beta^*$ from 0 to $b(1 + z)$ as long as they satisfy (25).

# C The $\dot{S} = 0$ and $\dot{S}^* = 0$ curves

Plugging the expressions of $\beta(v)$ and $\beta^*(v)$ in the corresponding case given in Appendix B into (28) and (29) gives the complete expression of the dynamic system that governs the two-country world. The phase diagram, with the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves on the $(S, S^*)$ plane, is useful in understanding the dynamics intuitively, which we explain how to draw in what follows.

**Case 1: Relatively small Home.** Consider the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 1, namely $z < \min\{(1-b)/b, b/(1-b)\}$. Plugging (101) into (28) and (29) yields the dynamic system in Case 1. Figures 15a and 15b depict Home's $\dot{S} = 0$ curve, with the former corresponding to the BT type and the latter to the CT type. Figures 15c and 15d show Foreign's $\dot{S}^* = 0$ curve, with the former corresponding to the BT type and the latter to the CT type. The following explains how to draw the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in this case.[28]

First, patterns (f,m), (f,d), (d,d), and (m,d) arise in Case 1, given, respectively, $v \leq (1-b)z/b$, $(1-b)z/b < v < 1$, $v = 1$, and $v > 1$. We can draw two boundary curves, $v = (1-b)z/b =$ constant and $v = 1$, which divide the $(S, S^*)$ plane into three regions, corresponding to patterns (f,m), (f,d), and (m,d), respectively. Pattern (d,d), however, arises along the $v = 1$ curve. Since $v \equiv A^*(S^*)a/A(S)a^*$ and both $A^*(\cdot)$ and $A(\cdot)$ are strictly increasing functions that satisfying $A^*(0) = A(0) = 0$, both the $v = (1-b)z/b$ curve and the $v = 1$ curve start from the origin and have positive slopes on the $(S, S^*)$ plane.[29] Moreover, because $(1-b)z/b < 1$, the $v = (1-b)z/b$ curve lies below the $v = 1$ curve.

Now, move on to the characterization of the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in each region.

In the (f,m) region, that is, the area above the horizontal axis and below the $v = (1-b)z/b$ curve, Home specializes in the resource good, that is, $\beta = 1$. Therefore, as long as $S = S_\infty(1)$, $\dot{S} = 0$ holds regardless of the value of $S^*$. That is, the $\dot{S} = 0$ curve is a vertical line given by $S = S_\infty(1)$ in the (f,m) region, as shown in Figure 15a (for BT Home) and Figure 15b (for CT Home). Similarly, the $\dot{S}^* = 0$ curve of Foreign is a horizontal line given by $S^* = S^*_\infty(0)$, as shown in Figure 15c (for BT Foreign) and Figure 15d (for CT Foreign).

In the (f,d) region, that is, the area above the $v = (1-b)z/b$ curve and below the $v = 1$ curve, Home specializes in the resource good and therefore, the $\dot{S} = 0$ curve remains being the vertical line; yet, Foreign becomes diversified and so the $\dot{S}^* = 0$ curve is no longer horizontal. To see this, assume to the contrary that the $\dot{S}^* = 0$ curve is a horizontal line in the (f,d) region. Moving left along this supposititious horizontal $\dot{S}^* = 0$ curve, $S$ decreases and $S^*$ remains the same, so the comparative advantage index, $v$, increases, leading to an increase in $\beta^*$ according to (101). This implies that the value of $S^*$ changes when moving along the $\dot{S}^* = 0$ curve, leading to a contradiction to the assumption that the $\dot{S}^* = 0$ curve is horizontal. Specifically, note that

[28] We draw the $v =$ constant curves as straight lines in Figures 15 to 18, which in general is not necessarily the case.

[29] For a simple exposition, the $v = (1-b)z/b$, $v = 1$, and $v = bz/(1-b)$ curves appear as straight lines in the figure. In general, this is not the case since $v$ is not necessarily a function of $S^*/S$.

$S^* = S^*_\infty(\beta^*(v))$ holds on the $\dot{S}^* = 0$ curve. Some algebra yields that

$$\left.\frac{dS^*}{dS}\right|_{\dot{S}^*=0} = \frac{S^{*\prime}_\infty \beta^{*\prime} \frac{\partial v}{\partial S}}{1 - S^{*\prime}_\infty \beta^{*\prime} \frac{\partial v}{\partial S^*}},$$

which suggests that, if Foreign is of the BT type, the $\dot{S}^* = 0$ curve is positively sloped and lies below the $S^* = S^*_\infty(0)$ line, as shown in Figure 15c; if Foreign is of the CT type, the $\dot{S}^* = 0$ curve lies above the $S^* = S^*_\infty(0)$ and, however, the sign of the slope is indefinite. The $\dot{S}^* = 0$ curve is drawn as negatively sloped in Figure 15d only for the purpose of simple illustration.

In the (d,d) region, which coincides with the $v = 1$ curve, both Home and Foreign are likely to be diversified. For Home, $\beta$ can vary in $[b - (1-b)/z, 1]$ as long as $\beta z + \beta^* = b(z+1)$ is satisfied, implying that the $\dot{S} = 0$ and $v = 1$ curves overlap between $S = S_\infty(b - (1-b)/z)$ and $S = S_\infty(1)$, as illustrated in Figures 15a and 15b. Similarly, for Foreign, $\beta^*$ can vary in $[b - (1-b)z, 1]$ as long as $\beta z + \beta^* = b(z+1)$ is satisfied. The $\dot{S}^* = 0$ and $v = 1$ curves overlap between $S^* = S^*_\infty(b - (1-b)z)$ and $S^* = S^*_\infty(1)$, as shown in Figures 15c and 15d.

Finally, in the (m,d) region, that is, the area above the $v = 1$ curve, Home specializes in the manufacturing good, implying that the part of the $\dot{S} = 0$ curve in the (m,d) region is the vertical line satisfying $S = S_\infty(0)$, as illustrated in Figures 15a and 15b. By contrast, Foreign remains diversified; yet the part of the $\dot{S}^* = 0$ curve in this region is now the horizontal line satisfying $S^* = S^*_\infty(b(1+z))$ since $\beta^* = b(1+z) =$ constant in pattern (m,d) according to (101).

**Case 2: Relatively small Foreign.** Consider the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 2, namely $z > \max\{(1-b)/b, b/(1-b)\}$. The dynamics in Case 2 can be obtained by substituting (102) into (28) and (29). Figure 16 presents the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 2. The following explains how to draw the figure.

First, patterns (d,m), (d,d), (d,f), and (m,f) arise in Case 2, given, respectively, $v < 1$, $v = 1$, $1 < v < bz/(1-b)$, and $v \geq bz/(1-b)$. We can draw two boundary curves, $v = 1$ and $v = bz/(1-b) =$ constant, which divid the $(S, S^*)$ plane into three regions, corresponding to specialization patterns (d,m), (d,f), and (m,f), whereas pattern (d,d) arises on the $v = 1$ curve.

Now, move on to the characterization of the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in each region. We have obtained the properties of the two curves in (d,d) in Case 1, and we can apply the argument over the (m,d) region in Case 1 to the (d,m) region here. In the following, we focus on the properties of the two curves within (d,f) and (m,f) regions.

In the (d,f) region, that is, the area above the $v = 1$ curve and below the $v = bz/(1-b)$ curve, Foreign specializes in the resource good and so the $\dot{S}^* = 0$ curve is the horizontal line satisfying $S^* = S^*_\infty(1)$. At the same time, Home becomes diversified. Note that $S = S_\infty(\beta(v))$ holds on the $\dot{S} = 0$ curve, from which it follows that

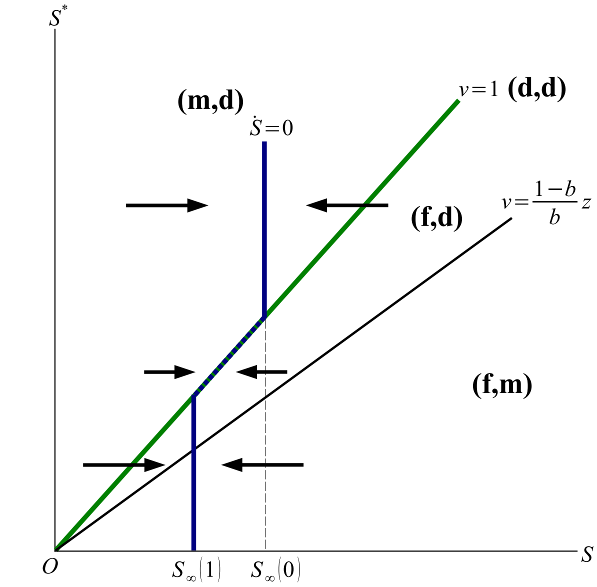
$$\left.\frac{dS^*}{dS}\right|_{\dot{S}=0} = \frac{1 - S'_\infty \beta' \frac{\partial v}{\partial S}}{S'_\infty \beta' \frac{\partial v}{\partial S^*}}.$$

It is then clear that, if Home is of the BT type, the $\dot{S} = 0$ curve is positively sloped and lies left to the $S = S_\infty(0)$ line; if Home is of the BT type, the $\dot{S} = 0$ curve lies right to the $S = S_\infty(0)$ line, and, however, the sign of the slope is indefinite.

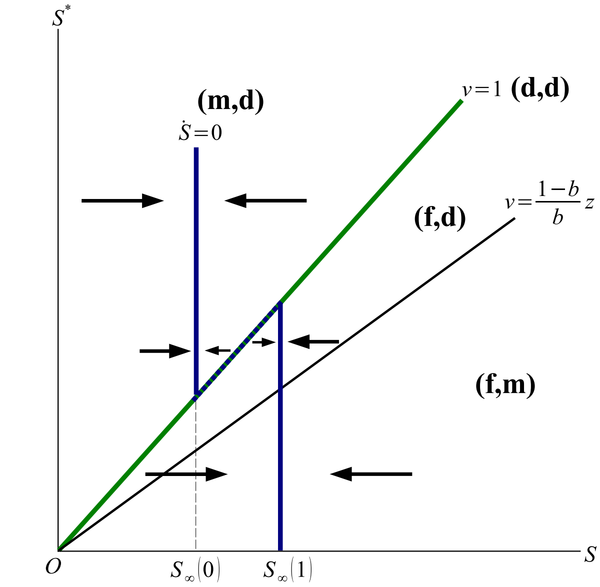
In the (m,f) region, that is, the area above the $v = bz/(1-b)$ curve, Home specializes in the manufacturing good. The $\dot{S} = 0$ curve turns into the vertical line satisfying $S = S_\infty(0)$ in the region, as illustrated in Figure 16a (for BT Home) and Figure 16b (for CT Home). Foreign remains specialized in the resource good and the $\dot{S}^* = 0$ curve remains a horizontal line, as illustrated in Figure 16c (for BT Foreign) and Figure 16d (for CT Foreign).

**Case 3: Similar sized with strong demand for resource goods.** Consider the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 3, namely $b > 1/2$ and $(1-b)/b \leq z \leq b/(1-b)$. The dynamics in Case 3 can be obtained by substituting (103) into (28) and (29). Patterns (f,m), (f,d), (d,d), (d,f), and (m,f) arise in Case 3, given, respectively, $v \leq (1-b)z/b$, $(1-b)z/b < v < 1$, $v = 1$, $1 < v < bz/(1-b)$, and $v \geq bz/(1-b)$. We can draw three boundary curves, which divid the $(S, S^*)$ into four regions, corresponding to specialization patterns (f,m), (f,d), (d,f), and (m,f). Again, pattern (d,d) arises on the $v = 1$ curve. Since we have obtained the properties of the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves within (f,m), (f,d), and (d,d) regions in Case 1, and those within (d,f) and (m,f) regions in Case 2. Applying these results yields Figure 17, which presents Home's $\dot{S} = 0$ curve and Foreign's $\dot{S}^* = 0$ curve in Case 3.

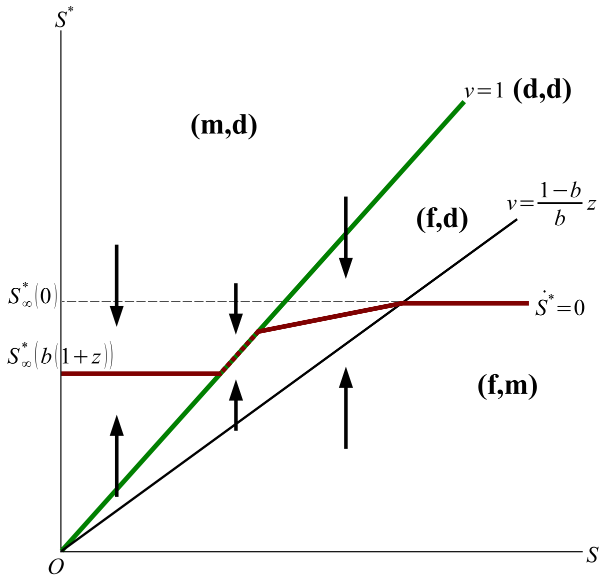
**Case 4: Similar sized with strong demand for manufacturing goods.** Consider the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 4, namely $b < 1/2$ and $b/(1-b) \leq z \leq (1-b)/b$. The dynamics can be obtained by substituting (104) into (28) and (29). Applying the results above, Figure 18 presents the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 4.



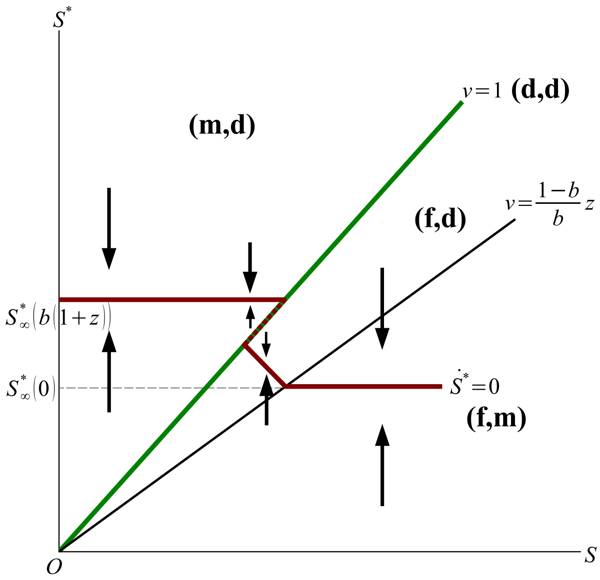

(a) $\dot{S} = 0$ (BT Home).




(b) $\dot{S} = 0$ (CT Home).

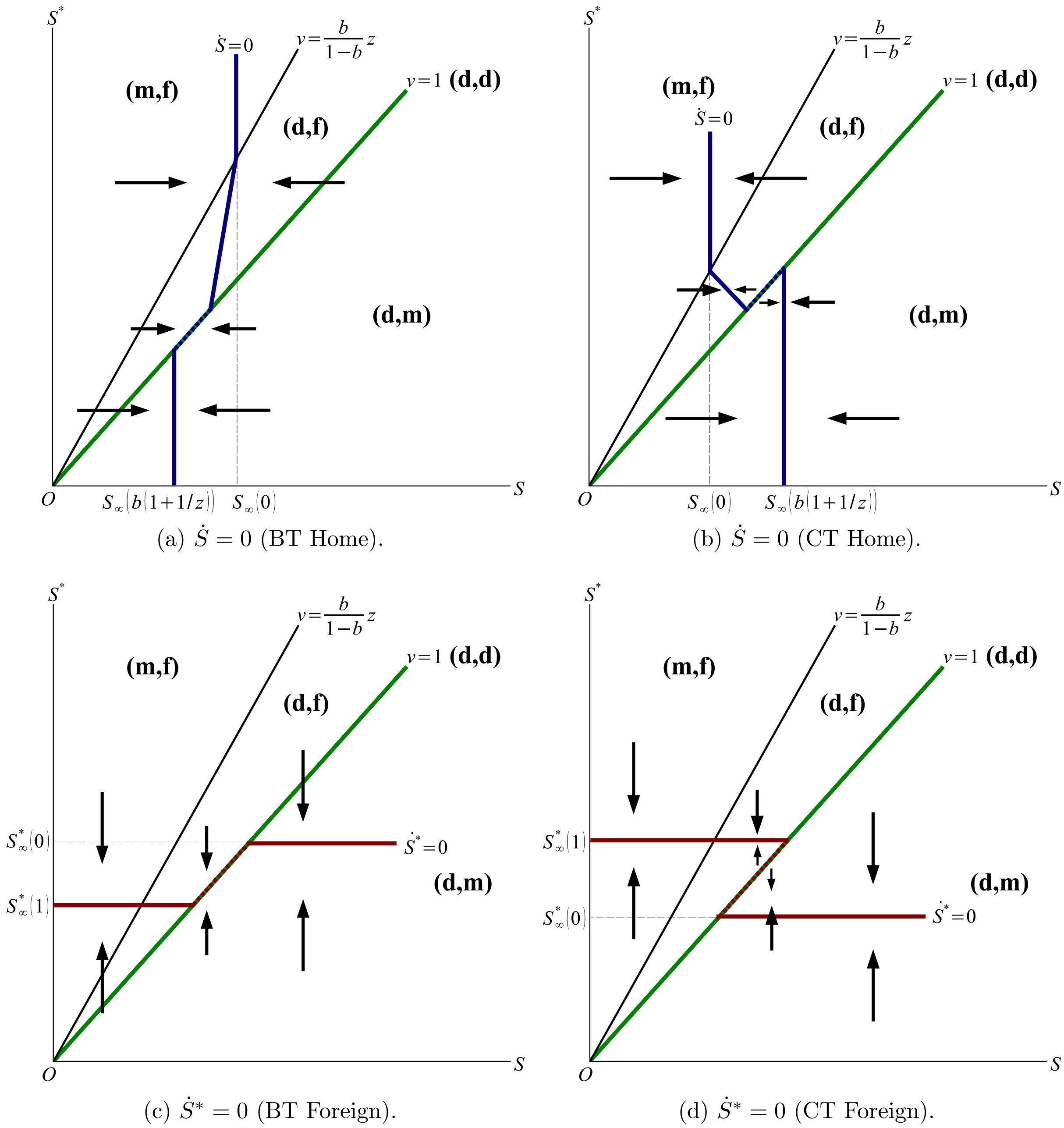



(c) $\dot{S}^* = 0$ (BT Foreign).




(d) $\dot{S}^* = 0$ (CT Foreign).

Figure 15: The $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 1. Panel (d) draws the $\dot{S}^* = 0$ curve in (f,d) region as negatively sloped, which in general is not necessarily the case.



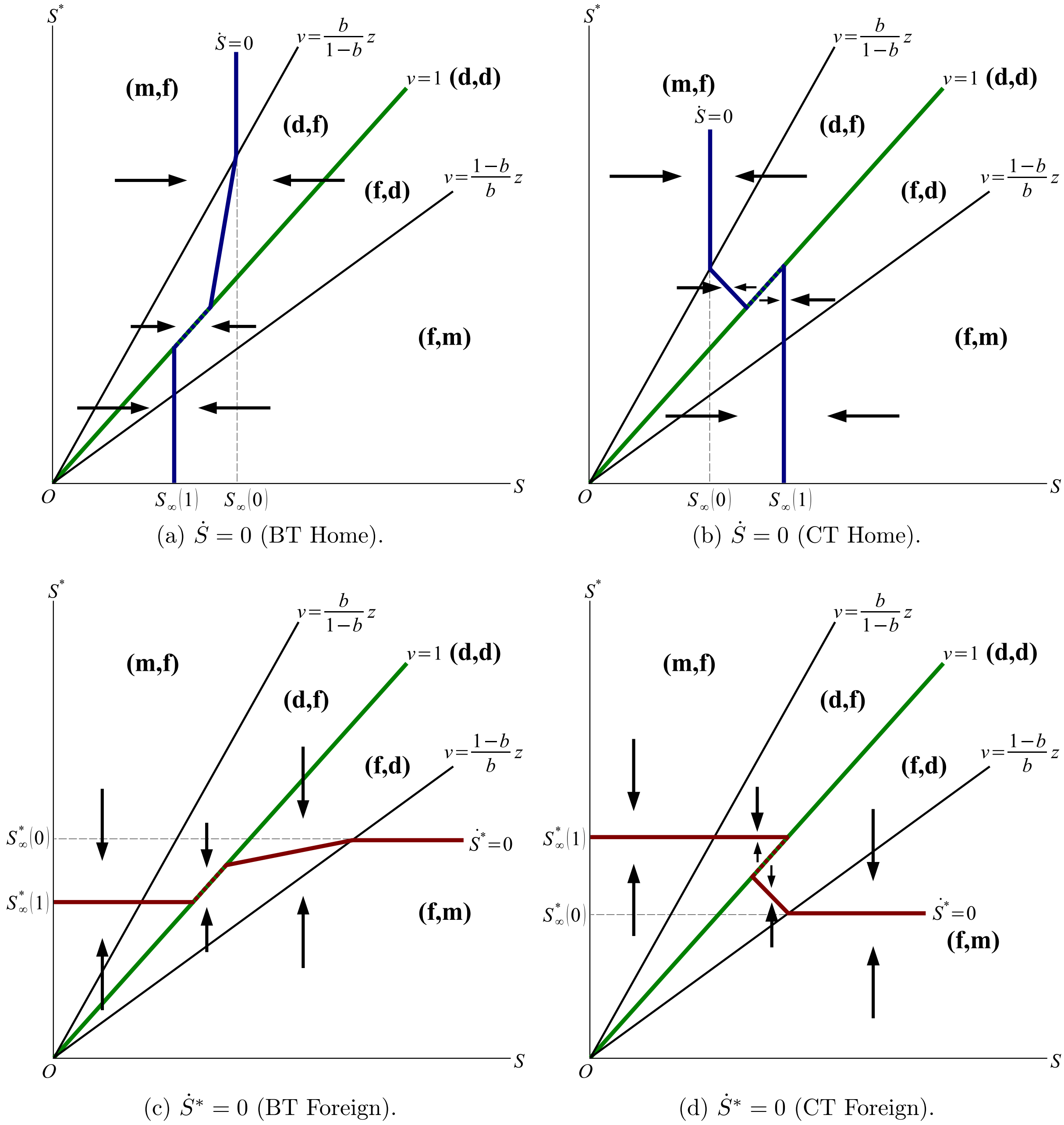

(a) $\dot{S} = 0$ (BT Home).

(b) $\dot{S} = 0$ (CT Home).

(c) $\dot{S}^* = 0$ (BT Foreign).

(d) $\dot{S}^* = 0$ (CT Foreign).

Figure 16: The $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 2. Panel (b) draws the $\dot{S} = 0$ curve in (d,f) region as negatively sloped, which in general is not necessarily the case.



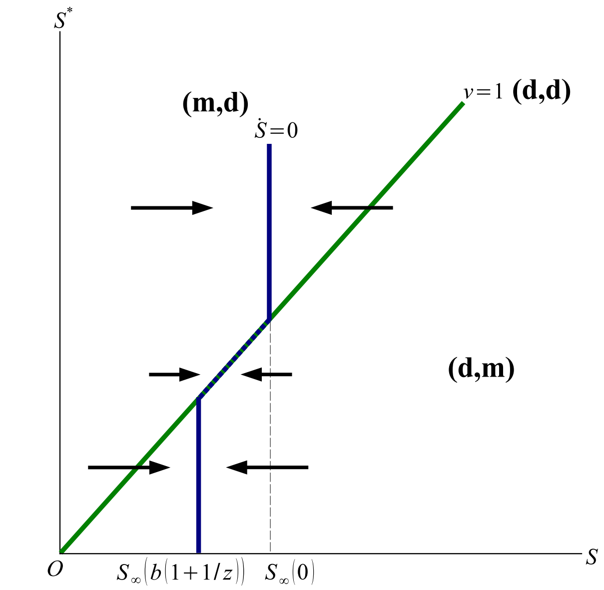

(a) $\dot{S} = 0$ (BT Home).

(b) $\dot{S} = 0$ (CT Home).

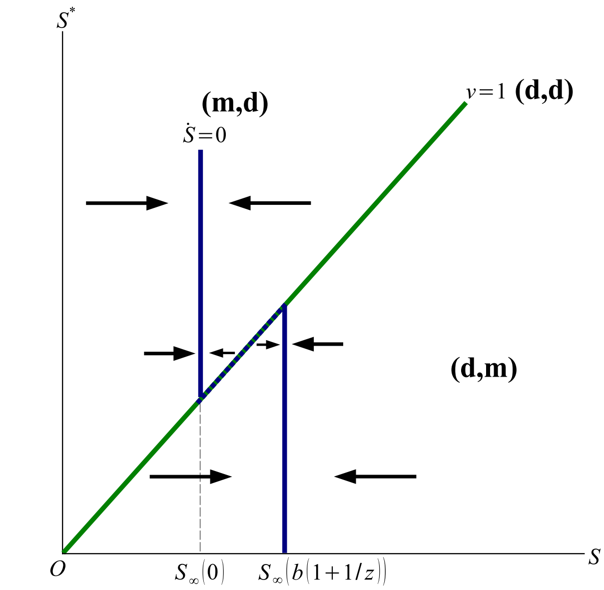
(c) $\dot{S}^{*} = 0$ (BT Foreign).

(d) $\dot{S}^{*} = 0$ (CT Foreign).

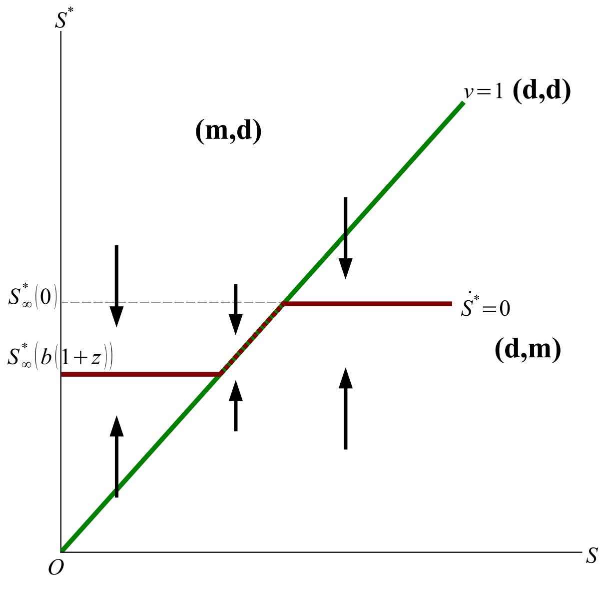
Figure 17: The $\dot{S} = 0$ and $\dot{S}^{*} = 0$ curves in Case 3. Panel (b) draws the $\dot{S} = 0$ curve in (d,f) region and panel (d) draws the $\dot{S}^{*} = 0$ curve in (f,d) region as negatively sloped, which in general are not necessarily the case.



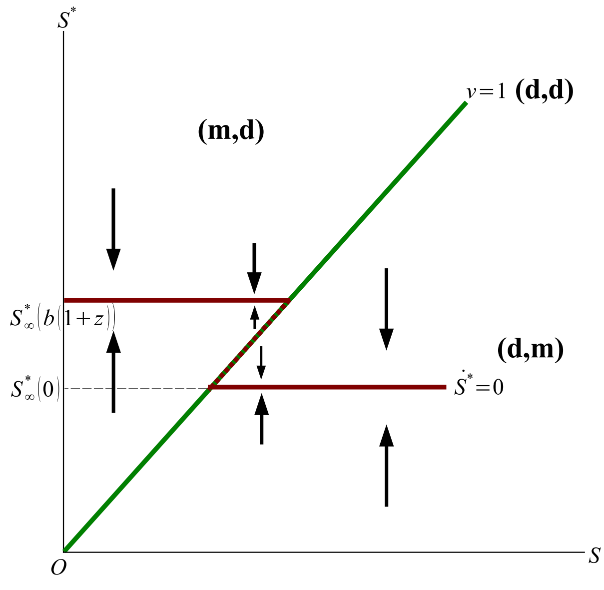

(a) $\dot{S} = 0$ (BT Home).




(b) $\dot{S} = 0$ (CT Home).




(c) $\dot{S}^* = 0$ (BT Foreign).




(d) $\dot{S}^* = 0$ (CT Foreign).

Figure 18: The $\dot{S} = 0$ and $\dot{S}^* = 0$ curves in Case 4.

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